CONNECTING INDUCTION MOTORS

The Practical Application of a Designing Engineer's Experience to the Problems of Operating Engineers, Armature Winders and Repair Men. Also the Presentation to Students of Practical Questions Arising in Winding and Connecting Alternating Current Motors

BY

A. M. DUDLEY, M.S., E.E.

Fellow American Institute Electrical Engineers; Member Society for the Promotion of Engineering Education; Engineer, Marine Electrical Engineering Department, Westinghouse Electric and Manufacturing Company

THIRD EDITION
FIFTEENTH IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK AND LONDON
1936

COPYRIGHT, 1921, 1925, 1936, BY THE
McGRAW-HILL BOOK COMPANY, INC.

PRINTED IN THE UNITED STATES OF AMERICA

All rights reserved. This book, or parts thereof, may not be reproduced in any form without permission of the publishers.

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE TO THE THIRD EDITION

In this prefatory note I wish to express my cordial appreciation of the support this book has received in the previous editions.

This edition is more than a revision. The type has been completely reset and many of the illustrations have been remade. Particular attention is called to the new series of standard diagrams in Chapter XVII which have been completely redrawn and greatly enlarged, making them much easier to read and follow.

In addition, I have added a number of new features which are commended particularly to old readers of the book. In Chapter II further explanation is given of the rotating magnetic field; in Chapter VI I have tried to make clearer the manner of making "stair-step" pictures of the profile of the magnetic field, since this is a useful means of analyzing the effect of winding changes; in Chapter VIII is inserted a method of calculating the effect on a motor's performance of operating it on a circuit which is unbalanced either as to voltage or as to a phase relation; to Chapter IX has been added an interesting scheme for checking the direction of rotation of a motor without actually running it. The most novel addition and the boldest attempt to reduce design to arithmetic has been made in an entirely new and long Chapter XX, which gives in detail, step by step, with specific ratings, a complete method of calculating the performance of a motor from its dimensions and the proposed winding. In connection with this new feature, I wish to disclaim emphatically any attempt to go "highbrow"; I do hope that some old friends will find it interesting to check the results they have gotten on rewinding jobs.

The acknowledgments of the previous edition are made to Messrs. Kincaid, Weber and Roe, and also to *Industrial Engineer*. In this edition thanks for help are extended to Messrs. R. E. Hellmund, M. M. Flanders and J. A. Wasmund; also acknowledgment is made to the National Electrical Manufacturers Association for the use of their Small Motor Classification.

I hope that the book may continue to be useful to practical men and to students.

OAKMONT, PA.,
October, 1936.

A. M. DUDLEY.

PREFACE TO THE FIRST EDITION

The material which later developed into this book appeared first in the "Electric Journal" in February, 1916. It was prepared as a general answer to questions which come to the Question Box Editor, regarding Induction Motor Connections and the possibility of making changes to meet varying conditions of voltage, phase, etc. This article came to the attention of Mr. F. A. Annett, Associate Editor of "Power," and at his request was elaborated into a series of articles appearing at intervals from January, 1917, for about 3 years. From the comments on these articles, there appeared to be a justification for a permanent form which is now presented in this book.

Owing to the fact that the articles appeared in this way and without definite plan at the start, the material lacks unity in some details, and also bears evidence of being viewed from a repair standpoint rather than as a book on winding. The author still cherishes the hope that the future may bring time and opportunity for a revision, which will permit a more orderly arrangement. In its present form it is offered for what it may be worth to practical men engaged in operating and repair work. It was these men who were always in mind and for whose use the material was intended.

The author takes this opportunity of expressing his gratitude to the Westinghouse Electric and Manufacturing Company for permission to present the material, and to the "Electric Journal" and "Power" for the use of cuts and material appearing in their columns. He wishes also to express a personal appreciation of the assistance and inspiration afforded by Mr. F. A. Annett, whose interest in the subject made this book possible.

A. M. DUDLEY.

EAST PITTSBURGH, PA.

CONTENTS

PAGE

PREFACE TO THE THIRD EDITION v

PREFACE TO THE FIRST EDITION vii

INTRODUCTION xv

CHAPTER I

WHAT THE WINDING OF AN INDUCTION MOTOR ACCOMPLISHES . . . 1

Counter E.m.f.—Functions of the Windings of Direct-current Motor—Synchronous Motor—Induction Motor.

CHAPTER II

THE ROTATING MAGNETIC FIELD 5

Why a Motor Drives Its Load—How Torque Is Produced—Setting Up a Rotating Magnetic Field by Alternating Current—Methods for Representing Alternating-current Magnetic Actions—Representing Single-phase and Polyphase Alternating Currents—Why the Middle Leg or *B* Winding Is Reversed—Setting Up a Magnetic Field with Two-phase Currents—Setting Up a Magnetic Field with Three-phase Currents—Drawing a Graphical Picture of the Magnetic Field—Interchanging Two Leads Reverses Direction of Rotation—The Counter E.m.f.—How the Single Phase Is Made to Work.

CHAPTER III

TYPES OF WINDINGS 26

Effect of Form of Slot—Windings Used in Partly Closed Slots—Windings Used in Open Slots—Master Diagrams for Polar Grouped Windings—"Wave" or "Progressive" Diagrams—Standard Direct-current Form of Wave Winding Adapted to Alternating-current—Voltage Relation of Individual Coils in This Winding—Concentric-coil Windings—Rearrangement of Concentric-coil Windings—Wave Windings—Passing to Open-slot Windings—Standard "Lap" Winding—Phase Insulation—Schematic Diagram—Check for Connecting Proper Ends of Phases to Star Point—How to Draw a Diagram to Suit Any Case—Three-phase Star Diagrams—Delta Diagrams.

PAGE

CHAPTER IV

ROTOR WINDINGS 55

Fundamental Conceptions—Squirrel-cage Rotors—Current Paths in Squirrel-cage Winding—Effect of Changing Number of Poles—Calculating Resistance of a Squirrel-cage Winding—Calculating Resistance of Rotor Winding in Terms of Stator Winding—Mechanical Construction of Squirrel-cage Windings—Wound Rotors—Winding Connections—Wave Windings—The True Wave Winding—Quick Method of Figuring Connections for Wave Winding—Modified Wave Winding.

CHAPTER V

SINGLE-PHASE WINDINGS 80

General Classification of Single-phase Motors—Forms of Windings in These Five Types—Schematic Diagrams Showing Outside Terminal Connections—Stator Windings—Method of Winding—Skein Winding—How the Skein Is Wound into the Slots—For Repulsion-type Motors—For Split-phase Motors—Connections of Main and Starting Windings—Mould-winding Method—Starting Windings.

CHAPTER VI

CHORDED WINDINGS OR THE EFFECT OF COIL THROW ON THE MAGNETIC FIELD. 102

Advantages of Chording the Winding—Changing Poles with Constant Throw—Explanation of Term "Chord Factor"—Effect of Chording—Distribution Factor Less Important—Phase Insulation Important—Plotting Pictures of the Magnetic Field—Getting from the Current and the Winding to the Magnetic Field—Effect of Chording Shown Graphically—Table of Chord Factors—How the Chord Factor Is Figured from the Throw of the Coil.

CHAPTER VII

DISTRIBUTION FACTOR AND THE USES THAT ARE MADE OF IT IN WINDING STATORS OF INDUCTION MOTORS 132

Concentrated Windings—Distributed Windings and Distribution Factor—How Current Bands Vary When Winding Is Connected for Different Number of Phases—Geometric Method of Finding Distribution Factor—Chord Factor—Distribution Factor—Graphic Solution—Arithmetic Formula—Two-phase Distribution Factor—Distribution Factor for Consequent Pole Winding on a Two-phase Motor—Three-phase Distribution Factor—Distribution Factor for Three-phase Consequent-Pole Winding—Distribution Factor for Fully Distributed Single-phase Winding—Why Single-phase Windings Are Often Concentric-coil Wound—Distinction between Chord Factor and Distribution Factor on Single-phase Windings—Practical Application of Distribution Factor.

PAGE

CHAPTER VIII

EFFECT OF VOLTAGE ON WINDINGS AND POSSIBILITY OF CONNECTING A WINDING FOR MORE THAN ONE VOLTAGE 148

Checking Insulation for New Voltage—Insulation Tests—Volts per Turn—Comparison of Motor Voltages with Various Three-phase and Two-phase Connections—General Tables Covering All Voltage Connections—Effect on Motor Performance of Unbalanced Voltages—How to Resolve Three Unbalanced Three-phase Voltages into Two Balanced Systems.

CHAPTER IX

HOW THE NUMBER OF PHASES AFFECTS THE WINDINGS AND THE RESULT OF CHANGING VOLTAGE AND PHASE AT THE SAME TIME 165

Two-phase to Three-phase—Scott Connection or "Tee"— Phase Changes and Voltage Changes Combined—Effect on Voltage between Collector Rings and Control—Matching Rotation of Induction Motors.

CHAPTER X

HOW THE FREQUENCY AFFECTS THE WINDINGS 183

Checking the Speed When Operating at Higher Frequency—Relation between Voltage and Frequency—Relation between Torque, R.p.m. and Horsepower—Starting a Squirrel-cage Motor by Bringing Up the Generator from Rest.

CHAPTER XI

THE NUMBER OF POLES AND THE R.P.M. AND THE POSSIBILITY OF VARYING THEM WITH THE SAME WINDING 191

Check Points in Changing Number of Poles—Slip—Chord Factor —Counter E.m.f.

CHAPTER XII

LESS COMMON CONNECTIONS USED FOR UNSYMMETRICAL CONDITIONS OR IN AN EMERGENCY 201

Number of Slots Not a Multiple of Phases Times Poles—Consequent Pole Windings for Two Speeds—"Split-group" Diagrams—"Tee" Connection.

CHAPTER XIII

RECONNECTING AN OLD WINDING FOR NEW CONDITIONS 213

General Fundamental Considerations—Cross Section of Copper and Iron—Generator Action of the Winding—Changing the Throw —All Changes Can be Handled as Voltage Changes—1. Change in Voltage—2. Change in Phase—3. Change in Frequency—4. Change in Number of Poles or Speed—5. Change in Horsepower—Example of Each.

PAGE

CHAPTER XIV

LOCATING FAULTS IN INDUCTION MOTOR WINDINGS 233

Noise and Vibration—Separating Air Noise from Magnetic Noise—Mechanical Vibration—Grounds—Short Circuits—Reversed Coil—Reversed Group—Wrong Grouping—Reversed Phase—Connected for Wrong Voltage—Wrong Number of Poles—Open Circuits—First Fault—Second and Third Faults—Fourth and Fifth Faults—Sixth Fault—Seventh Fault—Eighth Fault—Ninth Fault—Tenth Fault—Compass Test—Balance Test—Usual Order of Locating Defects.

CHAPTER XV

HOW TO FIGURE A NEW WINDING FOR AN OLD CORE. 262

Effect of the Winding on the Performance—Nineteen Points Considered in a Design—Output Coefficient and Table—Iron Below Slots—Flux per Pole—Conductors per Phase—Full-load Current per Lead—Insulation Space in Slot—Formula for Figuring Volts between Collector Rings.

CHAPTER XVI

CONNECTING TO CORRECT MAGNETIC SIDE PULL 282

Magnetic Conditions in Motor or Alternator—Stable Magnetic Equilibrium—Unstable Magnetic Equilibrium—Series Connection Does Not Correct Unbalanced Pull—Usual Parallel Connection Helps if Top to Top—Top-to-bottom Parallel Connection Has No Corrective Effects—Best Parallel Arrangement for Corrective Effect—How It Corrects—Equalizer Connections—Equalizer Connections on Direct-current Armature—Factors That Are Normally Corrective.

CHAPTER XVII

STANDARD GROUP DIAGRAMS FROM 2 TO 14 POLES 301

How Standard Diagrams Are Drawn—Changing from Star to Delta.

CHAPTER XVIII

LAP WINDINGS WITH UNEQUAL COIL GROUPINGS. 347

Possibilities of Windings with Unequal Coil Grouping—"Least Common Multiple" Scheme—How to Lay Out a Winding by the Least Common Multiple Scheme—With Such a Winding Parallel—Number of Coils in Each Path—Two-phase Condition to Look Out For—Three-phase Condition to Look Out For—Explanation of Coil Grouping Tables.

CHAPTER XIX

WAVE DIAGRAMS. 374

Why Rotor Winding Is Always Three-phase—How Wave Diagram Is Drawn.

PAGE

CHAPTER XX

PERFORMANCE CALCULATION 399

Hellmund's Method—Circle Diagram—Data to Be Calculated—No-load Current—Relation of No-load to Locked Current——Stator and Rotor Resistances—Iron Loss, Bearing Friction and Windage—Magnetic Field and Magnetizing Current—Leakage Coefficient—Iron Losses—Friction and Windage—Ohmic Resistances—Construction of the Circle Diagram—Use of the Circle Diagram—Practical Examples of Calculation—350-horsepower Wound-rotor Motor, Calculation and Diagram—200-horsepower Squirrel-cage Motor, Calculation and Diagram—Comparison of Precalculated Figures with Actual Tested Results.

INDEX 451

INDEX OF TABLES 460

INDEX OF DIAGRAMS 461

INTRODUCTION

The best text books for students usually are written by those most familiar with the art of teaching; so should the best technical books, for the active workers, be written by those who are in the midst of such work. Otherwise the text is liable to lag behind the actual practice. In the electrical art the growth has been so rapid and the changes in practice so numerous, that only those directly in touch with the many developments are able to tell the up-to-date story. Unfortunately, it is only in rare cases the *doer* is the *teller*, that is, too often he delegates the telling of his work to others, while he continues to *do*. Lack of practice in writing is often back of this. In Mr. Dudley's book we have a very positive exception to the usual practice, for here we have the case of a writer with fourteen years of active practical experience upon which to build his treatment of the subject. Consequently there is a sincerity in the facts presented and a logic in their treatment which appeal strongly to the practical man. The method given for checking phase rotation on a three-phase winding, is an example, as is also the table of voltages showing how connections may be changed for any combination of phases and voltages. Since the treatment does represent good engineering practice, it also makes an appeal to the student whose practical experience is still ahead of him.

Like all highly technical subjects, the Induction Motor, in the past, has been treated very completely from the theoretical standpoint, while comparatively little has been published concerning the really practical details, of which the windings are a prominent part. This type of motor, while much later "in the running" than its d.c. rival, has fairly pre-empted the field in general power work. Therefore a practical treatise on the winding characteristics of this apparatus, such as the author has presented, is not only most timely, but is really a practical necessity.

It is with the greatest pleasure that I recommend this work to those who are interested in both the theoretical and practical side of the Induction Motor problem.

(Signed) B. G. LAMME.

CONNECTING INDUCTION MOTORS

CHAPTER I

WHAT THE WINDING ON AN INDUCTION MOTOR ACCOMPLISHES

The simplest conception of any motor, either direct or alternating current, is that it consists of a magnetic circuit interlinked with an electrical circuit in such a way as to produce a mechanical turning force. A study of the reasons for this force and its results leads naturally to the consideration of the magnetic circuit and the way it is set up and of the electric circuit and the interrelation of the two. It was recognized a long time ago that a magnet could be produced by passing an electric current through a coil wound around magnetic material and the fact was established later that when a current is passed through a conductor or a coil which is situated in a magnetic field there is set up a force tending to produce motion of the coil relative to the field. Since it is equally true that a magnet is most easily produced by an electric current and that an electric current is most easily produced by employing a magnet it is not material which of these elements is considered the more fundamental and the better starting point for study. One thing which becomes apparent is that coils or turns of wire are essential to both the magnetic and the electric circuit and it is the form and combination of these coils in alternating-current motors which is the subject matter of this book.

Functions of the Windings in a Direct-current Motor.—In the familiar shunt-wound direct-current motor there are two separate and distinct windings each serving a special purpose. There are the shunt coils on the stator or field member whose function it is to establish the magnetic circuit or "field." There are also the

coils on the armature which constitute the electric circuit or the circuit carrying the working current.

In addition to carrying the working current the armature coils are also acting as generator coils and generating a voltage which prevents any more current flowing in the armature than is necessary to produce exactly the required amount of torque. A little consideration shows that this must be the case. The full-load current in a 5-hp. 230-volt motor is in the neighborhood of 20 amperes and the resistance of the armature between brushes may be 0.3 of an ohm. Since the armature brushes are put directly across the 230-volt line, if there was no other condition existing except Ohm's law, a current would flow in the armature

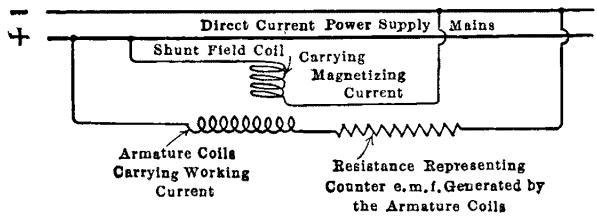


FIG. 1.—Windings of a direct-current motor and their functions.

having a value of $230 \div \frac{3}{10} = 767$ amperes. However, since the full-load current of the motor is only 20 amperes it is evident that only 6 volts are required to circulate this current in the armature and the remaining $230 - 6 = 224$ volts are absorbed or accounted for in some other way. As a matter of fact these 224 volts are taken care of by the armature which actually generates a voltage of 224 volts and opposes it to the line leaving only the difference between 230 and 224 or 6 volts available to force the needed working current through the armature. The name of this voltage generated in the armature is the "back e.m.f." or "counter e.m.f." and it is present in the case of all motors of any type whether direct or alternating current.

The foregoing is mentioned to show that on a shunt-wound direct-current motor the windings are exercising three distinct functions, viz., first, the field coils are setting up the magnetic field, second, the armature coils are carrying the working current and, third, the armature coils are generating a voltage which is opposed to the line voltage and which determines how much working current may flow in the armature and hence, directly, how much torque will be produced.

This condition is shown diagrammatically in Fig. 1 where the shunt-field coil is shown setting up the magnetic field and the armature coils carrying the working current. The counter-electromotive force which is generated by the armature coils is represented as a resistance in series with the armature since its action is to cut down the amount of current which would otherwise flow in the armature.

Synchronous Motor.—In an alternating-current motor of the synchronous type there are also two windings exercising these same three functions, viz., first, the direct-current winding

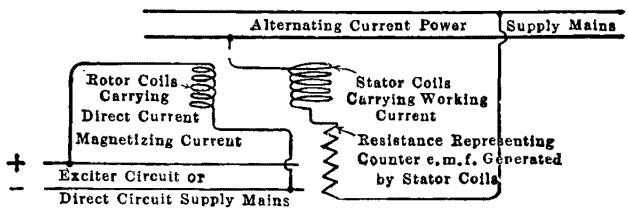

Alternating Current Power
Supply Mains
Rotor Coils Carrying Direct Current Magnetizing Current
Stator Coils Carrying Working Current
Resistance Representing Counter e.m.f. Generated by Stator Coils
+
−
Exciter Circuit or Direct Circuit Supply Mains

FIG. 2.—Windings of an alternating-current synchronous motor and their functions.

serving to set up the magnetic field, second, the alternating-current winding carrying the working current and, third, the alternating-current winding generating the counter-electromotive force nearly equal to the applied line voltage.

The condition is represented by the diagram, Fig. 2, which shows the magnetic-field circuit as separately excited from a direct-current source of supply. The stator winding or alternating-current winding is shown as carrying the working current and in addition generating the counter-electromotive force which is represented as a resistance in series with it.

Induction Motor.—In the case of the alternating-current induction motor there are again two windings, one in the stator and one on the rotor and these two windings are again exercising the same three functions but with a slight difference which is well worth noting. The rotor winding or secondary winding of a polyphase induction motor carries the working current. Since in this type of motor there is no electrical connection between the stator and rotor windings the only manner in which this current can be set up in the rotor is by transforming it from stator to rotor using the transformer action of the primary upon the secondary. This, then, sets up in the primary or stator

winding the very interesting condition that in one single winding or set of coils there exist three separate actions. First, the magnetizing current is flowing and setting up the magnetic field just as it does in the shunt direct-current or synchronous alternating-current motor; second, the working current is flowing and being transformed into the rotor and, third, there is a generator action taking place in the coils and generating a back or counter-electromotive force opposite in direction and slightly less in amount than the applied line voltage.

This condition is shown graphically in the diagram of Fig. 3 where the three separate actions are indicated and shown to be similar to the corresponding items in Fig. 1 and Fig. 2.

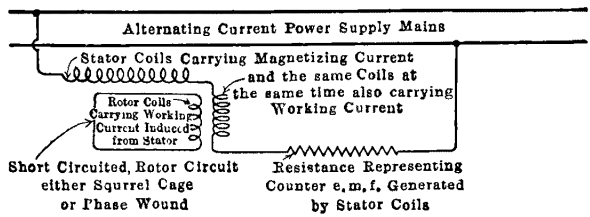


FIG. 3.—Windings of an alternating-current induction motor and their functions.

Since these three conditions do exist in the single winding it becomes evident that when changes in operating conditions occur such as are covered by reconnecting a winding for different phases and different speeds, etc., all three of these conditions must be satisfied if the operation of the motor is to be normal. That is to say, the cross section of the conductor in the windings must be great enough to carry the combined magnetizing and working current; the number of turns must be correct for setting up the required magnetic field and the combination of magnetic field and number of turns in the armature working together must generate the required counter-electromotive force, which in all cases is just slightly less than the applied line voltage. This also shows the reason why one of the simplest methods of figuring how many turns are required in the winding of a given motor is to consider it as an alternating-current generator rather than as a motor. This method is frequently referred to throughout the text and an effort made to have it appear as a physical picture of what is going on inside the motor rather than as a set of mathematical formulæ or an involved vector or circle diagram.

CHAPTER II

THE ROTATING MAGNETIC FIELD

Why a Motor Drives Its Load.—An induction motor rotates and drives its load because there exists inside the motor a magnetic field which rotates and pulls the iron of the rotor core and the rotor windings around with it. This magnetic field has a number of north and south poles and in its effect resembles several bar magnets riveted together in the center and spaced radially like the spokes of a wheel. The discovery that such a magnetic field could be established in an iron core and made to rotate by exciting a winding with alternating current is what made possible the development of the induction motor. With the proper conception of how this field is set up and caused to rotate and its effect upon the windings of the motor as it rotates it is easier to understand the working of the motor and also to form an opinion of the possibility of accommodating the motor windings to changes in operating conditions. It is the intent of this chapter to give a physical idea of the rotating magnetic field followed by a graphical explanation of how it is set up by alternating current.

How Torque Is Produced.—It is now generally understood that an electric motor produces torque or driving effort by utilizing the effect of a magnetic field upon a wire, or wires, which are carrying electric current. It is also understood that a magnetic field may be produced in an iron circuit by passing an electric current through a coil which surrounds or is interlinked with that iron circuit. The action of producing driving effort in a direct-current motor then becomes very simple. First the magnetic field is set up by passing a direct current through the field coils surrounding the poles. This direct current is drawn from the same source of supply that is to drive the motor. When the magnetic field is set up, another direct current is drawn from the source of supply and caused to flow through the armature coils which lie in the magnetic field just previously set up.

The action of the magnetism of the field on the current in the armature wires causes the rotor to develop torque and start to turn.

There is another conception which is equivalent to what has just been stated but which is sometimes easier to understand. It is that one magnet is formed by the field winding and the iron of the field or stator and a second magnet formed by the winding and the iron of the armature or rotor. The magnet formed by the armature is a little out of line with the field magnet and therefore has a tendency to be pulled into line, thus rotating the armature. However, just as soon as the magnets are in line the brushes on the commutator break the circuit with one armature coil and make contact with another, which forms a magnet of the armature iron in the original position before the armature was rotated. Thus by this action, which is similar to a kitten chasing its tail, the field magnet is constantly pulling the armature around, trying to get the rotating magnet in line, and is constantly being foiled by the brushes and the commutator bars which continually set up a new magnet in the right position to be pulled on. The situation can be visualized from Fig. 4.

The foregoing is elementary and exactly the thing that happens in the alternating-current motor, but in a little different way. In the direct-current motor just noted, two sets of coils were used. The first set—the field coils—was used to excite the magnetic field; the second set was the armature coils and was used to carry the working current. In the induction motor there is but one set of coils, which must at the same time exercise the two functions of setting up the magnetic field and carrying the working current. This fact is chiefly responsible for the condition in the motor which is called power factor and which is not present in the case of the direct-current motor.

It is worth while to consider as simply as possible the manner in which the magnetic field is set up in the induction motor and the reason it travels around the machine at a relatively high rate of speed.

Long before the days of Tesla and Feraris, it was known that if a magnet was passed over a sheet of copper close to its surface, a force was produced which tended to cause the copper to move in the same direction as the magnet. Although not then so recognized, this was the fundamental principle on which all

modern dynamo-electric machines are based. The contribution that Tesla and Feraris made was the discovery that such a moving magnetic field could be set up by an alternating current and need not rely on a permanent magnet or one excited by direct current.

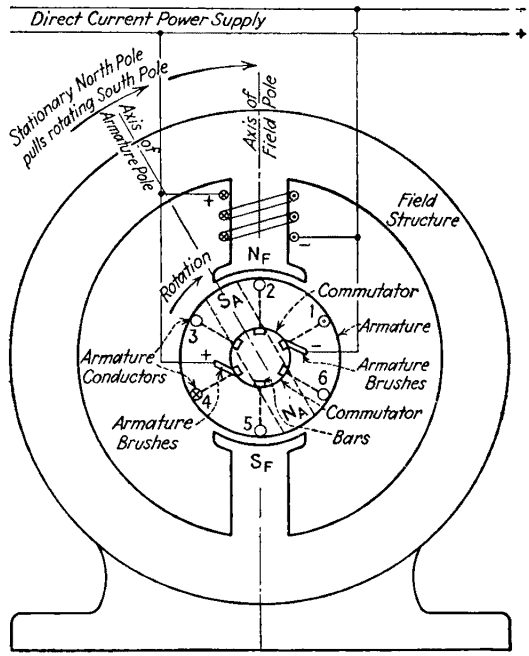


FIG. 4.

Setting Up a Rotating Magnetic Field by Alternating Current. Consideration of Fig. 4 shows that, regardless of the rotation of the physical armature, the magnetic field S_A-N_A stands still in space in the position shown. The magnetic field N_F-S_F, being in the stator, stands still of necessity. Then, thinking of the magnetic and mechanical relations, there are (1) two magnetic fields standing still in space, (2) one stationary mechanical structure and (3) one rotating mechanical structure. Contrast with this the condition that the armature is held stationary and the field structure is left free to rotate, bearings, brushes and all, around the armature. The two fields N_F-S_F and N_A-S_A and the field structure would now rotate in space and the only stationary element would be the physical armature which is being held from rotating by assumption. The rotating magnetic

field N_F-S_F is now similar to that which exists in any induction motor in that it is a succession of north and south poles rotating in space and cutting across the conductors in the armature. There is this difference, that the physical stator of the induction motor stands still, but invisibly in space the magnetic field is going around in it as surely as the magnetic field N_F-S_F in Fig. 4 is going around if the field structure rotates around the armature. But the magnetic fields in Fig. 4 are set up by direct current and an investigation must now be made to see how the magnetic field can be set up and rotated in space by two-phase or three-phase alternating currents, without rotating the physical structure, *i.e.*, the stator frame and punchings.

Since the entire action of the induction motor depends upon this phenomenon, one can well spend time and study on its understanding and mastery. A clear physical picture of what is taking place gives the ability to answer most of the questions which come up concerning induction-motor operation and incidentally helps much in winding problems.

Methods for Representing Alternating-current Magnetic Actions.—Consider a single-phase electric circuit with two leads and one current path in which an alternating current is flowing. If it is a 60-cycle alternating current, there are 120 times a second when the current is zero, 60 times a second when it is a maximum in the positive or plus direction, and 60 times a second when it is a maximum in the negative or minus direction. Suppose just one of these 60 cycles is considered and time is supposed to pass along a horizontal line. Then assume that as time passes arrows are set up above or below this horizontal line to represent the values of the current at particular instants. Such a picture would look like Fig. 5 and succeeding cycles would follow a similar pattern. If the tips of the various arrowheads are joined by a curved line as in Fig. 6, a smooth sinuous curve known in trigonometry as a "sine curve" results. This at once gives a clue to a means of finding the value of the current at any instant by drawing a sine curve. A sine curve is constructed by assuming that a vector or arrow as in Fig. 7 is fixed at one end and rotated in a counterclockwise direction around that end. In making a complete circle it passes through 360 circular degrees and if this circle is rolled out flat into a line as, for example, the line XYZ in Fig. 5, the distance XZ represents 360 circular

degrees or, as it is also called, 360 electrical degrees. Since XZ is divided into 12 equal parts, each part would represent 30 circular degrees of Fig. 7. An examination of Fig. 7 shows how the lengths of the arrows representing the values of current in Fig. 5 are obtained by taking the projection of the rotating

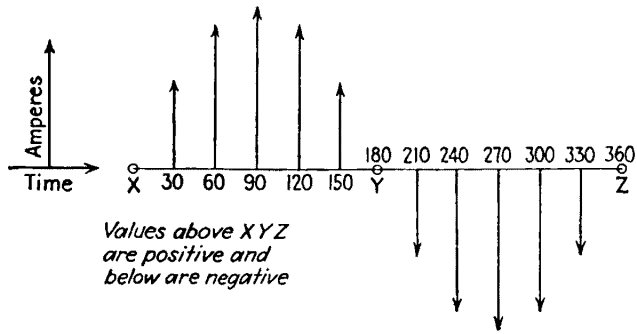


FIG. 5.

arrow on the vertical line YY at 12 points around the complete circle. For example, at 30 deg. as shown, the length "30 and 150" as marked is obtained. This is the same length as results when the vector has swung around to the 150-deg. position. Similarly the vertical arrow marked "60 and 120" shows the value of the current when the arrow is in the 60-deg. and 120-deg.

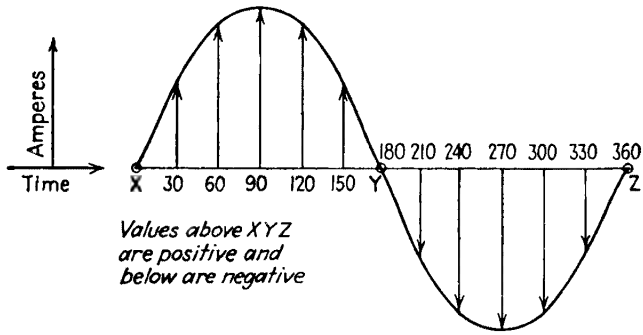


FIG. 6.

position and the vertical value marked "90" gives the current value when the rotating arrow is standing directly vertical. At this point the value is the same as the length of the rotating arrow or 1 and at all other points, except the corresponding value of -1 at 270 deg., the value of the currents is something less than 1. The ratio of the vertical arrow on YY' to the length

of the rotating arrow at any point is what is called in trigonometry the sine of the angle which the rotating arrow has reached. For example at 30 and 150 the value of the vertical arrow is one-half the rotating arrow and at 60 and 120 it is the $\sqrt{3}/2$ or 0.866. Likewise at 210 and 330 the value is $-\frac{1}{2}$ and at 240 and 300 the

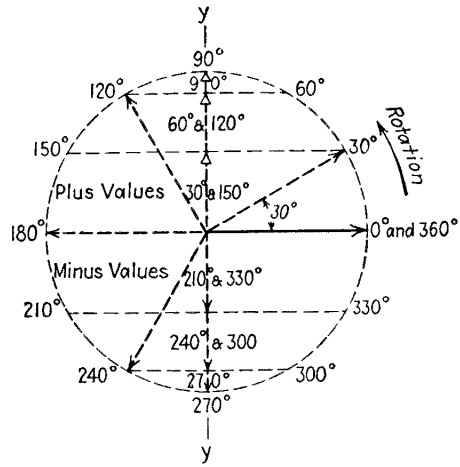


FIG. 7.—How sine curves are drawn.

value is -0.866. Reference to a table of natural sines such as that given in connection with the chapter on Chord Factor will show that these values are the natural sines of 30 deg., 60 deg., 90 deg., 120 deg., etc.

Representing Single-phase and Polyphase Alternating Currents.—The foregoing explains how it is possible to represent

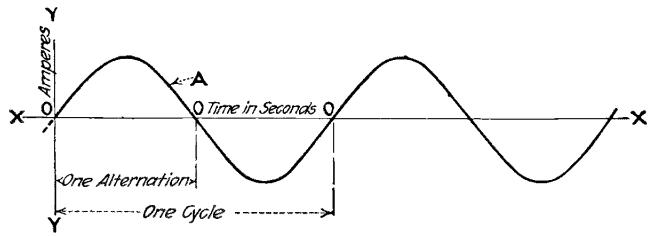


FIG. 8.—The "indicator card" of a single-phase alternating current.

single-phase current in value and time by a curve such as that shown in Fig. 8. Figure 9 shows what a two-phase current would look like. It is formed as would be two single-phase currents shown in Fig. 8 except that they are displaced in time by one-

quarter of a cycle. That is to say, if phase A starts in the positive direction at 0 deg. as shown in Figs. 6 and 8, phase B does not start in the same direction until 90 deg. is reached. This displacement by one-quarter of a cycle gave rise to an old expression for two-phase currents—"quarter phase." The two graphs or curves as shown in Fig. 9 are derived from two

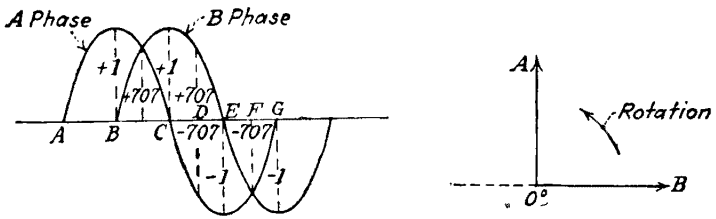


FIG. 9.—Sine-wave representation of two-phase alternating current.

rotating arrows as in Fig. 7 except that these arrows are at right angles to one another or 90 electrical degrees apart. That is, if the phase A arrow is shown vertically upright, the phase B arrow at the same instant would be pointing straight to the right as in Fig. 9. The corresponding picture of the currents in a three-phase circuit is shown by Fig. 10. This is similar again to three single-phase currents but the second or B phase starts 120 deg. later than the A phase and the third or C phase starts

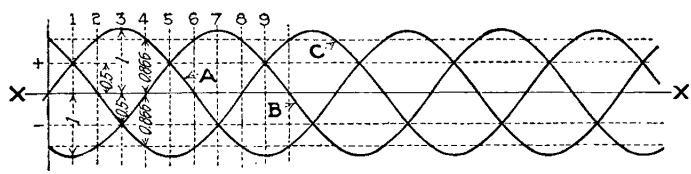


FIG. 10.—Sine wave representation of three-phase alternating current.

120 deg. later than the B and 240 deg. later than the A phase. Referring again to Fig. 7: If the arrow there shown pointing to 0 deg. is the A phase, the B phase arrow would be pointing to 240 deg. and the C phase arrow to 120 deg. and all three would be rotating counterclockwise. The result would be that when they passed any fixed point on the circle the A current would come first and one-third of a cycle or 120 deg. later the B phase would reach the same point and the same value; two-thirds of a cycle or 240 deg. later the C phase would reach the same value and the same point.

Why the Middle Leg or *B* Winding Is Reversed.—In the case of two-phase currents, referring again to Fig. 7, if the *A* phase arrow is pointing to 0 deg. as shown, it would not make any difference whether the *B* phase arrow was pointing straight down to 270 deg. or straight up to 90 deg. except that the motor would run in clockwise rotation in one case and in counterclockwise rotation in the other. All that it would be necessary to do, if the

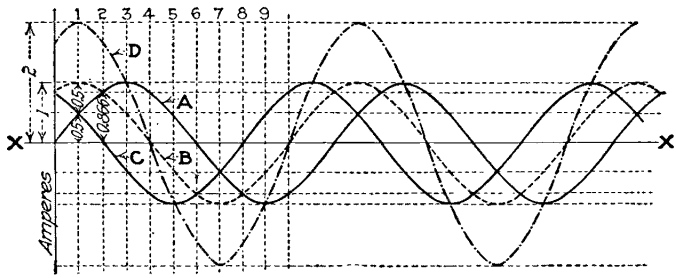


FIG. 11.—How the three phases combine to form one magnetizing current.

first try gave the wrong rotation, would be to reverse the two leads of either phase outside the motor and then the *A* phase arrow would lead the *B* phase arrow in rotation as the case might be. In the case of three-phase currents, however, something different happens which has to be taken care of inside the motor winding by always reversing the lead and star end of the middle leg or *B* phase. This condition can be explained as follows:

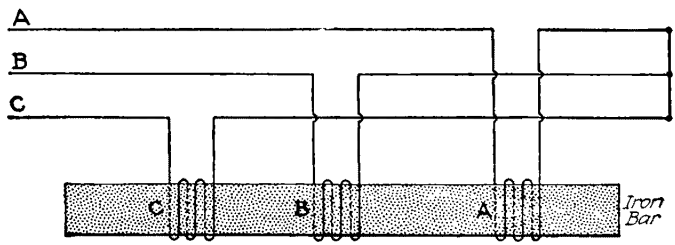


FIG. 12.—Iron bar acted upon by three-phase currents as arranged in Fig. 10. No resultant magnetism.

Assume that each one of the three-phase lines is wound an equal number of times around the same iron bar, as in Fig. 12. Whenever a coil is placed around iron and current flows in the coil, it sets up magnetic lines, or flux, and the iron becomes a magnet. It is evident, then, from Fig. 12, that any one of the three coils by itself would make a magnet of the iron bar which would have

its north pole at one end at one instant and a south pole at the same end the next instant as the current changed its direction according to the curve in Fig. 8.

However, when all three coils work together on the bar (Fig. 12) there is no magnetism set up, because at any instant the current in one coil is equal in amount and opposite in direction to the currents in the other two coils. This can be seen from Fig. 10. Take, for instance, the time marked by the vertical line 1. At this instant the A and C phases are measured above the horizontal line XX and hence are positive or plus in value and are each equal to $+0.5$, while the B phase is measured below the

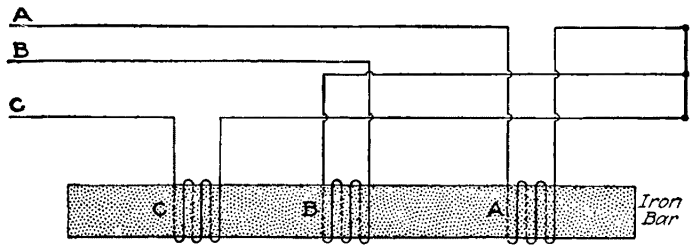


FIG. 13.—Iron bar acted upon by three-phase currents as in Fig. 11. Strong resultant magnetism alternately north and south.

X line and hence negative or minus value to -1. Therefore, the sum of all three currents is zero because $+(0.5 \times 2) - 1 = 0$.

At the instant 2, $C = 0$, $A = +0.866$ and $B = -0.866$ and the sum of the three currents is zero. At instant 3, $A = +1$, $B = -0.5$, and $C = -0.5$, total $= 0$; at the instant 4, $A = +0.866$, $B = 0$ and $C = -0.866$, total $= 0$; and so on at all points the sum of the three currents is zero. Therefore in Fig. 12 there will be no magnetism in the iron bar, since at all times there is an equal number of ampere turns in the coils trying to force the magnetism in each direction.

The next step is to reverse one coil, as shown at B in Fig. 13, and the bar immediately becomes a strong magnet, reversing its poles from instant to instant according to the change in direction of the curve D in Fig. 11. Reversing one coil in Fig. 13 is the equivalent of reversing the current in one phase of the generator. This is indicated in Fig. 11, in which curve B is shown plotted above the line where it is below the line in Fig. 10, and vice versa. The sum of the three curves A, B and C, Fig. 11, gives a resultant curve D, which represents the current that will be effec-

tive in magnetizing the core, Fig. 13. It will be seen that the A and C curves in Fig. 11 are the same as in Fig. 10, but the B curve is turned over, or reversed, since the B coil is reversed in Fig. 13. Curve D, Fig. 11, is obtained by adding the values of the three currents at any point. For example, at the time marked by the vertical line, 1, $A = +0.5$, $C = +0.5$ and $B = +1$, hence $D = +2$. At the time marked by the vertical line 3, $A = +1$. $B = +0.5$ and $C = -0.5$, hence $D = +1$. Also at time 4, $A = +0.866$, $B = 0$ and $C = -0.866$, hence $D = 0$. In this manner the curve D is obtained, and it serves as an indicator card of the magnetism in the iron bar in Fig. 13.

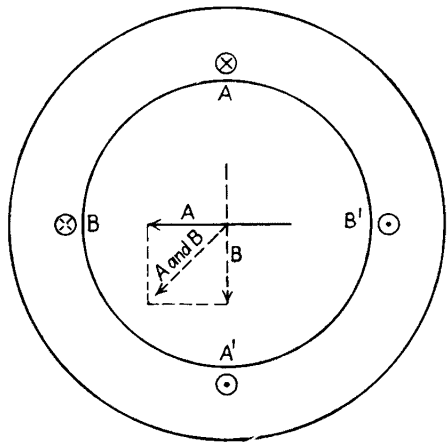


FIG. 14.

Setting Up a Magnetic Field with Two-phase Currents.—Suppose that Fig. 14 represents a cross section of the stator iron and coils of a two-phase two-pole induction motor. There is a single coil for the A phase and a single coil for the B phase. At the instant shown the current in the A phase is a maximum and that in the B phase is zero, as it would be at position B in Fig. 9. The current is shown going away from the observer at the top of the stator and coming back toward him at the bottom. Under the influence of this current a magnetic field is set up in the stator and rotor iron from right to left, as represented by the arrow A. One-quarter of a cycle later, or at C as shown in Fig. 9, the current in the A phase is zero and that in the B phase is a maximum. Under the influence of this current a magnetic field is set up in the iron from top to bottom, as shown by the dotted arrow B. One-

quarter of a cycle later, or at E shown in Fig. 9, the A phase current is at a negative maximum and the B phase current is zero. Under this condition the magnetic field would be horizontal again, but from left to right as the arrow A would be, with the head on the right-hand end. Finally at position 360 the A phase current is zero and the B phase current is a negative maximum and therefore the arrow is again vertical but with the head on the top end. Considering the changes that have occurred in the arrow in the center of Fig. 14, it will be seen that it has made a complete rotation around its center point and in so doing has represented one complete rotation of the magnetic field. This analysis has neglected any in-between points where current is flowing in both A and B as, for example, point F in Fig. 9. These would represent combined fields, as shown in the 45-deg. arrow in Fig. 14; this gives a still clearer view of how the magnetic field, owing to the combined action of phases A and B, is rotating counterclockwise.

One complete rotation of this center arrow, which represents the combined magnetic field due to both phases, takes place for each pair of poles. In the example shown a two-pole motor was chosen for simplicity, since a complete rotation of the mechanical field was the same as a complete physical rotation of the motor. In the case of a four-pole winding one complete rotation of the magnetic field would take place for one-half of a physical or mechanical rotation of the motor. That is why the synchronous or no-load speed of a two-pole 60-cycle motor is about 3600 r.p.m., that of a four-pole motor is 1800 r.p.m., that of a six-pole motor 1200 r.p.m., etc. The formula which combines the frequency, poles and r.p.m. is

$$\textit{Synchronous r.p.m.} = \frac{\textit{Cycles per second} \times 120}{\textit{Number of poles}}$$

A four-pole motor on 60 cycles would then have a no-load speed of $\frac{60 \times 120}{4} = 1800$ as just stated.[1]

In the same way a diagram similar to Fig. 14 could be drawn for a three-phase motor except for having three coils spaced 60

[1] Further detail regarding the rotating field in a four-pole two-phase motor is shown in Chapter II of the author's book "Induction Motor Practice," McGraw-Hill Book Company, Inc. New York.

mechanical degrees apart; as that introduces some additional complication, it is approached in another way.

Setting Up a Magnetic Field with Three-phase Currents.—This conception of three-phase coils making a magnet whose flux or field varies in value and direction according to the curve *D* in Fig. 11 can be readily transferred to the stator of an induction motor, as shown in Fig. 15. Here is shown part of a laminated core slotted on the inner periphery, and in two of these slots are shown three coils, *A*, *B* and *C*, to correspond to the coils in Fig 13. Assume the three coils to be connected in star and to a three

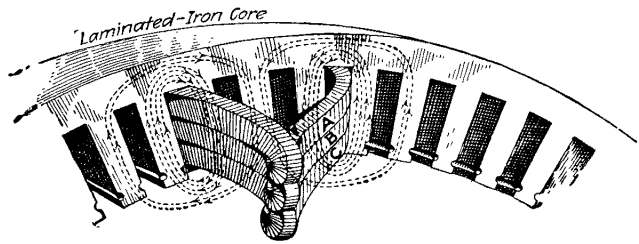


FIG. 15.—Cross section of stator core with three coils similar to Fig. 13.

phase circuit. A magnetic field will then flow into the air gap and back through the core, as shown by the curved dotted lines and arrowheads. This magnetic field will flow in the direction of the arrows for a fraction of a second, then fall to zero, and increase to a maximum in the direction opposite to the arrowheads, and so on. In other words, the three coils working together would make first a north pole and then a south pole on the inner periphery, and repeat, and the amount and direction of the magnetism in the iron between the two sides of the coil could be measured by taking the distance from points on the curve *D*, Fig. 11, from the horizontal reference line and calling all points above that line north values and below the line south values.

For example, at the position marked 1 the magnetic value would be a maximum north value; at 3 it would be 0.5 north; at 4 it would be zero; at 5 it would be 0.5 south; at 7 it would be a maximum south value, etc. There would be no tendency, however, for this magnetic field to rotate or travel around the stator as it does in an induction motor. It would simply stand still in space and alternate backward and forward through the coil as described. In order to get the rotating motion, it will be neces-

sary to separate the three coils and put each one in a separate slot, as shown in Fig. 16, as they would be in any normal induction motor.

A section cut through the core and coils, Fig. 16, is shown in Fig. 17 with one side of each coil in the bottom of slots 1, 2 and 3

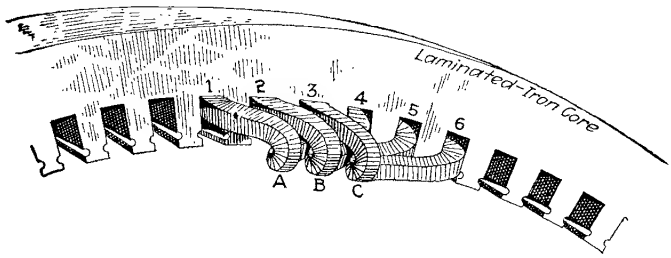


FIG. 16.—The three coils of Fig. 15 distributed as in normal induction motor.

and marked A, B, C, respectively, and their other sides in the top of slots 4, 5 and 6 and marked A', B', C', respectively. By means of Figs. 11 and 17 taken together; it is possible to build up small pictures of the magnetic field from instant to instant and show how it moves or rotates around in the stator core and air gap. These small pictures, of which one series is shown in Fig. 18 and another in Fig. 19, can be very well compared to the individual small pictures on a moving-picture film as they appear when the film is at rest, and the rotating magnetic field as it really exists could be compared to the same film when in motion

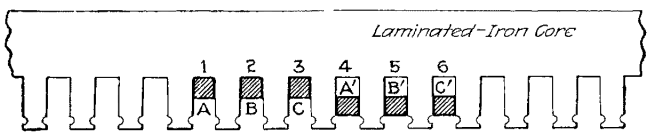


FIG. 17.—Cross section of core and winding in Fig. 16.

and thrown on the screen. The method of making these small pictures is very simple and is as follows.

Drawing a Graphical Picture of the Magnetic Field.—At the top of Fig. 18 is a section through the coils and core, Fig. 16, the same as that given in Fig. 17. A current is assumed to be flowing in each coil, and the value of that current is taken from the curve marked with the same letter in Fig. 11. For example, at the time represented by the vertical line 1 in Fig. 11, curve B is at its maximum value, which is called $+1$, because it is

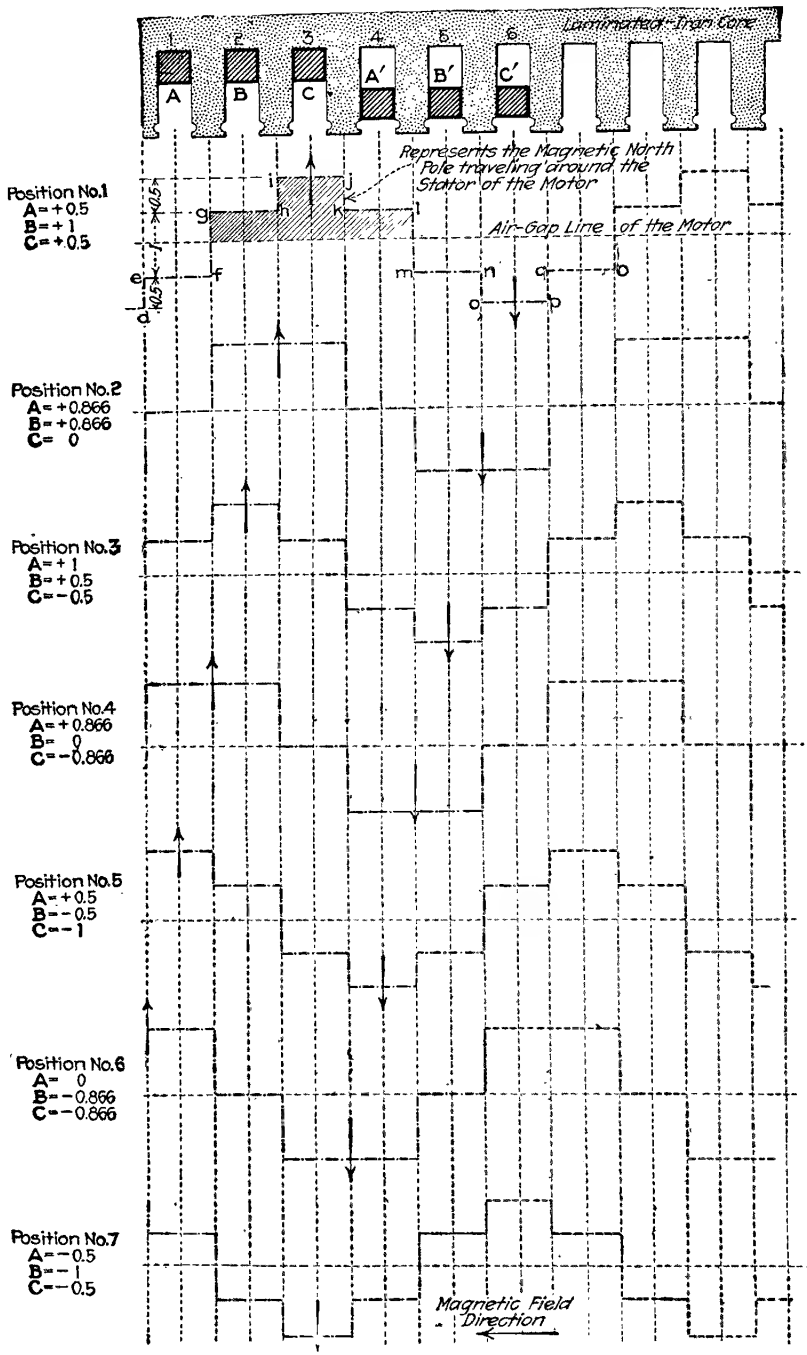


FIG. 18.—Instantaneous values of the magnetic field set up by the coils of Fig. 16. Note field traveling from right to left.

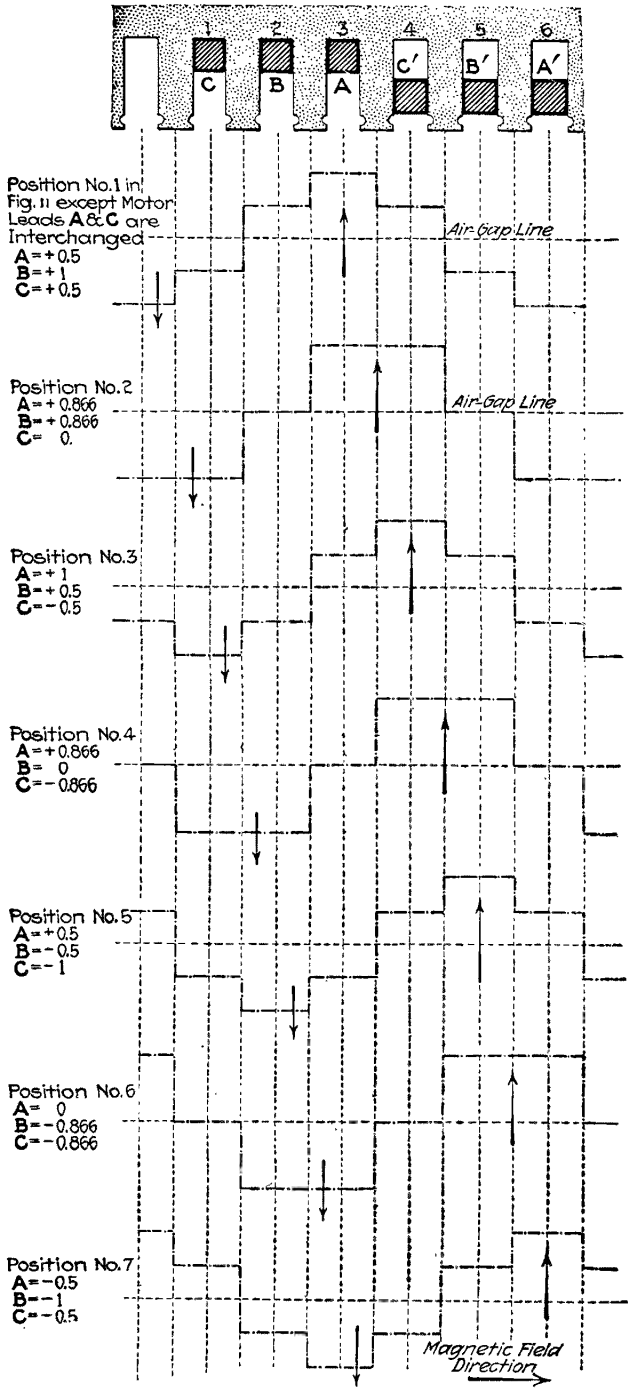


FIG. 19.—Similar conditions to Fig. 18 except two leads reversed causing field to travel from left to right and hence reversing direction of rotation of motor.

above the horizontal reference line, and curves A and C are each at a value of $+0.5$, since they are half their maximum value and are also above the reference line XX. Similarly, at the time represented by the vertical line 2, which is called position 2, in Fig. 11, the value of the A and B curves is $+0.866$ and the C curve is zero. The value 0.866 is obtained because these current curves are all what are known as "sine curves" and the reference points or positions 1, 2, 3, etc., are taken as one-twelfth of a complete cycle apart.

A complete cycle is known as 360 electrical degrees similar to the 360 mechanical degrees in a circle, and hence the reference positions 1, 2, 3, etc., are one-twelfth of 360 deg. or 30 deg. apart. From a table of natural sines such as is found in any handbooks, it will be found that the sine of 30 deg. = 0.5, sine of 60 deg. = 0.866, sine of 90 deg. = 1, sine of 120 deg. = 0.866, sine of 150 deg. = 0.5 and sine of 180 deg. = 0. Continuing from 180 deg. to 360 deg., the same values recur with a minus sign since they are measured below the horizontal reference line. So that it is these values which are used in plotting the pictures in Fig. 18, and the values for different positions are given in the left-hand column in the figure.

From Fig. 16 we have the position of the coils, and from Fig. 11 we have the value of the current in each coil as given in the column on the left of Fig. 18. Then if the values of these currents are plotted or drawn, the resulting curve is a measure of the magnetic field, since such a field depends on the number of turns of wire and the current flowing in the coil. It remains, then, only to draw the small figures or curves in Fig. 18 in the following manner:

Starting from any arbitrary point as at d, Fig. 18, the line moves in direction and amount according to the value of the current in slot 1. Slot 1 contains the A coil and the value of the current is $+0.5$ as is shown on the left; since the direction of plus is up, the line is drawn upward from d to e and ef is drawn horizontally, representing by its height above d the current in slot 1 and the magnetic field at that point. From f the line goes up to g, making fg twice as long as de because the B coil is in slot 2 and the value of the current in the B coil is $+1$, or twice that in A, and the line gh is drawn horizontally, representing by its height above d the current in slot 1 + slot 2 and therefore

the magnetic field at that point. From h the line goes up to i because the C coil is in slot 3 and the current in the C coil as shown at the left at that instant is $+0.5$. The line ij is drawn horizontally, representing by its height above d the combined currents in slots 1 plus 2 plus 3 and therefore the magnetic field at that point. From j the line drops down to k because the A' conductor is in slot 4 and the A' conductor is the other side of the A coil and hence the current in it is in the opposite direction to that in the A side. By referring to the column at the left of the figure, if the current in the A side was considered $+0.5$, the current in the A' side must be -0.5 and hence the curve drops down for a minus value from j to k. Similarly, it drops twice as far from l to m, since $B = +1$ and therefore the other side of the B coil or B' must $= -1$. Following the curve in this manner to n and o, it completes one cycle or one north and south pole. The north pole is considered as that part above the horizontal reference line and under the line g, h, i, j, k, and l, which is shown shaded, and the center of this north pole is indicated by the vertical arrow.

In an actual machine the magnetic field would not have such sharp corners, but would be smoothed out by the rotor winding into a smooth curve practically a sine curve such as the current curves in Fig. 11, but for purposes of illustration the "stair-step," or square-shouldered curves, may be considered as shown. In a similar manner the little stair-step picture may be drawn for each position and the center of the north pole marked by an arrow pointing up as shown. After drawing seven positions, the very interesting fact may be noted that the center of the north pole has traveled three slots to the left, which in this case means 180 electrical degrees, or a half revolution on a two-pole motor or a quarter revolution on a four-pole machine.

Interchanging Two Leads Reverses Direction of Rotation.—Figure 19 is drawn to show the effect of interchanging the leads to the coils A and C, or in other words, the line lead that was connected to A is now connected to C and vice versa. For this reason in the little sketch at the top of Fig. 19, taken from Fig. 16, the C coil is now in slot 1 and the A coil is in slot 3, the B coil remaining in slot 2 unchanged. The numerical values of the currents are again taken from Fig. 11 just as it stands, because it must be remembered that the curves in Fig. 11 represent cur-

rents in the line and that they depend on the generator and are not changed by the change in the motor leads. These assumptions give the current values for the different positions, as shown in the left-hand column in Fig. 19, and the small stair-step pictures show the magnetic field in the same manner as in Fig. 18. The interesting thing to note is that the center of the north pole has now traveled from the center of slot 3 to the center of slot 6, or the magnetic field has now traveled three slots to the right, which discloses the well-known fact that interchanging two leads on a three-phase motor will reverse the mechanical direction of its rotation. As a problem the reader might attempt to produce the same result for a two-phase motor and will find, as previously

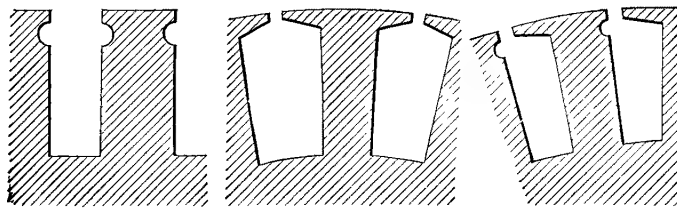


FIG. 20.—Open slots. FIG. 21.—Partly closed slots—center opening. FIG. 22.—Partly closed slots—side opening.

Common forms of induction-motor stator and rotor slots.

pointed out, that this field plotting becomes a fascinating mental diversion.

A comparison of Figs. 10 and 11 shows at once why the middle leg of a three-phase winding is reversed in all the common diagrams that will be shown in this book. Figures 18 and 19 show how the magnetic field may be studied and how reversal follows exchange of two leads.

After a designing engineer has determined how many turns are required in the winding which he is calculating, the largest single factor which decides the form or type of windings to be used is the mechanical form of the slots; that is, whether they are open, Fig. 20, or semiclosed, as in Figs. 21 and 22, and the width of the opening if they are semiclosed. The factor of next importance is whether the winding is on the rotor or on the stator.

The Counter E.M.F.—Having considered the manner of setting up the field and causing its rotation, there is another action, easily understood, which is perhaps as useful as any in giving a clear idea of how many turns are required in a motor

winding under different conditions. This is what is called the generation of the counter e.m.f. force. Since the coils of the motor are standing still and the magnetic field is rotating past them and threading through them, there is of necessity a voltage generated in the coils by the rotating field. This is the voltage which is referred to as counter e.m.f. and is in all cases equal to the voltage of the supply line which is applied to the motor, except for a small loss in the motor caused by producing the necessary torque or driving force.

With this conception and the fundamental formula for the generation of an electromotive force, it is a simple matter to write expressions showing how the turns in a motor should vary with different line voltages and for different speeds, etc. For example, a motor to operate on 440 volts must have twice as many turns in the coils as the same motor when operating on 220 volts, and a motor operating at 900 r.p.m. in general would require twice as many turns as the same motor when operating at 1,800 r.p.m. These are matters with which the designing engineer is chiefly concerned, but they are sufficiently simple to be borne in mind at all times, and in themselves offer the readiest first-hand answer as to the probable result of operating a given motor under changed conditions.

Having in mind this physical conception of the rotating magnetic field the next step is to be able to draw a picture of this field as it would look if it might be arrested in space at any instant and photographed. This can be most easily accomplished by the simple graphical method explained above and sometimes called "stair-step" pictures. By means of this method the rotating magnetic field can be explained and studied and the readiest possible answer given to such questions as, *Why does reversing two leads of a three-phase motor reverse its direction of rotation? Why is a phase-wound rotor always three-phase, whether the stator is for two-phase or three-phase? Also* such questions as the effect of chording the coil and changing the number of poles are readily analyzed.

The confidence that will be gained in the understanding of induction-motor operation and troubles will well repay the amount of study required to master it, and the amount of electrical knowledge required is not so great as to discourage anyone who has even a speaking acquaintance with alternating

current and its behavior. No claim is made that this is a new method.

How the Single-phase Field Is Made to Work.—The foregoing text gives an explanation of how the rotating magnetic field is set up by two-phase or three-phase currents, but it does not explain how the same thing is possible when only single-phase current is available. For this refer to Fig. 23 which shows a cross section of the cores and windings of a single phase motor.

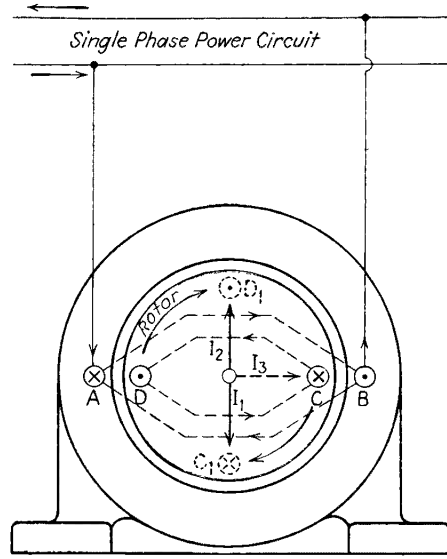


FIG. 23.—Principle of single-phase motor.

The motor is assumed to be standing still and a current is coming from the power circuit at the left into the conductor *A*, away from the observer, through a connector at the back of the motor, forward through the conductor *B*, toward the observer, and back into the power circuit as shown.

As a result of the current in this stator circuit, a transformer magnetic field or flux is set up in the rotor and stator as shown by the straight vertical arrow OI_1 pointing downward. In the squirrel-cage rotor directly opposite the conductors *A* and *B* are the conductors *C* and *D*, which are short circuited by the rings at the end of the rotor. These act, then, like a short-circuited secondary turn on a transformer and a current is set up flowing toward the rear in *C* and forward in *D*, as indicated by the cross and the dot. The magnetic effect of the current in *C*

and D is to set up a magnetic field, shown by the dotted arrow OI_2 pointing upward. This is practically equal and opposite to the field OI_1, set up by the current in A and B and hence cancels it out and there is no torque and no tendency to turn the rotor. As the alternating single-phase current pulses in A and B the currents reverse in C and D so that the fields OI_1 and OI_2 reverse in direction but are at all times practically equal and opposite. This is the well-known phenomenon of a motor with a straight single-phase winding which has no starting torque and whose rotor can be turned by hand in either direction of rotation without any opposing torque.

Suppose now the rotor is given a smart spin mechanically in the clockwise direction at somewhere near the full speed of the motor. This causes the currents in C and D, which were induced in a horizontal line, as shown in Fig. 23, to be mechanically carried into a vertical line, as shown by the dotted circles C_1 and D_1 of Fig. 23. These currents then set up a magnetic field from left to right as indicated by the arrow OI_3 and immediately it is recognized that the currents I_1 and I_3 are no longer in a single-phase relation but are 90 electrical degrees apart and hence are in a two-phase relation. From that point on, the motor continues to run as a two-phase motor and the magnetic field, instead of pulsating back and forth, up and down, now rotates around the machine just as in a two-phase or three-phase motor. This explains why a single-phase motor continues to run after it is artificially brought up to speed. In order to get it to start from rest by its own action instead of spinning it mechanically, it is necessary to "split the phase" artificially. This means taking two supply circuits from the same single-phase circuit and either putting a reactance coil in one of them or a condenser, as in the capacitor motor, thus producing a kind of imitation two-phase, which makes enough torque to get the motor started. After it is started, the picture in Fig. 23 gives an understanding of how a second phase is created by carrying the currents around in space until they are almost in a true two-phase relation. This is perhaps the simplest conception of what is often made to appear as a most difficult and involved mathematical and physical phenomenon. It also illustrates the widely described "cross field" theory which is used to explain many actions in alternating-current commutator machines.

CHAPTER III

TYPES OF WINDINGS

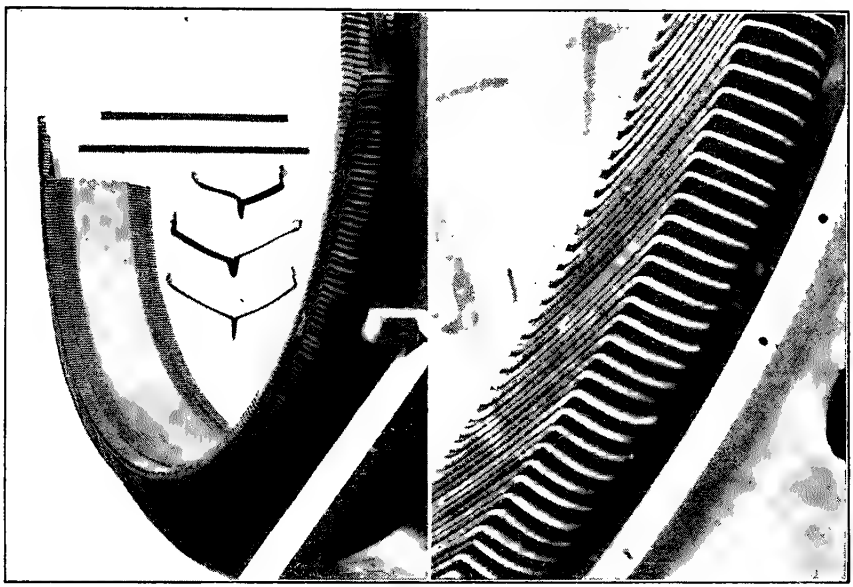
Effect of Form of Slot.—The question of open versus semiclosed slots has out-lasted many controversies and is still open to argument. It is enough to say that, other things being equal, the designing engineer favors semiclosed slots. Slots of this type usually give the highest performance and the maximum efficiency in the use of material. The repair man prefers open slots on account of the greater accessibility of the windings and the consequent ease of repair. These factors will always remain somewhat divergent and must be adjusted to suit the times and the local conditions. The reason why a machine cannot be built with as good a performance or as economically with open slots in both members is that, broadly, its capacity and excellence may be measured by the square inches of laminated-iron surface on the rotor periphery or in the bore of the stator core. Since the slot openings subtract directly from this useful surface, it is desirable to make them as small as possible. If the slot is made wide open, it subtracts the maximum amount from this useful working surface, hence the core must be made longer axially or the rotor increased in diameter to bring back the useful working surface to somewhere near the value it would have if entirely inclosed or if semiclosed slots were used. This problem is of more interest to the designer than to the repair man, but is mentioned to explain the use of a mechanical construction that is apparently undesirable from an operating standpoint.

Windings Used in Partly Closed Slots.—The types of windings adapted to semiclosed slots and most generally employed are:

1. Straight bars with involute end connectors.

2. Pushed-through coils. In this type the coils are formed in a U-shape and pushed through two slots at once in a direction parallel to the shaft. After the coil is in place, the separate wires are bent around and connected together at the other side of the core.

3. Hand-wound or threaded coils. In this construction each coil is formed in place in the machine itself, from a single piece of wire, by the process of passing the wire through the length of one slot, bending it around a wooden former to make a suitable end and threading it back through another slot and repeating until the coil is complete with the desired number of turns. When completed, it resembles the pushed-through coil.



(*a*) (*b*)

FIG. 24.—Bar-and-end-connector winding: (*a*) bars and connectors, and a partially completed winding; (*b*) completed windings.

4. Fed-in, or dropped-in coils. In this type the coil is formed complete into a so-called diamond shape and then the turns are fed one at a time through the opening at the top of the slot.

The first of these types, bar and end connector, has been widely used for both stators and rotors. The bars and connectors and a typical assembled winding are shown in Fig. 24. This winding gave excellent satisfaction, the only real criticism, from a mechanical standpoint, being that it was difficult to brace the coil ends mechanically owing to their form and relation to other parts. It has been almost abandoned on modern machines for the reason that it limited the winding to one conductor or two conductors per slot, and also because modern practice has demon-

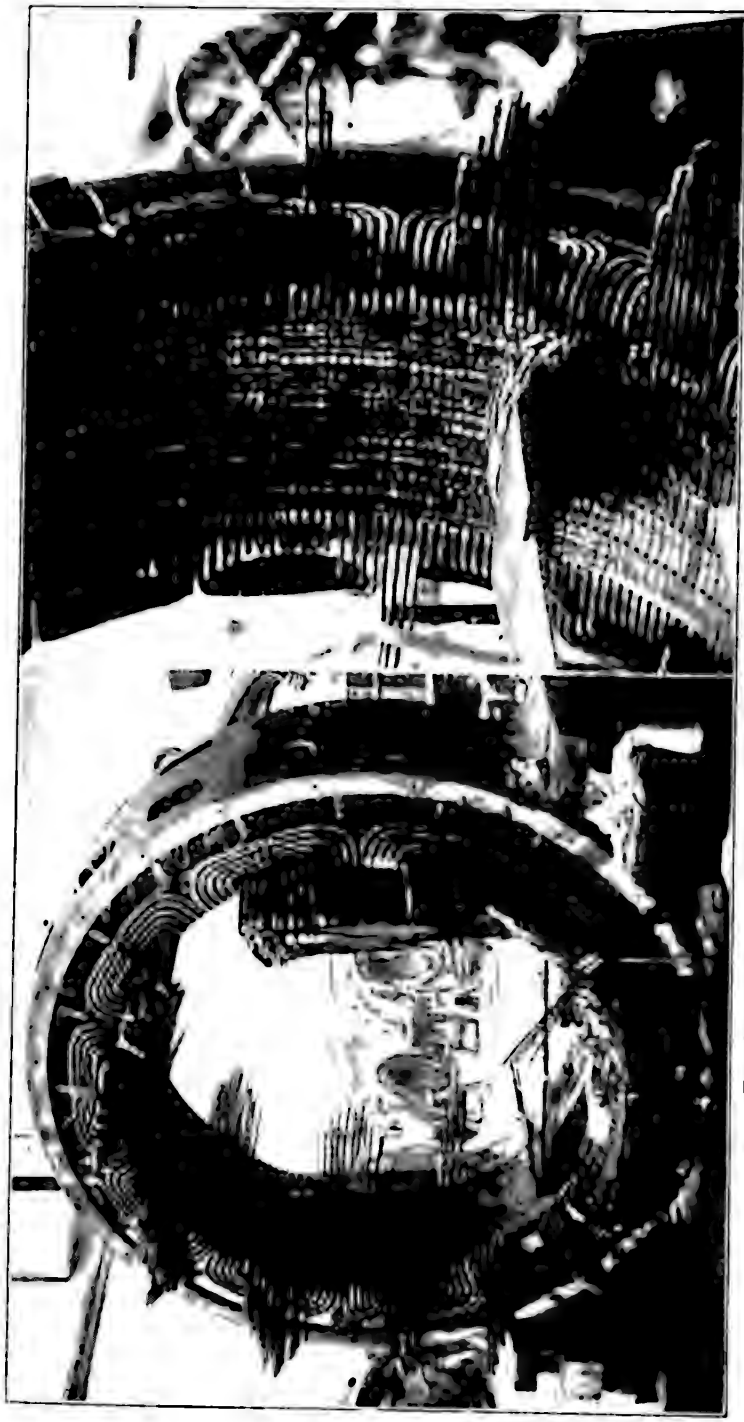


FIG. 25.
FIG. 26.
FIGS. 25 and 26.—Partially wound stators showing method of forming and connecting coils. Pushed-through windings.

strated that the use of single very heavy conductors gives rise to additional copper losses which are not present when several smaller conductors in parallel are used to carry the same current.

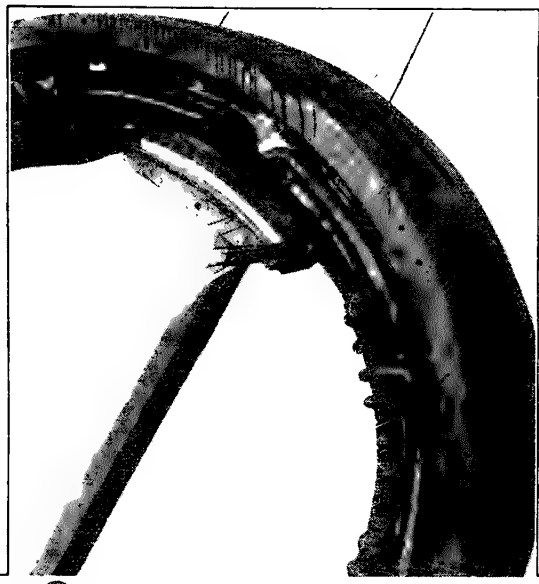


FIG. 27.

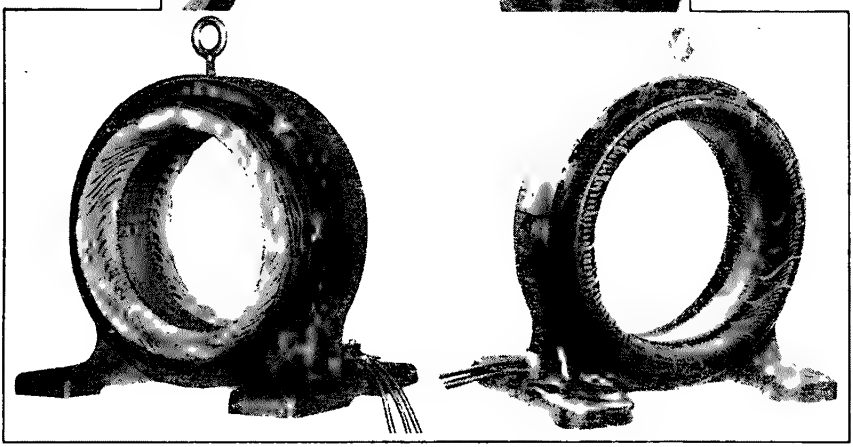


FIG. 28. FIG. 29.

FIG. 27.—Hand-wound, threaded type of winding.
FIG. 28.—"Fed-in" type—"mush coil" or one coil per slot.
FIG. 29.—"Fed-in" type—"diamond" or two coils per slot.
Stators with partly closed slots.

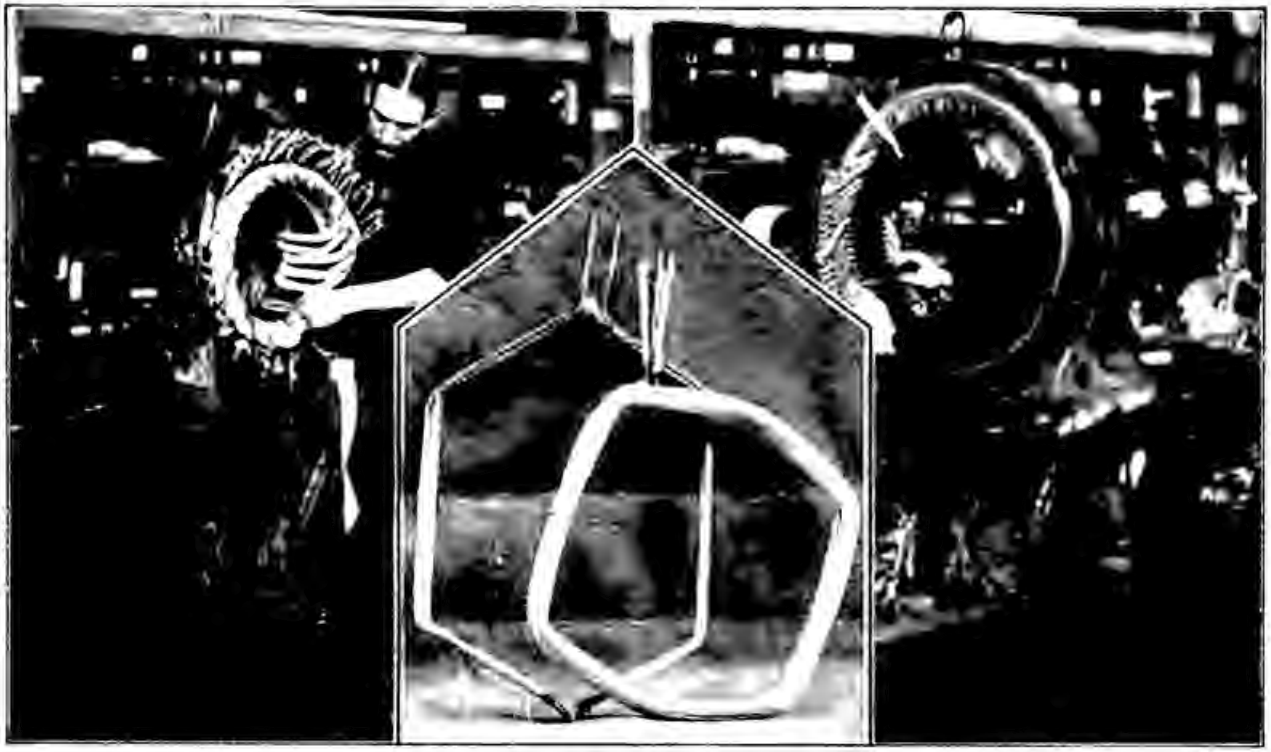
Examples of the different methods of connecting up such windings will be given in a later chapter.

The second type, or pushed-through winding, is illustrated in Figs. 25 and 26. It will be observed that the labor of bending

the coil ends and soldering each turn separately was considerable. This construction required somewhat more copper than the hand-wound type, but had the advantage that it could be better insulated. It has practically become obsolete in this country for induction motors, owing to the difficulty of making repairs.

The third type, or hand-wound, is illustrated in Fig. 27. It requires greater skill than any of the other types shown and somewhat more space in the slot than the pushed-through type, and great care has to be used to avoid skinning the wire in winding. It has an advantage over the pushed-through winding in having no soldered joint anywhere throughout the entire length of the coil. Both the pushed-through and hand-wound types require considerably more handwork than other types and are consequently better fitted for use abroad, where hand labor is cheaper than in this country. For this reason these types of windings are still very generally used in Europe, but are practically superseded by other forms in the United States.

The fourth type, or fed-in coils, is illustrated in Figs. 28 and 29 and is used almost universally for the stators of motors up to 15 and 20 hp., at voltages of 550 and under. It has been widely employed as a stator winding in larger capacities, but the present-day tendency is to confine its use to smaller ratings and make use of open slots on the stator above this classification. It has still a considerable field as a rotor winding where the mechanical forces acting on the winding make desirable the use of a semi-closed slot with overhanding tooth tips which give greater support to the coils than is possible with open slots with wedges or bands. This type of winding is employed in two forms: First, as shown in Fig. 30, the coil for which is shown at *A*, Fig. 31; and second, as shown in Fig. 32, with the corresponding coil at *B*, Fig. 31. In the first of these forms there is but one coil per slot and the shape of the ends of the coils is controlled largely by the winder as he puts them in place. In the second form there are two coils per slot, which have a definite and final form before being placed in the core and which resemble exactly, when completed, the well-known diamond-shaped coils wound into open slots. The first of these forms is suited to small and the second to larger machines. A modification of the second form makes use of a slot shaped as in Fig. 22 and is shown in place in Fig. 33.



B *A*

Fig. 30. One coil per slot.

Fig. 31.—Right hand or "A" coil for winding in Fig. 30. Left hand or "B" coil for winding in Fig. 32. Winding "fed-in" type.

Fig. 32.—Two coils per slot.

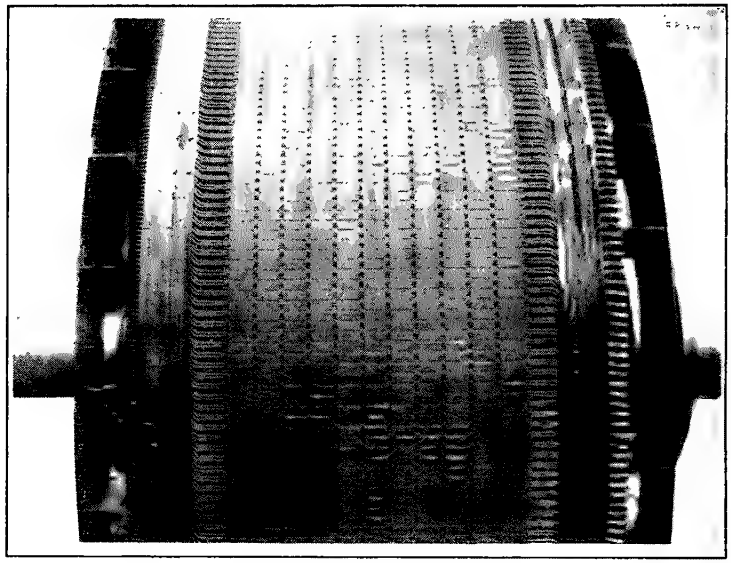


FIG. 33.—Rotor with slots as in Fig. 22 wound with strap coils of the "wave" type.

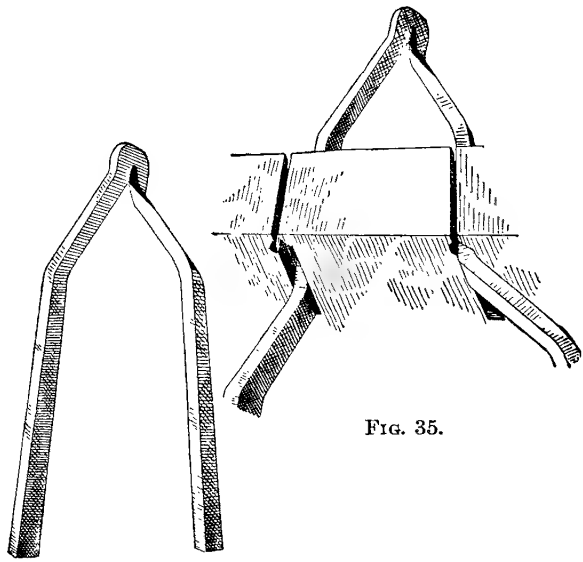


FIG. 35.

FIG. 34.

FIGS. 34 and 35.—Strap coil in partly closed slot bent to form after pushing through slots.

Each coil is completely insulated from ground and inserted in the slot as a unit, so that it might be considered as a combination of the coils from two adjacent open slots brought together and securely held by the overhanging tooth tip, which leaves an opening large enough for the passage of one complete coil while winding. It is considered one of the most satisfactory forms for use on the rotating part of machines up to the largest capacity. A similar winding has been made by forming the coil of one or two straps bent on one end only, as shown in Fig. 34, and insulating it. The straight sides of this coil are then pushed through two partly closed slots in an axial direction, and the two ends are bent to the proper form to connect with other coils, as shown in Fig. 35. This makes a good mechanical job, but is rather difficult to repair owing to the fact that several straps must be straightened out to get at the damaged coil.

Windings Used in Open Slots.—With open slots, as illustrated in Fig. 20, the most popular and widely used form of winding is that shown in Figs. 36 and 38, for which the coil is shown in Fig. 37. This is the well-known diamond coil, so-called from its shape, and is entirely formed and insulated before placing in the slots. It is also the simplest and easiest coil to wind and is used by designers wherever the conditions permit. The greater number of typical connection diagrams shown in this book have reference to windings of this general type, since they lend themselves so readily to changes of arrangement and various reconnections.

There have been many other modifications of coils or windings employed with both open and closed slots in making special machines or where unusual conditions justified their use, but the forms described cover the great majority of machines found in use today.

Master Diagrams for Polar Grouped Windings.—In discussing windings, frequent reference is made to the usual forms of connection. For this reason much space in this and the following chapters is devoted to illustrations of the typical forms of diagrams that are employed by all manufacturers in connecting induction motors. A passing consideration will indicate that there would have to be an indefinitely large number of these diagrams to cover all possible combinations. For example, machines are usually connected either two-phase or three-phase.

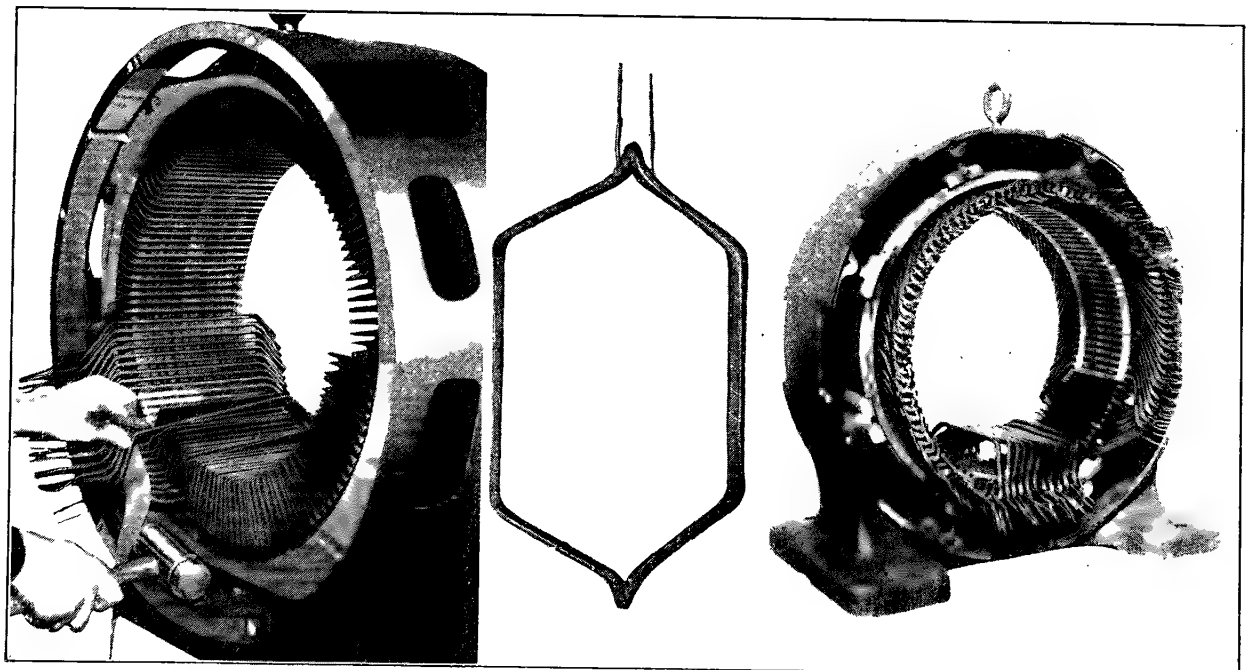


FIG. 36.—Method of placing coils in the core.

FIG. 37.—Typical "diamond" coil extensively used for induction-motor windings.

FIG. 38.—How the last group of coils is put in place.

Open-slot "diamond" coil windings.

The three-phase machines may be either Y (star) or Δ (delta), any winding may be in series or two groups in parallel, a six-pole machine may be three parallels or six parallels or a ten-pole machine may be five parallels or ten parallels, etc. With these fundamental elements alone, if speeds from two poles to fourteen poles are considered, there are 81 diagrams of connections required. These diagrams are shown in Chapter XVII. Then follows the fact that any one of these 81 diagrams may be used on a core having an indefinite number of different slots such as 24, 36, 60, 72, 90, etc., so it becomes evident that it would require a book of considerable proportions to include even the usual combinations encountered in ordinary commercial machines. Fortunately a simple system has been developed, and will be explained later, by which the number of slots can be eliminated from consideration in the group connections, since it affects only the number of coils that are connected together to form a pole-phase group and does not affect the connection of the ends of this pole-phase group to neighboring groups. As stated earlier in the chapter windings are divided, in general, into two classes with reference to whether they are used in partly closed slots or in open slots. The partly closed slot windings are again divided into four classes: (1) Bar and involute-end connector, (2) pushed-through, (3) hand-wound and (4) fed-in, or dropped-in.

So far as the polar connections are concerned, the bar and involute-end-connector windings may be handled in the same manner as a formed-diamond-coil winding used in open slots, since the bar may be considered as the straight portion of the coil and the involute connector as its diamond-shaped end. This would mean that these windings could be connected up into pole-phase groups and these groups in turn cross-connected according to any of the standard group connections in common use, of which a number will be shown in Chapter XVII.

"Wave" or "Progressive" Diagrams.—In addition to such connections, this type of winding has been often used as a wave or series-circuit winding such as is commonly employed in direct-current armatures. Windings of this type are illustrated in Figs. 39 and 40. It will be seen at once that a desirable feature of such a winding is its simplicity and compactness and comparative freedom from group cross connections. These connections are limited to the connections for the star or the corners of the

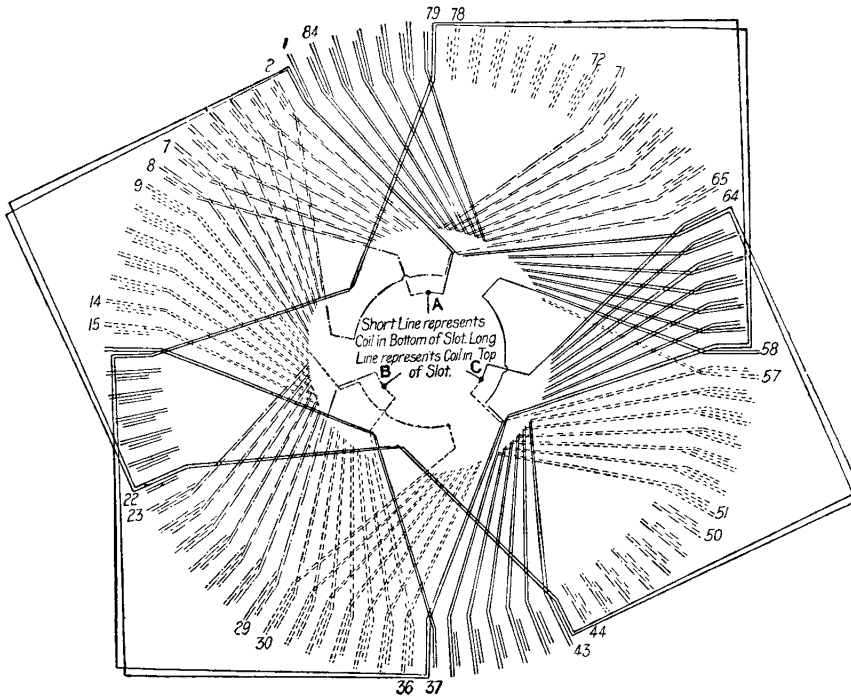


FIG. 39.—Typical "wave" diagram for three-phase, four-pole, series-delta connection.

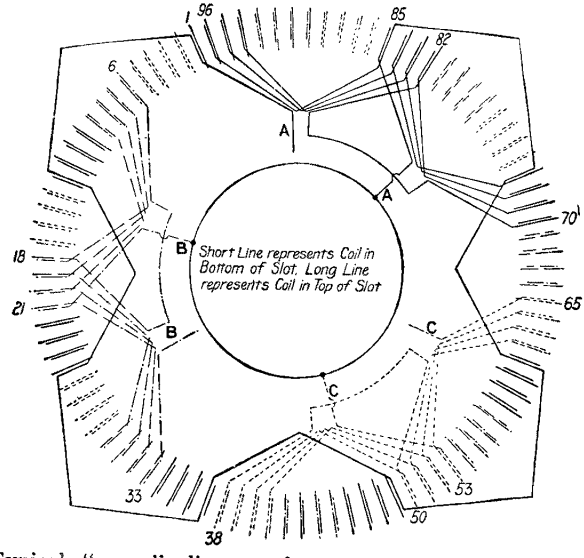


FIG. 40.—Typical "wave" diagram for three-phase, eight-pole, series-star connection.

delta, the leads, and one series connection in the middle of each phase. When compared with the mass of cross connections for the simplest form of pole-phase group winding, the advantage is apparent. It will be noticed that the windings shown in Figs. 39 and 40 are perfectly symmetrical and balanced at all points, the number of slots being an exact multiple of the number

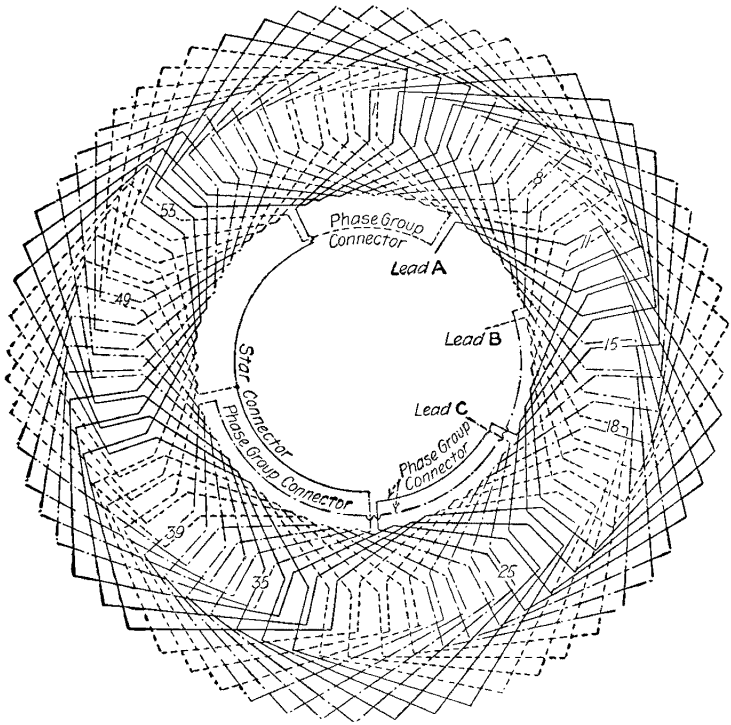


FIG. 41.—Special form of wave diagram for three-phase, six-pole, series-star connection.

of phases times the number of poles; this is true in practically all cases for this type of winding.

Standard Direct-current Form of Wave Winding Adapted to Alternating Current.—An interesting variation from the foregoing type is illustrated in Fig. 41 and is typical of a method of connection that has been widely employed, particularly on the rotors of motors of the phase-wound type. Here it will be seen that the number of slots, 62, is not an even multiple of 18 (3×6, phases times poles), but follows the same law as a direct-current, series or wave, armature winding; namely, the number of slots ± 1 divided by the number of pairs of poles must equal an integer,

and this integer divided by 2 is equal to the proper pitch, or throw, of the connector. In the case shown in Fig. 41,

$$\frac{\textit{Number of slots} \pm 1}{\textit{Pairs of poles}} = \frac{62 \pm 1}{3} = 21.$$

The proper pitch of the coil is $21 \div 2 = 10.5$ slots; that is, the throw should be 10.5. Since this is not physically possible, the throw is made 10 slots or 1 to 11 on one end, and 11 slots or 1 to 12 on the other end, giving an average of 10.5.

Assume that a bar in the bottom of the slot 1 is connected by the connector on the back of the core to the bar in the top of slot 12, and that the front end of the bar in slot 12 is in turn connected on the front end of the core to the bar in the bottom of slot 22 and this again on the back end to the bar in the top of slot 33. Tracing the winding through in this manner, after one complete circuit has been made around the core it will be found to connect to the bottom bar in slot 2 and for the second round to the bottom bar in slot 3 and so on, until finally, when all the slots are traced through both top and bottom, the last throw will close the winding on itself by connecting to the front end of the bottom bar of slot 1. This can be proved easily by setting down a table of numbers 1–12, 12–22, 22–33, 33–43, 43–54, 54–2, 2–13, etc., representing the path of the winding around the core as described until each number has appeared two times, or until $2 \times 62 = 124$ bars have been passed through. This would then give a completely closed winding, and if the middle point of each end connector were attached to a bar on a suitable commutator, it would represent exactly a direct-current series armature winding.

To employ this winding on alternating current, the proper phase leads must be brought out, and this can be accomplished in several ways. One method of doing this would be to leave the winding closed and bring out three-phase taps 120 deg. apart, as shown in Fig. 42, or four taps, as in Fig. 43. The first would give three-phase and the second two-phase. A second method is to open the winding at three proper places and use these three pieces to form the usual star or delta connection. This is indicated in Fig. 44. It must not be assumed that the winding is actually interrupted at the points A, B and C since each portion of the winding between these points actually runs com-

pletely around the core several times. This can be readily grasped if the table as set down in the foregoing is separated into three parts, each part having one-third of the total bars in it, or $\frac{62 \times 2}{3} = 41\frac{1}{3}$; say 40 bars in one section and 42 bars in the other two. The slight unbalancing so caused is, in this case, of no consequence.

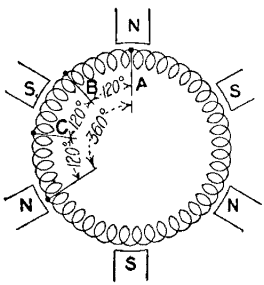


FIG. 42.

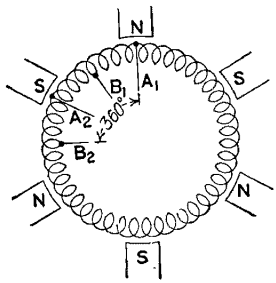


FIG. 43.

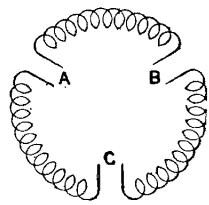


FIG. 44.

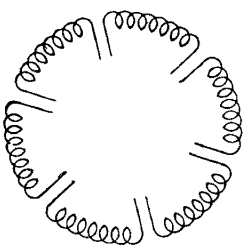


FIG. 45.

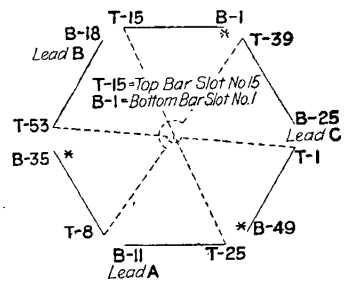


FIG. 46.

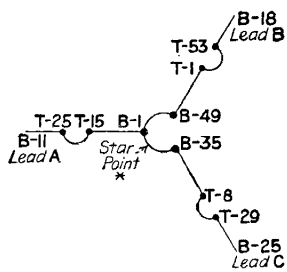


FIG. 47.

FIGS. 42, 43, 44, 45, 46 and 47.—Manner of connecting and bringing out leads of the winding in Fig. 41.

It is necessary to keep an even number of bars in each section for the reason that the connections are all on one end of the core and an odd number of bars in any section would mean ending that section on the back end of the core. Or, in other words, in tracing through the winding on the odd-numbered bars one is always going from the front to the back and on the even-numbered bars always coming from the back to the front, hence to end on the front an even number of bars must have been passed through.

In the connection shown in Fig. 41 a still different method is adopted by separating the winding into six sections, four of which have 20 bars in them and two have 22 bars each. These six

sections are connected in pairs in series and the three pairs connected in series star to form a three-phase winding. The reason for this is a more efficient use of the copper than either of the two preceding methods. This follows from the fundamental idea brought out in the first chapter that every induction motor is at the same time an alternating-current generator, due to the fact that the stationary windings are cut by the rotating field. The output of an alternating-current generator is measured by the product of the volts times the amperes. In Fig. 41 the copper will carry a certain maximum current. It then follows that to get the most out of it as an alternating-current generator, the windings must be made to generate the maximum practicable voltage, and this is the result accomplished by the connection in Fig. 41.

Voltage Relation of Individual Coils in This Winding.—In the complete closed winding, Fig. 41, each coil is generating a small voltage which is slightly out of phase with all its neighbors. The situation can be described as a polygon having 62 equal sides, each side representing the voltage of a single coil. Obviously, the maximum voltage would be obtained if we could roll out this polygon into a straight line and use one-third of its length for each of the three phases. This cannot be done in practice, but it can be approached as shown in Fig. 45. Here the circle represents the 62-sided polygon just mentioned. By dividing the winding into six pieces, the effective voltage of each piece is reduced to the equivalent of one side of a hexagon. By putting the opposite side of the hexagon in series and then the three pairs in series star, the winding is made to develop almost the maximum voltage. A slight gain could still be made in the same way by dividing the winding in 12 pieces and using these 6 pairs as a six-phase winding, but this is too complicated for ordinary use.

The connection shown in Fig. 41 is obtained practically by setting down a table, as previously stated, including all the bars and then dividing it into 6 pieces as nearly equal as possible, keeping an even number of bars in each. In this case sections 1, 2, 3 and 4 have 20 bars and 5 and 6 have 22 bars each. Section 1 is then connected with 4 for phase *A*, section 2 with 5 for phase *B*, and sections 3 and 6 for phase *C*. The proper ends of these connectors for star and the leads can be determined from Figs.

46 and 47, the numbering on which corresponds to that on Fig. 41. A little practice in this way will suggest how different three-phase connections could be made for star or delta or series or parallel to accommodate different voltages and how corresponding two-phase or even six-phase connection, could be obtained.

Concentric-coil Windings.—In the pushed-through type of winding, previously described, the coil is formed in the shape of a U and the two branches are simultaneously pushed through the proper slots in the core, after which the ends are bent toward each other and the individual conductors connected in series. In the hand-wound type a single long wire is threaded around and back through two slots until the complete coil is formed. The completed coil is practically identical in the two types, and the completed winding takes the form shown in Figs. 48 to 53 inclusive. Figure 49 is typical of a two-phase arrangement. The coils are concentric and there are two shown per group, but in practice on induction motors as many as five or even six have been used. The coils that are inside on one end of the core are outside on the other end, thus insuring symmetry and equal resistance in the two phases. Figure 48 shows a cross section of the core and coils on the line XX and indicates the relative position of the two banks of coil ends.

Figure 51 shows a three-phase winding similar to the two-phase Fig. 49 except that only one coil per group is shown; however, there might be four or more concentric coils per group, as in the two-phase. It will be noticed at once from this figure and Fig. 50 that the winding is not so simple as the two-phase. Owing to the passing of the coils at the ends of the core, three banks, or tiers, are necessary instead of two, and the coil ends are correspondingly longer. It will be noticed that the A phase occupies the middle tier all the way through and the B and C phases are alternately in the inside and outside tiers. In this manner the resistance is kept nearly equal in the three phases.

In order to be able to wind the three-phase with a two-bank winding similar to the two-phase, the scheme shown in Fig. 53 is employed. It can be seen that there are the same number of slots as in Fig. 51 and that both are three-phase four-pole series-star windings. However, Fig. 53 has only two tiers at the ends and has two coils per group instead of one, but only two

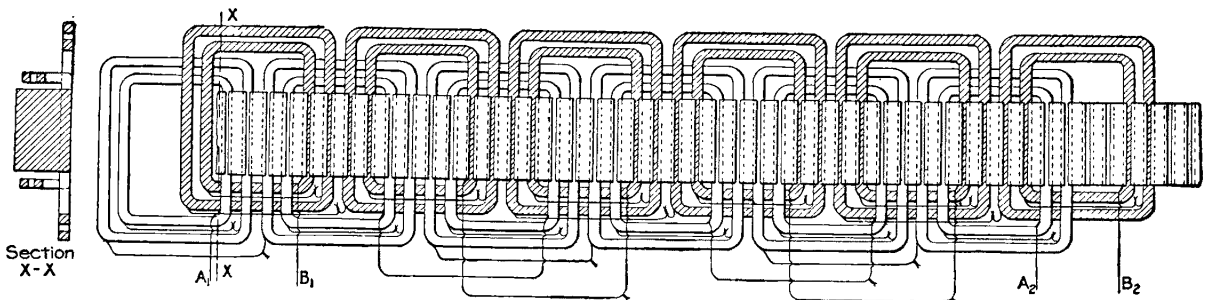


FIG. 48.—Cross section of winding in Fig. 49.

FIG. 49.—Typical two-phase, two-bank winding.

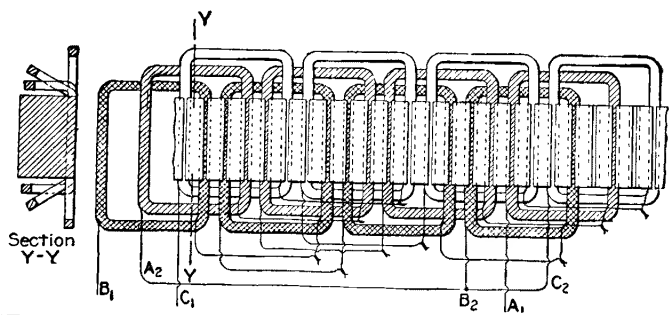


FIG. 50.

FIG. 51.

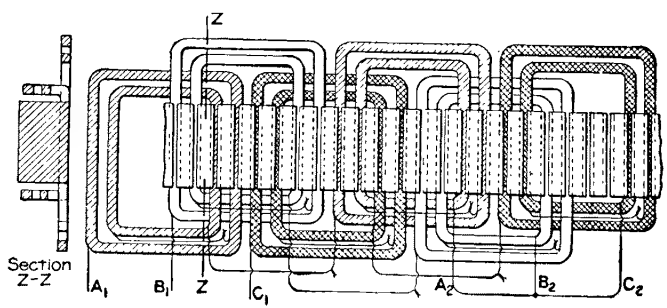


FIG. 52.

FIG. 53.

FIG. 50.—Cross section of winding in Fig. 51.

FIG. 51.—Typical three-phase, three-bank winding.

FIG. 52.—Cross section through winding in Fig. 53.

FIG. 53.—Three-phase winding with only two banks, accomplished by connecting for consequent poles.

Pushed-through windings.

groups per phase instead of four. This is what is called a "consequent-pole winding," because the current passes in the same direction through all the coils forming, for example, two north poles in each phase. Since there cannot be a north pole without a corresponding south pole, the magnetism returns between the groups in each phase, thus forming the two south poles, or four in all. This winding is simpler to make than Fig. 51, mechanically, but has some slight electrical disadvantages. Figure 52 is a section through the core and winding on the line *ZZ* and indicates the relative positions of the two banks of coils.

Rearrangement of Concentric-coil Windings.—It will be seen that these concentric-coil windings do not lend themselves readily to rearrangement or reconnection for different poles or phases, and this is one reason why they have gradually fallen into disuse. Two-phase windings such as Fig. 49 can sometimes be connected in "T" and run on three-phase, and mention of this will be made in a later chapter. Also, a comparison of Figs. 49 and 53 indicates that the winding in Fig. 49 might be connected for three-phase eight poles by a consequent-pole connection similar to Fig. 53, since the total number of groups, being 12, is half of 3×8, and this lines up with Fig. 53, where the total number of groups is 6, or half of 3×4.

Where the coils are of the closed type similar to "diamond" coils used in open slots, they may be grouped and connected by the usual diagrams for that type, which will be discussed under open-slot windings. There is, however, a large class using one- or two-turn coils of the open end, or "wave," type which form very interesting windings, two of which are shown in Figs. 39 and 40. This type of winding is believed today to be the form best adapted to the rotating member of phase-wound motors up to the largest sizes. Since they are perfectly symmetrical, they can be equally well employed in the stator, where the design permits a number of conductors not exceeding four per slot. These diagrams are practically self-explanatory, but their great utility and wide employment merit a brief comment. They are typical three-phase diagrams connected both star and delta. Three-phase is chosen as it is suitable for either stator or rotor and is oftenest met with. Figure 39 shows a four-pole series-delta winding, but it may be equally well connected parallel-star. The winding, Fig. 39, has four conductors per slot.

In Fig. 40 is an eight-pole series-star connection where the two wires in the top of the slot are connected in parallel, also the two in the bottom of the slot, to form one conductor, or a total of two conductors per slot.

Wave Windings.—In these windings it is of interest to note that the number of cross connections is a minimum, being reduced to the star or delta connection, the leads and one short connection in the middle of each phase. Such conditions are ideal for a rotor, and when the coils are placed in a slot with the tip overhung from one side, the winding forms one of the best mechanical jobs for a rotor that is known at the present time.

Passing to Open-slot Windings.—It is the object of the rest of this chapter to explain the method of connecting up these windings with sufficient examples to make it possible to lay out such a diagram when one is not immediately available. It should be borne in mind that such diagrams can also be used with partly closed slot windings when they are of the same form as "diamond coils." Such for example are the so-called "fed-in" or "dropped -in" coils, which are really "diamond" coils except that they are placed in partly closed slots, one wire at a time, through the small opening at the top of the slot. Such also are the strap coils referred to earlier, where the slot is half open and the tooth tip overhangs from one side. While there are four separate coils in such a slot, each coil is insulated from ground and for purposes of connecting up may be considered the equivalent of an open-slot winding laid in twice the number of slots. Such a winding is shown in Figs. 22 and 33. Bar-and-end-connector windings when of the "lap" and not the "wave" type are also connected in the same manner.

Standard "Lap" Winding.—A completely developed picture of an open-slot winding is shown in Figs. 54*a* and 54*b*. The straight radial lines are shown in pairs. These radial lines represent the straight parts of the "diamond" coils. The shorter line of each pair represents the side of the coil lying in the bottom of the slot and the longer line the side of the coil in the top of the slot. Taking Fig. 54*a*, for example, before any cross connecting was done there were 24 separate coils with the beginning and ending of each coil projecting at the end of the winding as shown in Fig. 55, which is the winding represented in Fig. 54*a* in place in the stator except laid out flat. Since it is to be connected for

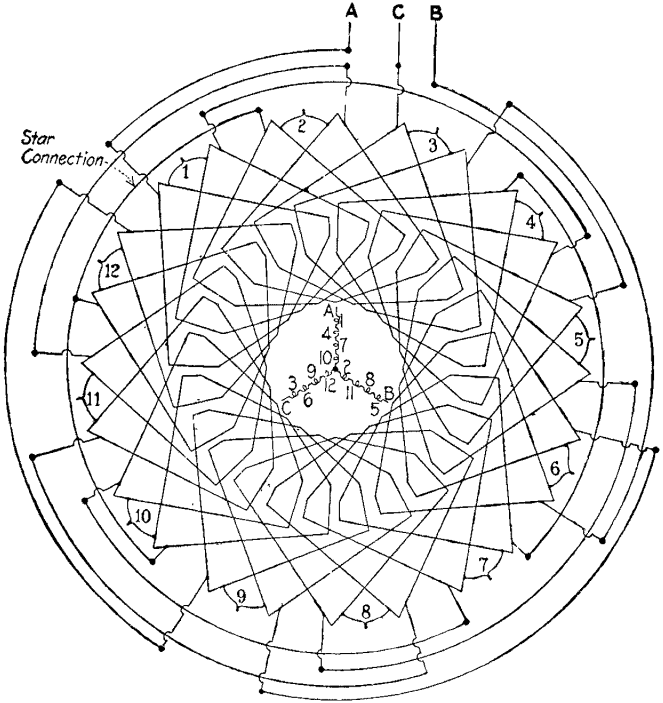


FIG. 54*a*.—Typical three-phase, four-pole, series-star winding for an open-slot stator.

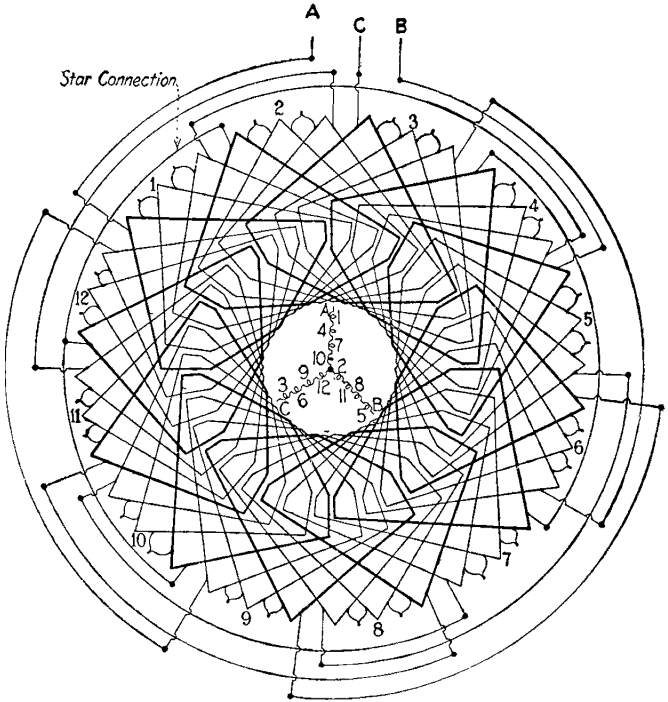


FIG. 54*b*.—Similar connection to Fig. 54*a* except coils with heavier insulation where phase changes are shown in heavy lines.

three-phase four poles, there is a total of $3 \times 4 = 12$ pole-phase groups required and this results in $24 \div 12 = 2$ coils per group. The first step, therefore, is to connect the coils in pairs, each pair forming a pole-phase group, as in Fig. 56. These coil-to-coil

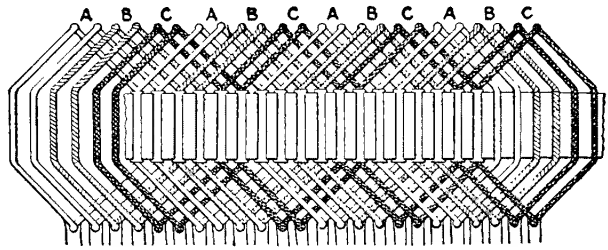


FIG. 55.—Coils for the winding in Figs. 54*a* and 54*b* shown in place ready to connect.

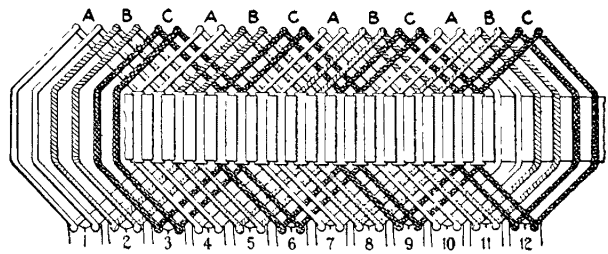


FIG. 56.—Same coils "stubbed up" or connected into pole-phase groups.

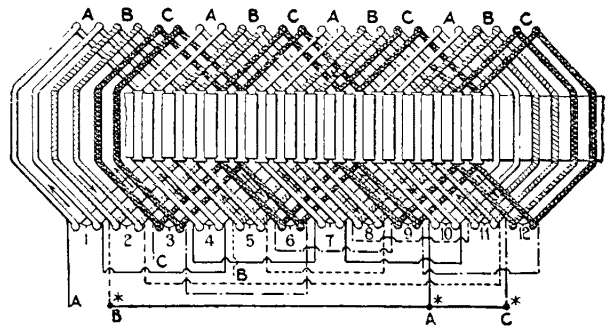


FIG. 57.—Completed winding same as Figs. 54*a* and 54*b*.

connections, or stubs, are shown at the group numbers. The resulting 12 pole-phase groups are then cross-connected to form the completed winding as in Figs. 54*a* and 57.

A comparison of Fig. 54*a* with Fig. 54*b* shows that the cross connections or pole-phase-group connections are identical, the only difference between the two being that Fig. 54*b* has 36 coils

total instead of 24 and hence there are three coils in each pole-phase group instead of two. The coils shown in heavy lines, Fig. 54*b*, represent the coils having heavier insulation, where the phases change between adjacent coils and will be referred to in a later chapter. A consideration of these figures leads at once to two conclusions: First, that such a form of diagram as Figs. 54*a* and 54*b* is entirely too complicated for use by the average winder and a diagram like that in Fig. 57 requires too much time to make and is therefore too expensive. Second, since the actual cross connections themselves are not affected by the number of individual coils in the pole-phase group, the entire picture shown

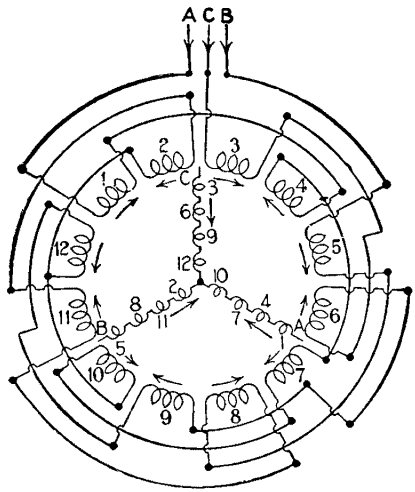


FIG. 58.—Schematic, four-pole, series-star diagram exact equivalent of pictured winding in Figs. 54*a*, 54*b* and 57.

in Figs. 54*a* to 57 may be replaced by the simple diagram shown in Fig. 58. The spiral lines representing the pole-phase groups, which are numbered to correspond with Figs. 54*a*, 56 and 57, can be imagined as being the coils which form the pole-phase groups. It is obvious that there might be any number of coils connected in series to form the groups. If, for example, the complete machine instead of having 24 or 36 slots had 48, 60, 72 or 96 slots, the cross connections of the groups in any case would be as shown in Fig. 58. A diagram of this type is therefore always used for such windings, since it can be used for any three-phase four-pole machine independently of the number of slots in a particular machine.

Schematic Diagram.—Attention is called to the small "Y" diagram in the center of Figs. 54*a* and 54*b* which is also reproduced in Fig. 58. It has no electrical connection with, but is the "schematic equivalent" of, the rest of the diagram. It is the designing engineer's imaginary conception of the cross connections reduced to their simplest terms. By comparing the numbers of the groups on this small diagram with the corresponding numbers on the larger diagram, it will be seen that each pole-phase group is shown in its proper phase and with the proper direction of its ends toward the lead or toward the star connection. The arrows shown on the larger diagram, Fig. 58, and also on the small schematic equivalent represent a simple and positive check as to whether the connections to the different groups are correct.

Check for Connecting Proper Ends of Phases to Star Point.—There is a danger in a three-phase winding that the three phases may be connected in a 60-deg. relation instead of a 120-deg. relation, or as it might be expressed on the diagram, Fig. 58, there is danger that the wrong end of the *B* phase, for example, may be connected to the star point. As a check against this each phase is traced through, starting from the lead or terminal and proceeding to the common, or "star," point at the center of the winding. As the successive groups are passed through, an arrow is placed on each as shown, indicating in which direction that group was passed through. When all three phases have been traced through and the arrows on the groups are inspected, the diagram is correct if the arrows on adjacent groups reverse; that is, if they are alternately clockwise and counter-clockwise in passing around the winding. This check should be studied over and thoroughly mastered, as it is the one check that the author has found in fifteen years of practical experience is always reliable and easily applied. The only exception to this check is the case of consequent-pole machines, to be described in another chapter, but these are so special and so infrequently met with that they may be practically put out of the consideration and the check be regarded as almost universal.

It is the common practice of all manufacturers to send out machines that can readily be connected for either one or two voltages. This is accomplished by a series or parallel arrangement and can be understood by comparing Figs. 58 and 59.

By looking at the small "equivalent" diagram in the center, it will be seen that there are twice as many groups in series between the terminal leads in Fig. 58 as there are in Fig. 59. This means that, if Fig. 58 is proper for 440 volts, Fig. 59 would be right for 220 volts. The idea was given in an earlier chapter that one function of the winding was to generate the counter-electromotive force. It can be seen at once that, if the coils as connected in Fig. 58 are generating 440 volts, they will obviously generate only half as many, or 220 volts, connected as in Fig. 59. As

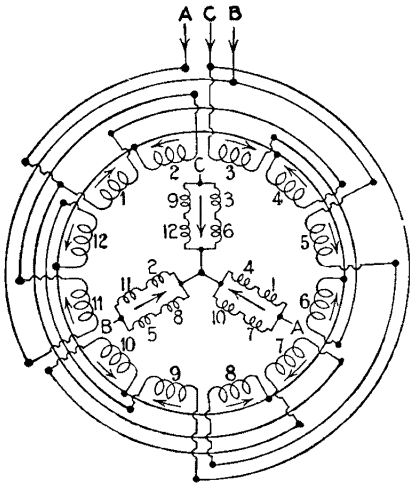


FIG. 59.—Showing the diagram Fig. 58 reconnected from series to parallel star.

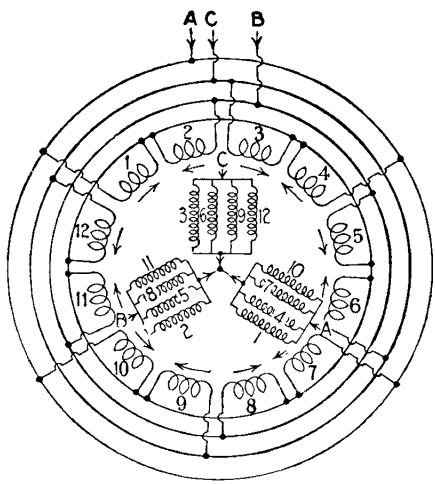


FIG. 60.—Showing the diagrams of Figs. 58 and 59 reconnected to four-parallel star.

another consideration, it is seen that if the motor has the same horsepower at both voltages, it will have twice the number of full-load amperes at 220 as it has at 440 volts. This is properly taken care of, as will be seen from Fig. 59, since the winding being doubled has twice the copper cross section in Fig. 59 that it had in Fig. 58.

If the number of poles in the machine is divisible by 4 as, for example, 4, 8, 12, 16, etc., the winding may be put in 4 parallels as shown in Fig. 60 and by comparison with Figs. 58 and 59 would be good for 110 volts at the same horsepower. The increased current at 100 volts is again taken care of by providing 4 times the copper section, as shown. This same principle can be extended, and when the number of poles for which the machine is wound can be divided by 6, it is possible to have the winding

connected for 3 parallels or 6 parallels, as shown in Figs. 61 and 62, respectively. If divisible by 8, there could be 2, 4 or 8 parallels, and if divisible by 10, there could be 2, 5, or 10 parallels. It will be explained in a later chapter on "Changes in

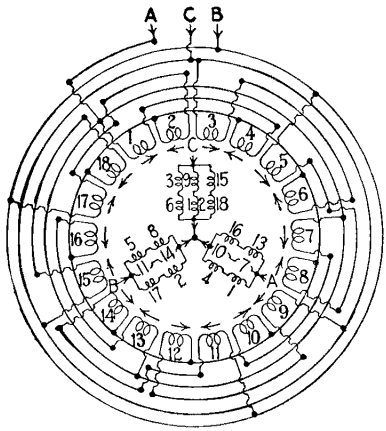


FIG. 61.—Three-phase, six-pole winding connected three-parallel star.

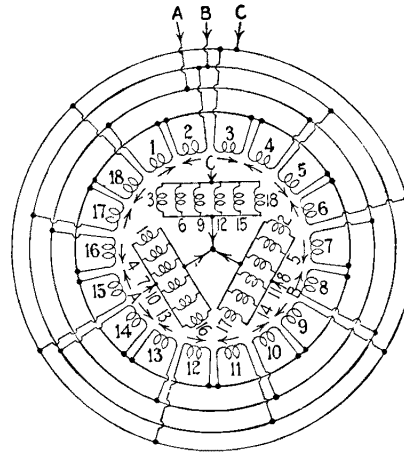


FIG. 62.—Three-phase, six-pole winding connected six-parallel star.

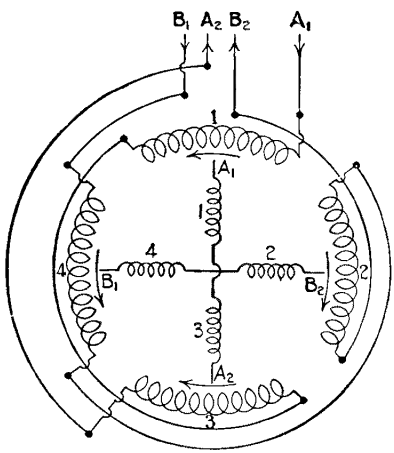


FIG. 63.—Two-phase, two-pole winding series connection.

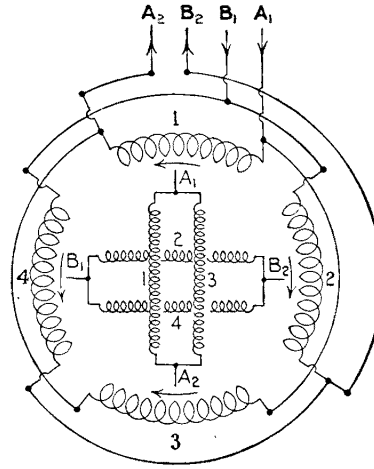


FIG. 64.—Two-phase, two-pole winding connected in two parallels.

Voltage" that these possible changes when considered with the possibility of "star" or "delta" allow in many cases the reconnecting of motors for new conditions.

How to Draw a Diagram to Suit Any Case.—As regards the number of possible diagrams, these multiply very fast. As an

instance are shown the diagrams, Figs. 63, 64, 65, 66, 67 and 68. Here the simplest case is studied—that of two poles—and when two- and three-phase are considered, series and parallel, and

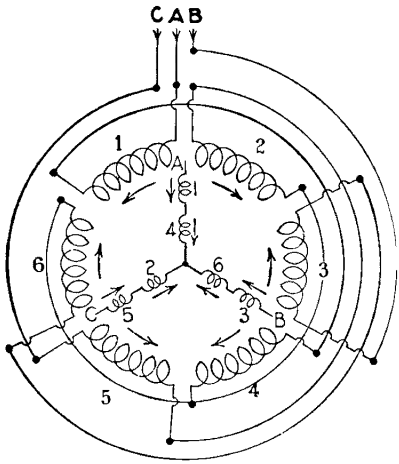


FIG. 65.—Three-phase, two-pole winding connected series star.

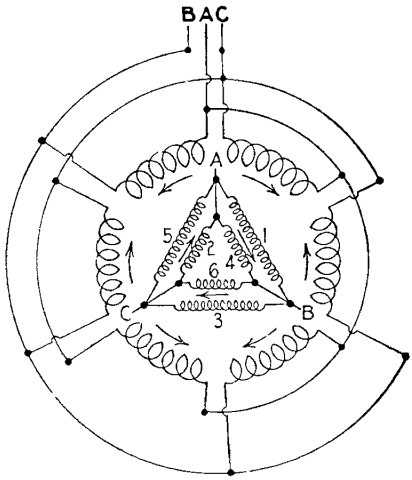


FIG. 66.—Three-phase, two-pole winding connected parallel delta.

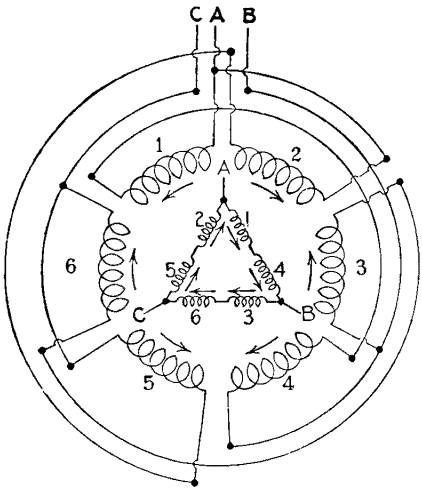


FIG. 67.—Three-phase, two-pole winding connected series delta.

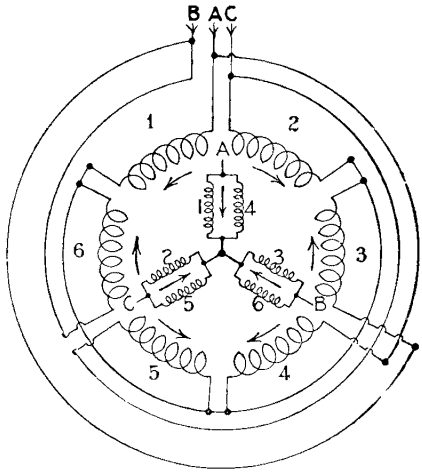


FIG. 68.—Three-phase, two-pole winding connected two-parallel star.

star and delta, there are six possible diagrams of connection, as indicated. Considering for the moment a 12-pole winding, there are possibilities for series, 2-parallel, 3-parallel, 4-parallel, 6-parallel and 12-parallel groups, which with two- and three-

phase and star and delta give 18 diagrams, just for 12 poles It becomes plain that it is desirable to analyze these diagrams and arrive at a simple scheme by which any one can be drawn

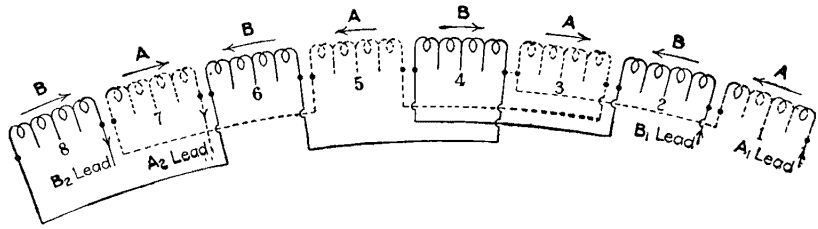


FIG. 69. Two-phase, four-pole series connection.

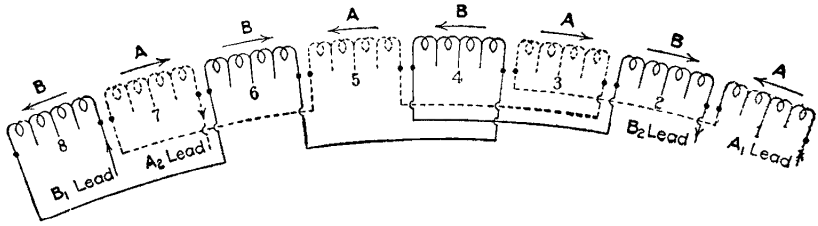


FIG. 70. —Same as Fig. 69 except "B" phase reversed.

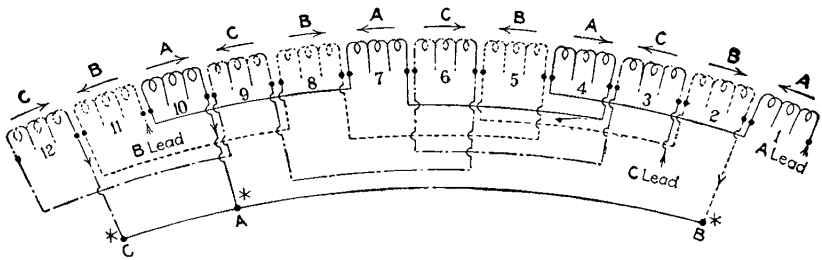


FIG. 71.—Three-phase, four-pole series-star connection.

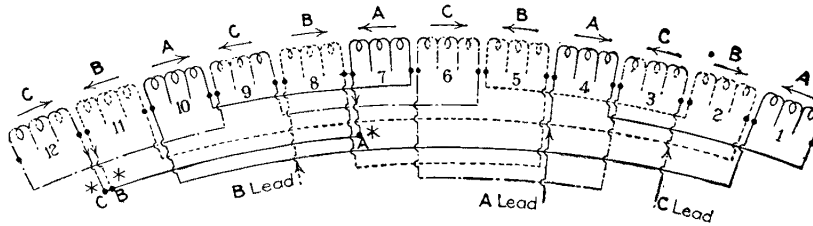


FIG. 72.—Same as Fig. 71 except leads brought out from different groups. General scheme of laying out pole-phase-group diagrams.

at need without the necessity of relying on a bulky collection of diagrams which may not be available when needed. In Figs. 69, 70, 71 and 72 is shown the method of laying out diagrams of this general nature. The first operation in making the con-

nection is to connect the individual coils into pole-phase groups. There are as many coils in series in each group as the total number of coils in the winding divided by the number of phases times the number of poles. In Figs. 69 and 70 this is assumed to be four coils, and hence each pole-phase group is shown as having four individual coils in series. The next step, as shown in Figs. 69 and 70, for a two-phase machine is to letter the alternate groups *A*, *B*, *A*, *B*, etc., to designate the groups in the *A* phase from those in the *B* phase. The next step is to put on the arrows, as shown, in groups of two pointing in the same direction on two successive groups of coils. It does not matter what group is used to start with. The only essential is that there shall be first two arrows pointing clockwise and then two arrows pointing counterclockwise. The third step is to show the connections to the different groups so that the current at any given instant will pass through the groups in the same direction as the arrows. If this method is followed in laying out the connections of two-phase windings, the result will always be a diagram that shows the pole-phase groups connected in their proper relation.

Figure 70 is produced to compare with Fig. 69 to verify the statement already made that the arrows may be placed beginning with any group. In Fig. 69, beginning at the right, there are two arrows counterclockwise on groups 1 and 2, whereas in Fig. 70 the first two arrows counterclockwise are on groups 4 and 5. The only effect of this is to reverse the *B* phase, or in other words, the motor in Fig. 70 would have the opposite rotation of the motor in Fig. 69. Since this is at once corrected by reversing the leads of one phase outside the motor, it will be seen that, if the internal connections are made according to Fig. 69 or 70, the motor will operate properly in all respects.

Three-phase Star Diagrams.—The three-phase winding shown in Fig. 71 is even simpler. Here there are three coils per pole-phase group, and as in the two-phase winding the individual coils are first connected into pole-phase groups and the groups lettered consecutively *A*, *B*, *C*, *A*, *B*, *C*, to separate the phases. Then the arrows are put on as shown, first clockwise and then counterclockwise, alternately, beginning with any convenient group, it matters not which. The lines are then drawn in for the group connections as shown, following the convention that the arrow enters the lead or terminal of each phase and goes toward the

star or common connection at the center of the winding. If this rule is followed, the connection will be correct and it is applicable to any combination of numbers of slots and poles. By keeping in mind either Fig. 69 or Fig. 70 for two-phase and Fig. 71 for three-phase, all diagrams of this type are mastered and can readily be reproduced at a moment's notice.

Delta Diagrams.—In checking a delta diagram, check it first as if it were a star diagram and then form the delta by connecting the star end of the *A* phase to the *B* lead, the *B* star to the *C* lead and the *C* star to the *A* lead. These three connections will be the delta points from which the three external leads are brought out. Another method of checking where it can be handled without confusion is to imagine the current flowing around inside the closed delta. The arrows on adjacent pole-phase groups will then alternate in direction as in the check on a star winding. This latter check may be applied to Figs. 66 and 67 by starting from terminal *A*, or any terminal for that matter, and following around through all the pole-phase groups back to *A*. For example, in Fig. 67, starting from *A* terminal, follow through group 1, then through 4, 3, 6, 5 and 2 back to *A*; thus a closed circuit has been made through all the groups in the direction of the arrows.

A further consideration of the arrows on the pole-phase groups of Fig. 71 shows that there might be a number of different connections, all correct, which check with these arrows and differ only as to the particular group from which the lead or the star connection is taken off. In fact, the lead or the star connection may be taken off from the proper end of any pole-phase group in a given phase so long as the cross connections, when followed through, give the alternate arrows as shown. Fig. 72 is added to show one of these possible connections just as correct as Fig. 71, but with the leads and stars taken off from different pole-phase groups. Referring to the winding, Fig. 57, and again applying this rule, it will be found to hold good as indicated by the arrows. This demonstrates conclusively the correctness of this method of checking three-phase diagrams of this type.

CHAPTER IV

ROTOR WINDINGS

Fundamental Conceptions.—Much has been written about the induction motor, but the greater part has dealt almost exclusively with the primary winding and its modifications, and has assumed the rotor to be of only passing interest. However, just how and why the rotor operates is still a mystery to many people, especially since it has no electrical contact with the line.

The rotor receives all its power from the stator through the action of the magnetic lines of force; and at standstill it corresponds to the secondary winding of a transformer. In the transformer the primary and secondary windings are placed as close together as possible, and are held firmly in place so that they have no relative motion when current flows in them. All the power which is supplied to the primary as electrical energy is delivered by the secondary as electrical energy, neglecting losses. Although a force is exerted, no mechanical power is delivered by the secondary coil, since the coil is kept from moving; both force and motion together being required to produce power.

If the secondary coil of the transformer were not held in position, but allowed to move, mechanical power would be delivered by it, since there would be both force and movement. The electrical output from the secondary coil would then be less than the electrical input to the primary by the amount of mechanical power delivered. In other words, the secondary coil is inherently able to deliver both mechanical and electrical power, the sum being equal to the power received from the primary, losses neglected. The transformer is designed to get electrical power from the secondary with no mechanical power, while the reverse is true of the induction motor. For this reason, a transformer may be called a special case of the induction motor; or an induction motor may be called a special case of the transformer.

Since no movement is desired between the two windings of the transformer, and as much as possible of the magnetic flux should link both windings, the transformer coils are usually interlaced and built up on a laminated core without an air gap. This construction reduces to a minimum the value of current necessary to produce the required flux in the core. On the other hand, the induction motor requires mechanical movement between the primary and secondary windings, and so must be constructed with an air gap between the two windings. This air gap is made as short as is mechanically practical, so that the current required to produce the necessary flux is as small as possible. This magnetizing current has a direct influence on the power factor of the motor, and is therefore one of the principal elements to be considered in the design of the induction motor.

In the usual construction of the induction motor, the primary winding is put in axial slots on the inner cylindrical surface of a laminated-steel core, known as the "stator" because it is stationary in operation. The secondary winding is put in similar slots on the outer cylindrical surface of a drum-shaped laminated-steel core which is mounted on a shaft, and known as the "rotor" because it rotates during the operation of the motor.[1] The radial clearance between rotor and stator cores is known as the air gap; it varies from 0.01 in. in small motors to 0.17 in. in very large motors. Most induction motors are made with an air gap of less than 0.047 in. By putting both windings as near the air gap as possible, the relative amount of flux which links one winding and not the other is kept to a minimum. This flux is called "stray" or "leakage" flux; it is the factor which determines the maximum overload torque of the motor, usually called the "pull-out" torque, its value being stated in per cent. of the full-load torque.

The primary winding is connected so that the alternating current flowing through it produces rotating zones of flux which correspond to poles; the flux crosses the air gap from primary to secondary under one pole and from secondary to primary under the next pole. This rotating field induces in the rotor conductors a voltage which is in one direction under one pole and in the

[1] The locations of the primary and secondary windings may be, and have been, reversed in commercial types of motors; this construction, however, is obsolete at the present time.

opposite direction under the next pole. This voltage is greatest at standstill and would become zero if the rotor were to revolve at synchronous speed.

Squirrel-cage Rotors.—Induction motor secondaries are of two general types—squirrel cage and wound rotor. The squirrel cage is the simpler and cheaper in construction. It consists

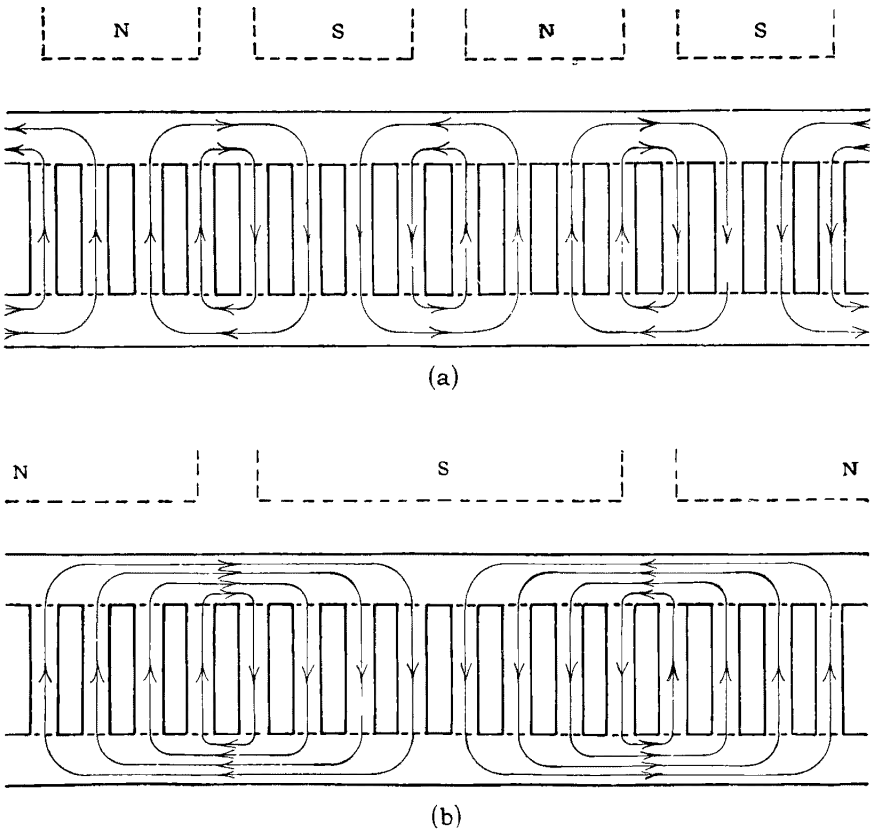


FIG. 73.—Current paths in squirrel-cage rotor winding. (*a*) With four-pole stator winding. (*b*) With two-pole stator winding. Note the increase in current density in the end rings with a decrease in the number of poles.

merely of conducting bars, usually of copper, connected together at each end by conducting rings. At standstill the voltage induced in each bar is seldom over ten volts, and in most motors is much less, so that insulation between bars and core is generally unnecessary.

The bars under each pole are in parallel, and the currents under adjacent poles are in opposite directions. The current flowing

out from the bars under one pole passes into the end ring, and dividing equally returns to the other side of the core through the bars under each of the two adjacent half poles, as shown in Fig. 73.

Since the squirrel-cage rotor is symmetrical and has no connections within itself which determine the relative directions of current flow in the rotor, the currents induced in the bars and the number of poles produced are determined only by the primary winding. Hence, a squirrel-cage rotor will operate in any primary of the correct mechanical dimensions, and will run at the speed of the primary field, minus the slip. However, the starting torque, which is dependent upon the secondary resistance, will not be the same when the rotor is used in stators with different numbers of poles, since the current paths in the rotor winding are changed; this alters the effective resistance of the secondary, as explained below.

The rotor resistance is made up of bar resistance and end-ring resistance. Since the bars under each pole are in parallel, the bar resistance per short-circuit path will be less as the number of poles is decreased and the number of bars per pole increases. But the number of parallel paths decreases as the bars per pole increase. These two effects balance one another, the net result being that the equivalent bar resistance does not change as the number of poles is varied. This may be illustrated by the following numerical example in which, for simplification, the number of bars assumed per pole is considerably below the number used in commercial machines.

Current Paths in Squirrel-cage Winding.—Assume that in Fig. 73 (*a*) represents a complete rotor winding which is being used with a four-pole primary, as indicated by the dotted poles. In Fig. 73 (*b*), the same rotor, is assumed to be used with a two-pole primary. The current paths are indicated by closed loops with arrows to show the relative instantaneous directions. There are 16 bars in the complete winding, and a resistance of 1 ohm per bar will be assumed. With four poles, there are four parallel current paths in the rotor, each path containing two series groups of two parallel bars. Each path has a bar resistance of $0.5 \times 2 = 1$ ohm; the total bar resistance of the rotor is then $1 \div 4 = 0.25$ ohm. With two poles, there are two parallel current paths, each path containing two series groups of four parallel

bars. Each path has a bar resistance of $0.25 \times 2 = 0.5$ ohm; the total bar resistance of the rotor is then $0.5 \div 2 = 0.25$ ohm, the same as for four poles.

Effect of Changing Number of Poles.—Changing the number of primary poles changes, however, the effective resistance of the portion of the winding which is included in the end rings. The cross-sectional area of the ring is constant, while the length of ring included by each path decreases and the number of parallel paths increases as the number of poles is increased. The net result is that the portion of the rotor resistance due to the end rings is approximately inversely proportional to the square of the number of poles. This is shown in the following example.

Calculating Resistance of a Squirrel-cage Winding.—Assume the same rotor as before, Fig. 73, used in the same primaries. Let the total resistance around each ring be 0.4 ohm. With four poles, approximately one-eighth of the resistance of each ring is included in series in each of the four parallel current paths of the rotor. (This is approximately the end-ring resistance for the mean path of the current; the approximation becomes more accurate as the number of bars on the rotor is increased.) The end-ring resistance in each path is then $\frac{0.4}{8} \times 2 = 0.1$ ohm; and the total end-ring resistance of the rotor is $\frac{0.1}{4} = 0.025$ ohm.

With two poles, approximately one-fourth of the resistance of each ring is included in series in each of the two parallel paths. The end-ring resistance per path is then $\frac{0.4}{4} \times 2 = 0.2$ ohm; and the total end-ring resistance for the rotor is $0.2/2 = 0.1$ ohm. This is four times the resistance for four poles; thus, using one half the number of poles causes the effective resistance of the end rings to be increased four times.

In general, the greater the resistance of a squirrel-cage winding, the greater is the starting torque, and the slower is the speed corresponding to any torque. High-resistance rotors are sometimes required for applications which require a high-starting torque, or a large drop in speed when the load comes on. Some machines with a small normal drop in speed can be changed to give a large drop in speed by machining the end rings to a small cross section; but from the above discussion it is readily seen

that only motors with a small number of poles can have the total resistance of the motor changed to any extent by changing the cross section of the ring. Machines which have large numbers of poles have relatively little resistance in the ring, and the ring would have to be machined to a very small section to cause any appreciable increase in the total rotor resistance. If this were done, the ring would be mechanically weak, and the high losses in the small amount of material would soon burn it up. High-resistance windings for slow-speed multipolar machines are made by using bars of small cross section or of low conductivity, and rings made of a low-conductivity alloy.

In actual practice, the size and number of bars and the size of the rings are more or less standardized, so that the required total resistance is obtained by choosing different combinations of available parts. Thus, no definite percentage of the total resistance is put in the bar or ring. For this reason, one motor can have its secondary resistance, and consequently its starting torque, increased by machining the end rings, while another motor may show but little increase in torque after machining off a large portion of the rings.

Calculating Resistance of Rotor Winding in Terms of Stator Winding.—If the effective resistance of the rotor is to be changed, either by machining the end rings or by a change in number of poles, it is desirable to know the relative resistances before and after the change. This can be determined in terms of the primary resistance by the following equation. Usually, the equivalent secondary resistance of a squirrel cage is of the same general order as the primary resistance, *i.e.*, the primary and secondary copper losses are usually about equal. The equivalent primary resistance of the secondary winding is the actual resistance of the complete rotor winding multiplied by the square of the ratio of the effective conductors on the primary and secondary. This equivalent resistance is also used, together with the primary resistance and primary reactance, to determine the starting torque and starting current. It is calculated by means of the following formula:

$$r_2 = \frac{(K \times W_1 \times CF)^2}{P \times 10^7}(B + R + J)$$

where $B = \dfrac{l \times c}{A_B \times W_2 \times Cond.}$ = secondary bar resistance $\times 10^7$.

$R = \dfrac{2 \times D \times c}{A_R \times \pi \times p^2 \times Cond.}$ = secondary ring resistance $\times 10^7$.

$J = \dfrac{2c_1}{W_2}$ = $10^7 \times$ resistance of joints between bars and ring. (Can be neglected with brazed or welded joints.)

K = primary distribution factor (0.955 for three phase; 0.901 for two phase).

W_1 = number of primary series conductors (all phases).

W_2 = number of secondary bars (total).

CF = chord factor of primary coils.

P = number of phases in primary winding.

l = length of bar in inches.

c = constant = specific resistance per inch-cube of copper, at correct temperature, times 10^7.

$Cond.$ = relative conductivity (copper = 1.00).

D = mean diameter of ring in inches.

A_B = cross-section area of bar in square inches.

A_R = cross-section area of ring in square inches.

p = number of poles.

c_1 = constant = $10^7 \times$ resistance of one bar-ring joint.

Mechanical Construction of Squirrel-cage Windings.—In the mechanical development of the squirrel-cage winding, many different designs were brought out, and a few of those which have been used are shown in Fig. 74. Some of these types readily allow changes in the number of slots and size of rings, while others involve a large tool cost when such changes are made.

In the type shown in Fig. 74 (*a*), the bars are of square or rectangular copper, drilled and counter-bored at each end. Paper insulation is used on the portion in the core. The end rings are usually of some resistance alloy and are cast with separate lugs for each bar. Vanes for cooling are often cast integral with the ring. Bolts and nuts hold the bars tightly to the outside of the ring; lock washers are used to keep them tight, and to take up the difference in expansion of bolt and ring with temperature changes. A rotor of this type is shown in Fig. 75.

In the type shown in Fig. 74 (*b*) the bars are the same as in (*a*), except that they are tapped for the bolts, usually two per bar for each ring. Plain rings are used, and these are drilled to correspond with the holes in the bars. Tap bolts and lock washers are used without nuts. The ventilating vanes are mounted on the spider, as shown in Fig. 76.

In some small motors, the construction shown in Fig. 74 (*c*) is used. The bars are keystone-shaped or rectangular, and are

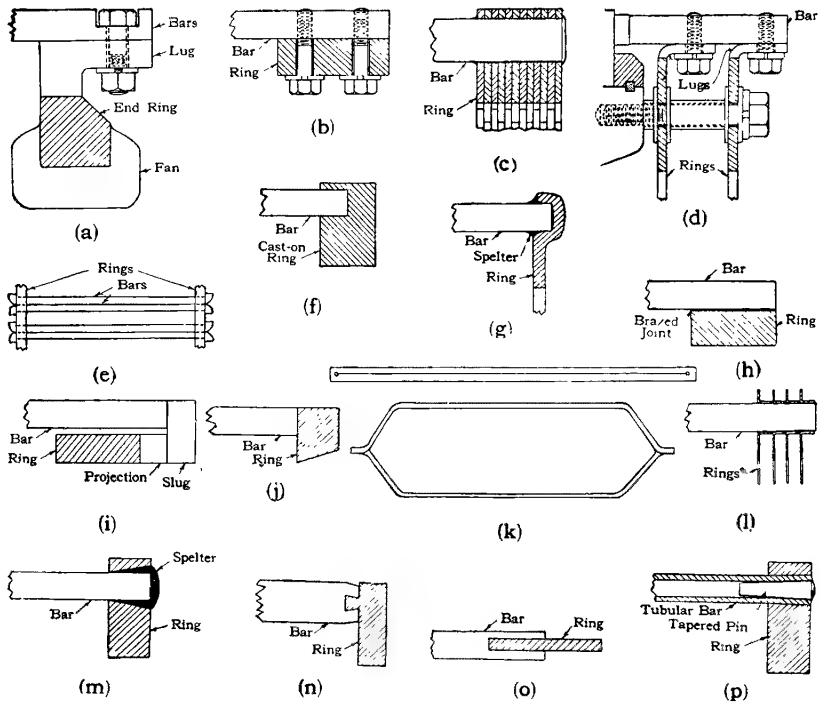


FIG. 74(*a*) to (*p*).—Some of the mechanical constructions which have been used in squirrel-cage windings.

insulated from the core with paper. The rings consist of several layers of sheet-copper segments having holes to fit the bars. These segments are pressed onto the bars one after the other until built to the desired thickness. The ends of the bars are peened, and the rotor is afterwards dipped in solder to give good electrical contact.

In the type shown in Fig. 74 (*d*) sheet metal is used for the rings, with usually two rings at each end. The rings for all but

the largest sizes are punched from a solid sheet, with the outer edge notched to produce lugs for bending over and bolting to the bottoms of the bars. The ends of the bars and the tips of the rings are tinned to give good electrical contact. This construction gives a large radiating surface, as shown in Fig. 77.

FIG. 75.—An early type of squirrel-cage winding. The end rings are cast with vanes for cooling and with separate lugs for bolting to the bars. This construction is shown in Fig. 74(*a*).

Some very small rotors use for the end rings a heavy punching at each end of the laminations, and have bars consisting of two straps side by side in each slot, as shown in Fig. 74 (*c*). The ends of the two bars in each slot are separated like the ends of a cotter pin, and are then bent over to lie flat on the end rings. The rings also act mechanically as end plates, and the bars as rivets, as shown in Fig. 78. The bar length is such that the ends from

adjacent slots do not interfere. After assembly, the rings and bar ends are tinned to give good electrical contact.

Another type of construction, shown in Fig. 74 (*f*), uses various alloys of different conductivities for the rings, which are cast to

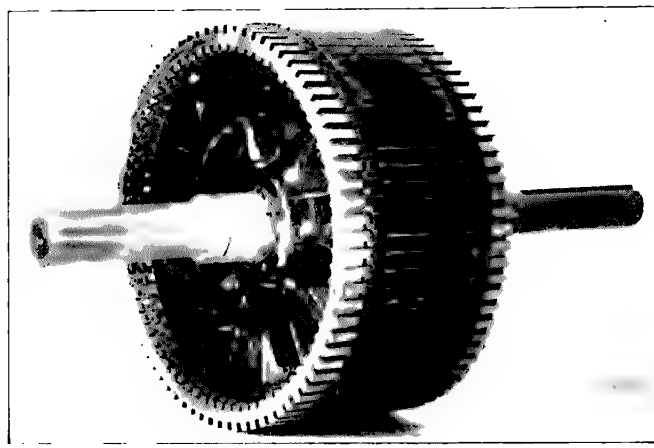


FIG. 76.—Squirrel-cage rotor with bars tap-bolted to machined end rings. This construction is shown in Fig. 74(*b*).

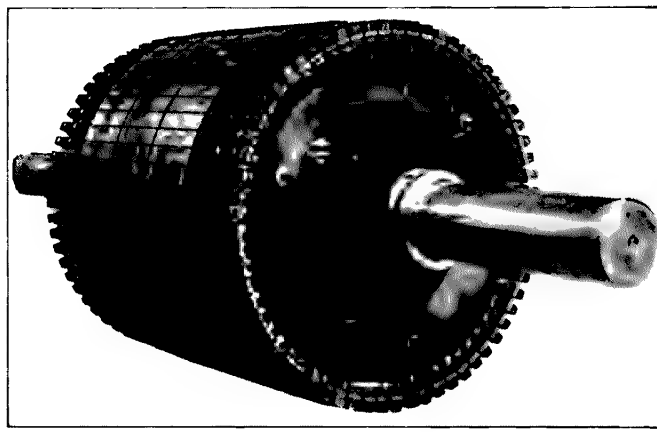


FIG. 77.—Squirrel-cage rotor with bars tap-bolted to bent-over lugs on punched sheet-metal end rings. This construction is shown in Fig. 74(*d*).

the ends of the bars after they have been assembled in the rotor slots. After casting, the rings are machined to the correct dimensions.

In the construction shown in Fig. 74 (*g*) and Figs. 79 and 80, the bars are driven into the slots without insulation. The end rings

are punched and formed from sheet metal, usually copper or brass. The annular groove, which is formed near the outside edge of the ring, fits over the ends of the bars and is filled with a

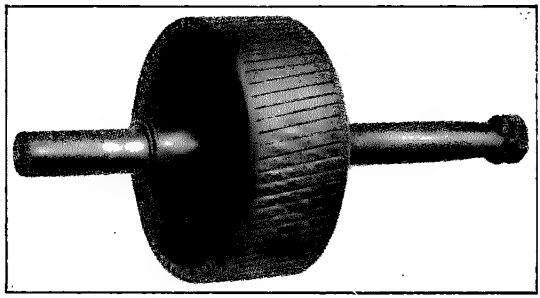


FIG. 78.—A small squirrel-cage rotor. The rings serve mechanically as end plates and the bars as rivets [see Fig. 74(*e*)].

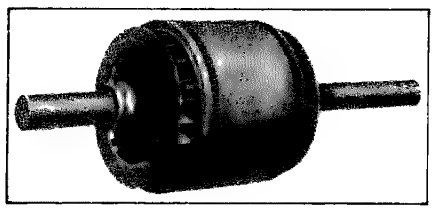


FIG. 79.—A medium-sized squirrel-cage rotor. The rings are punched from sheet metal, and are formed with an annular groove which fits over the bar ends and is filled with spelter [see also Fig. 74(*g*) and Fig. 80].

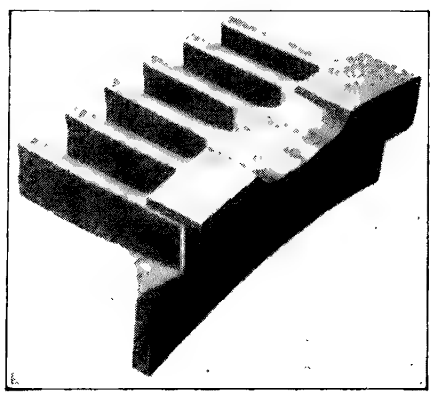


FIG. 80.—Section of the winding on the rotor of Fig. 79. With a portion of the ring removed to show the spelter completely filling the space around the bar ends.

special spelter, which makes a good electrical and mechanical joint between the bars and the ring. This spelter is melted into the groove in special furnaces which heat the ring uniformly; this prevents strains which would result from uneven heating.

It is not advisable to extend the construction last described to the larger machines, as the demand for them does not warrant the expense of tools; also, the sheet would be difficult to obtain in the necessary widths. The larger sizes are constructed, as shown in Fig. 74 (*h*) and Fig. 81, by brazing the bars to the outer cylindrical surfaces of flat rings which are made of some alloy having the desired conductivity. The heat for brazing each bar to the ring is produced by passing an electric current through the contact resistance.

In the type shown in Fig. 74 (*i*), the end rings have projections on the outer sides which match the bars. On special welding machines, copper slugs are welded, one at a time, so as to con-

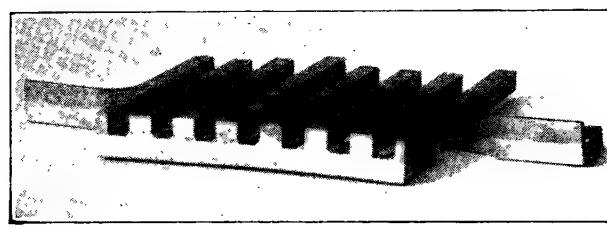


FIG. 81.—A squirrel-cage construction used on large rotors. The bars are brazed by the heat of an electric current to the outer surface of a plain end ring [see Fig. 74(*h*)].

nect the ends of each bar with corresponding projections of the rings.

On some small-sized rotors, the complete winding, including the bars and end rings, is cast at one time. The alloy used is usually one containing aluminum, and after casting requires practically no machining, making a very cheap and rugged design. The end-ring section is generally of the form shown in Fig. 74 (*j*).

Some rotors use a winding made of a slit strap of metal, which is pulled into a shape resembling a primary coil as shown in Fig. 74 (*k*). Usually two or more such coils are wound per slot. One advantage of this construction is in the variable chord possible, which is desirable on two-speed motors to permit the proportioning of the starting torque on the different speeds. However, this type of winding is difficult to support mechanically and is seldom used.

In the type shown in Fig. 74 (*l*), a number of thin end rings are used at each end, with the holes for the bars pierced so that the

material displaced rides on the top and bottom of the bars and is welded to the bars. This construction gives good ventilation but is expensive on account of the many joints made.

For the construction shown in Fig. 74 (*m*) the rings are of sheet metal, punched with tapered holes for the bars. The bars project through the ring, which has the small ends of the holes toward the core, and the spaces around the ends of the bars on the outer side of the ring are filled with a brazing spelter.

In the type shown in Fig. 74 (*n*) the end ring has a circular dovetail machined on the inside, and the bar ends are notched to extend over this dovetail. The bar projections are then pinched together and brazed to the ring.

In a common foreign construction, the ends of the bars are slotted, and a relatively flat end ring is partially inserted in this slot, as shown in Fig. 74 (*o*).

For very high resistance rotors, tubes of copper or brass have been used. The tubes project through holes in the rings and are swedged tight by the use of tapered pins driven into the ends of the tubes, as shown in Fig. 74 (*p*).

Wound Rotors.—Wound-rotor induction motors have characteristics which make them desirable for certain applications. The speed can be varied under load, the current taken at start can be regulated, and the starting torque can be adjusted to any value up to the maximum torque of the motor. These changes are effected by varying the amount of resistance inserted into the secondary circuit. This type of rotor has coils of copper wire, ribbon, bar or strap, wound into the slots, insulated similarly to the stator winding, and connected for the same number of poles as the stator. The different types of wound rotor may be roughly classed, according to the kind of conductor used in the winding, as wire, ribbon, strap and bar-wound rotors.

The wire winding is used on small-sized machines where the slots are small and must be tapered, as shown in Fig. 82 (*a*), to give a good tooth section. The use of round wire in these small slots allows a large number of conductors to be put into each slot, so as to give a secondary voltage and current suitable for the controller resistance. An increase in the number of conductors raises the secondary voltage and enables the required ampere turns to be produced with a smaller secondary current to be collected from the slip rings. Very small rotors have the wire

wound directly into the slots, while larger rotors use form-wound coils similar to stator coils. The windings are connected into pole-phase groups, cross-connected for three phase, and the leads are connected to three collector rings.

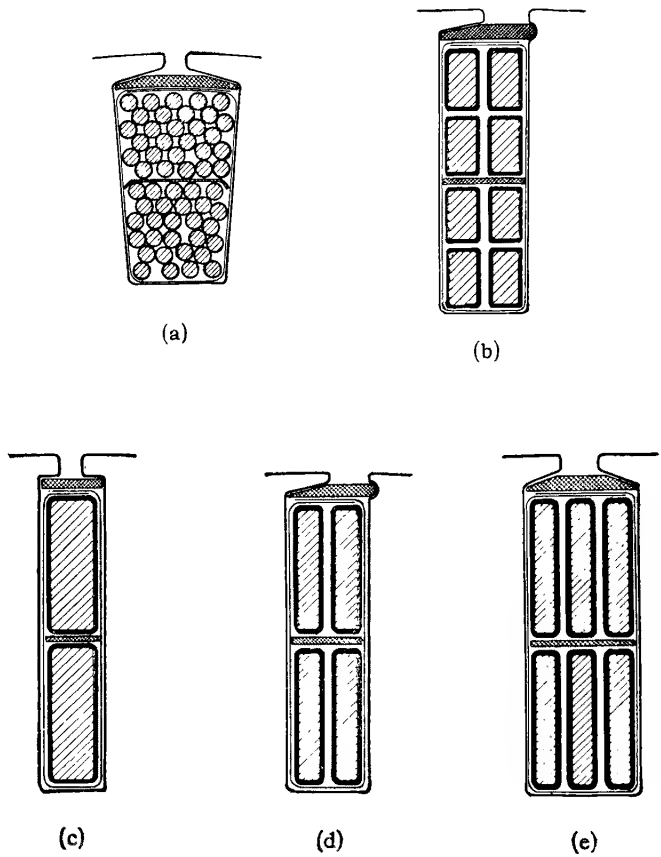


FIG. 82.—Cross-sectional diagrams of typical wound-rotor slots. (*a*) Round-wire winding used on small rotors; conductors placed hit-or-miss in slot. (*b*) Ribbon winding used on intermediate-sized rotors. (*c*) Strap winding, two conductors per slot. (*d*) Strap winding, four conductors per slot. (*e*) Strap winding, six conductors per slot.

The ribbon coil is used on rotors next larger than the wire-wound type, where the use of parallel-sided slots does not materially weaken the tooth at the root. The slots in the ribbon-wound rotor have lips on one side which close them about one-half their width, as shown in Fig. 82 (*b*). The coils are made two ribbons in width and two or more in depth. They are wound into the

slots one conductor at a time through the top openings, similarly to the round-wire type. After they are in the slots, however, they are arranged in their definite positions –not hit-or-miss as are the round-wire coils. This construction gives a number of conductors per slot intermediate between the round-wire and strap coils. Ribbon windings are connected in the same manner as round-wire windings.

As the power of the motor increases, the secondary voltage induced in each conductor increases, so that at approximately a 75- to 100-hp. rating, the open-circuit secondary voltage with eight conductors per slot is greater than the insulation with this type of coil is good for. At this point, the design changes to strap coils with only two, four or six conductors per slot.

When **two straps per slot** are to be used, as shown in Fig. 82 (*c*). the winding may be accomplished by either of two methods. One method uses straight copper straps which are insulated for most of their length. Two of these are pushed, one above the other, into each slot from one end. The part of the strap which extends beyond the core on each end is bent to shape by hand—the lower strap in one direction, and the upper strap in the opposite direction. Connectors are then placed on both ends of the straps to complete the coils and make the proper connections, which are usually of the "wave" or "two-circuit" type; in two- and four-pole motors, where the coil extensions for this type of winding would be very long, a group connection is used. The other method uses a semiformed coil, one end of which has the loop and diamond part formed, and the other end has two straight leads. These straight ends are pushed through the slots in the same manner as the separate straps. With this type, all the coils must be started at once and pushed into place as a unit. Coils cannot be inserted singly, as the final throw of coils could not be put in, due to interference with those already in place. After the coils are in place, the front ends are bent and connected by hand in the same manner as are the straight-strap coils.

Four-strap-per-slot coils are fully formed and insulated before winding, and are wound into the slots through the slot openings. The straps are arranged two wide and two deep, as shown in Fig. 82 (*d*). The lower strap under the lip is first wound into the slot, followed by the other lower strap, which pushes the first one into place. The upper straps are wound in the same manner.

These coils are usually connected to give a wave winding. However, group connections are sometimes used where end space is limited or where a special connection is required. A typical coil

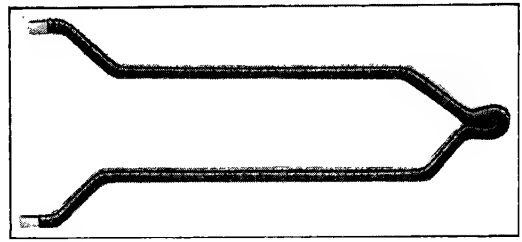


FIG. 83.—Typical coil for strap-wound rotor with four conductors per slot.

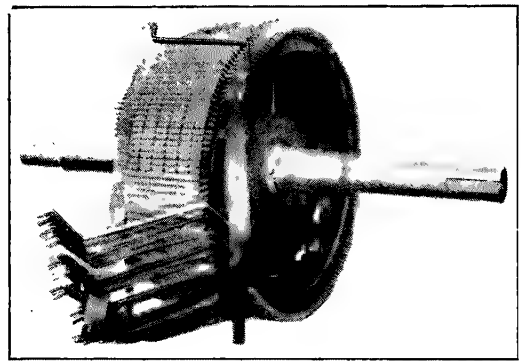


FIG. 84.—Partly wound rotor with coils as shown in Fig. 83.

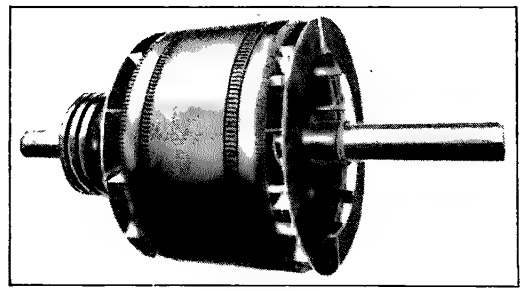


FIG. 85.—Completed strap-wound rotor with four conductors per slot. The slot cross section is shown in Fig. 82(*d*).

for a four-strap-per-slot winding is shown in Fig. 83. Figure 84 shows a partly wound rotor, and Fig. 85 the completed rotor using this type of coil.

Six-conductor-per-slot coils also consist of straps which are fully formed and insulated before winding. The slot has a central opening with overhanging lips on each side. In winding

this type, the side straps are inserted first and the central strap last. This winding is usually of the closed-coil type and is used principally on high-speed motors, where the increased number of straps per slot helps to counteract the small number of slots on the rotor. It affords a greater number of conductors on the complete rotor, and hence gives better secondary voltage

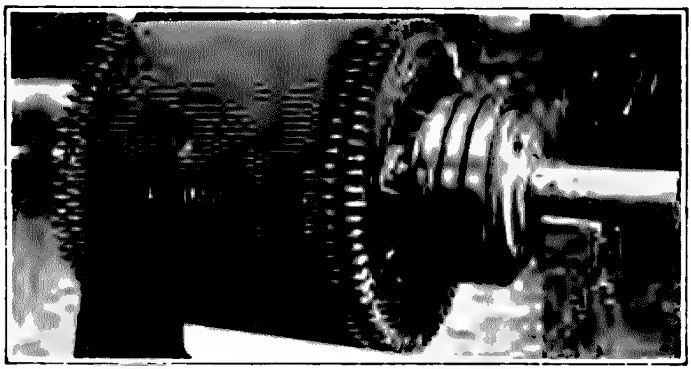


FIG. 86.—Bar-wound rotor with one conductor per slot. Alternate bars are different in length to make room for the end connectors.

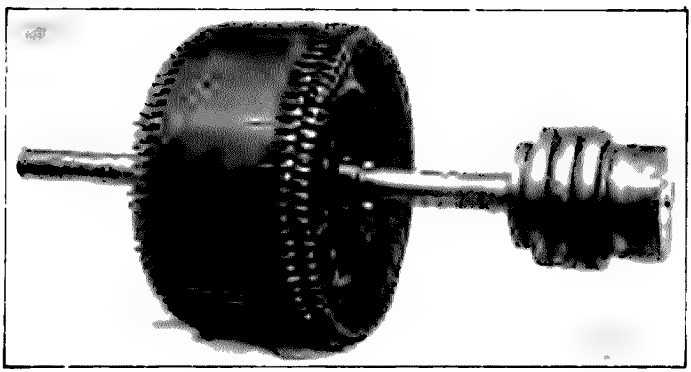


FIG. 87.—Bar-wound rotor with two conductors per slot. Top and bottom bars are of different lengths to prevent interference of end connectors.

conditions than four straps per slot. A further advantage is that the closed coil can be chorded and hence made with shorter end extension, which is also desirable on high-speed motors.

The bar winding is so called from its construction, which is of straight, rectangular bar conductors, connected on the ends by specially shaped strap connectors. This type of winding is very seldom used now, being superseded by the other types. Either one or two bars per slot were used. With one bar per slot, alternate bars were of different lengths, as shown in Fig. 86; with

two bars per slot, the top and bottom bars were of different lengths, as shown in Fig. 87. Both types used strap connectors of the involute shape. These required special forming dies and were difficult to support.

The almost universal practice for supporting the ends of rotor windings is to put on bands of steel wire under tension, the operation being known as "banding." These bands hold the coils down tightly on a coil rest which is usually supported from the core.

As the number of phases in the rotor circuit is not determined by the supplied power, but by the method of connecting the rotor coils, the three-phase connection is nearly always used in order to reduce the number of collector rings to the minimum for a polyphase circuit. Six-phase windings with six collectors are sometimes desirable when the motor is to be used in connection with other apparatus, as in speed-regulating sets.

A given rotor may be used in all stators which have the same number of poles, the correct mechanical dimensions, and the proper relation of primary and secondary slots. No changes in the rotor connections are necessary when the primary winding is changed as to voltage, phase or frequency, without a change in the number of poles. However, when the frequency is increased, the open-circuit voltage may exceed the insulation limit, and this would require a change in the connections.

Winding Connections.—Small rotors are connected three-phase in the same manner as primary windings; the groups in each phase being connected either all in series or in two or more parallel paths, and the three phases being connected either in star or in delta, so as to give a collector-ring voltage of a value suitable for the controller.

Wave Windings.—In large motors with strap coils and a large number of poles and coil groups, the connections necessary to make group connections would be very cumbersome, difficult to support and balance, and would also require a large amount of copper. On these motors, the wave winding is generally used, as only twelve leads from the winding are necessary to obtain a three-phase circuit. With group connections there are twelve leads which must be connected together for every pair of poles. Wave windings are of either the true or modified types. The true wave winding has a pole and slot combination such that all

coils have the same slot throw; while in the modified winding there are six places where a group of coil ends are bent to a shorter throw than the other coils.

The wave winding advances around the rotor, passing through each successive pole in opposite directions. After one complete circuit has been made around the rotor, the first conductor in the second circuit is adjacent to and on either side of the starting conductor. If it is just beyond the first conductor, the winding is progressive; if it is in the rear of the first conductor, the winding is retrogressive. In any wave winding, this is one of the most important checks; *i.e.*, to see that the first conductor of the second circuit lies adjacent to the first conductor of the first circuit.

Assume a four-pole 61-slot rotor. The bottom conductor of slot 1 is connected at the rear to the top of slot 16; thence at the front to the bottom of slot 31; at the rear to the top of slot 46; at the front to the bottom of slot 61; etc. The conductor at the bottom of slot 61 is the first conductor of the second circuit and is next to the first conductor on the near side; it is therefore a retrogressive winding. This is a true wave winding.

Assume four poles, 60 slots. Beginning with the bottom conductor of slot 1 and using the same span of 15 slots, the other conductors are top of slot 16, bottom of slot 31, and top of slot 46. Fifteen slots beyond 46 would be 61, or 1, which is the same slot used in the beginning. To avoid joining 46 to the first conductor, the last end connection is shortened a slight amount and conductor 46 is joined to 60. This shortening is done at this place in every circuit around the rotor until one-sixth the conductors are connected up, completing half of one phase. This is a retrogressive winding of the modified wave type.

The throw of the coils usually is made full pitch, as the maximum rotor voltage possible for the winding is desired. The pitch of the coil, as indicated by the connection between conductors on either the front or rear, can be varied from full pitch, but the sum of the front and rear pitches must be equal to the number of slots per pair of poles. In the four-pole, 60-slot rotor, the rear throw can be other than 1 to 16, but the next conductor must be 31; *i.e.*, the sum of the front and rear pitches must be 30, or the number of slots per pole pair.

The complete winding is divided into six sections—two in each phase—which can be connected either in series or, if the

number of coils and the phase relation in both are exactly the same, in parallel. The three phases can then be connected in star or delta, depending on the secondary voltage desired.

The True Wave Winding.—The true wave winding can be divided into sections by going through the complete winding in order, and dividing it into six equal parts; or by the following short-cut method. This method assumes two conductors per slot, so that for four conductors per slot, the number of slots is to be assumed doubled. To get an odd number of two-conductor slots with four straps per slot, one coil, *i.e.*, a top conductor in one slot and a bottom conductor in a slot which is one coil pitch away, is left idle. If a tap must be taken off any slot which is spanned by the dead coil, care must be taken to get the correct strap. The first slot which has a dead conductor (say the top conductor) has only the bottom conductor active, so that the top conductor of the next slot should be considered as belonging in the first slot. In other words, the new slots in the zone included by the dead coil are made up of the bottom conductor in one slot and the top conductor of the next slot. If a tap is not taken from any of these slots, the above need not be considered, as it will then take care of itself.

Quick Method of Figuring Connections for Wave Winding.—The short-cut method is as follows:

$$\text{Let } B = \frac{slots \pm 1}{pole\ pairs} = \text{front pitch} + \text{rear pitch.}$$

(B must be an integer.)

$$B/2 = \text{average pitch.}$$

$$K = \frac{2 \times slots}{6} = \text{number of conductors per section.}$$

K must be an even integer or made so; *i.e.*, if K is odd, alternately add and subtract one to obtain six even values K_1, K_2, K_3, K_4, K_5 and K_6; or if K is not an integer, odd or even, divide twice the number of slots into six even values as nearly equal as possible.

Multiply K_1 by $B/2$ to obtain the number of slots advanced by the first section; add to this product 1, or the number of the first conductor of the section. Divide the result by the number of slots, and the remainder is the number of the first conductor of the second section. To this remainder add K_2 multiplied

by $B/2$, and divide by the number of slots; continue in this manner until all six values of K have been used. The remainders are the numbers of the beginning conductors of the sections; the sixth remainder should be 1, or the number of the first conductor of the first section. The beginnings are taken as being in the bottoms of the slots. The end conductor of the preceding section is obtained by subtracting the front pitch from the beginning slot found as above. The ends are top coils.

For a series-star connection, the beginnings of the first, third, and fifth sections are taken as leads. The beginnings of the second, fourth, and sixth are star points. The ends of the first and fourth, second and fifth, and third and sixth are joined together. This is shown in the following example.

Assume a rotor with 6 poles, 119 slots, and two conductors per slot. The coil span will be 20 slots and the first circuit around the rotor will be bottom conductor 1—top 21—bottom 41—top 61—bottom 81—top 101; the next conductor is 121, or 2. This being beyond 1, the winding is progressive.

$$B = \frac{slots + 1}{pole\ pairs} = \frac{119 + 1}{3} = 40.$$

$$K = \frac{2 \times slots}{6} = \frac{2 \times 119}{6} = 39\tfrac{2}{3}.$$

$$K_1 = 38;\ K_2 = K_3 = K_4 = K_5 = K_6 = 40.$$

$$B/2 \times K_1 = 20 \times 38 = 760.$$

$$B/2 \times (K_1 \text{ to } K_6) = 20 \times 40 = 800.$$

The sections are calculated by the short-cut method as follows:

```
                    1  = first slot of first section
add B/2 × K1 =    760
                  ---
  divide by 119) 761 (6
                 714
                 ---
remainder =       47  = first slot of second section
add B/2 × K2 =    800
                  ---
  divide by 119) 847 (7
                 833
                 ---
                  14
```

remainder = 14 = first slot of third section
add $B/2 \times K_3$ = 800

divide by 119) 814 (6
714

remainder = 100 = first slot of fourth section
add $B/2 \times K_4$ = 800

divide by 119) 900 (7
833

remainder = 67 = first slot of fifth section
add $B/2 \times K_5$ = 800

divide by 119) 867 (7
833

remainder = 34 = first slot of sixth section
add $B/2 \times K_6$ = 800

divide by 119) 834 (7
833

1 = first slot of first section (check)

The first section begins with bottom conductor 1 and ends with top conductor 27; which is the beginning of the second section less the front pitch, which in this case is $B/2$; thus $47 - 20 = 27$. Similarly:

The second section begins with bottom 47 and ends with top 113.

The third section begins with bottom 14 and ends with top 80.

The fourth section begins with bottom 100 and ends with top 47.

The fifth section begins with bottom 67 and ends with top 14.

The sixth section begins with bottom 34 and ends with top 100.

The leads are bottom conductors 1, 14, and 67. The star points are bottom conductors 47, 100, and 34. The following pairs of top conductors are joined: 27 and 47; 113 and 14; 80 and 100.

If the six sections are plotted in order as sides of a hexagon as shown in Fig. 88(*a*), the phase relations of the various sections can be seen, and the connections given above verified. Each section can be treated as a unit and combined with the others in various ways as, for example, to form a series-star connection as shown in Fig. 88(*b*) or a series-delta connection as shown in Fig. 88(*c*); but when making these shifts, the sides of the hexagon must not be turned from their original directions. The length of each side should be made proportional to the voltage induced in its section. Sections of different length or angle, however slight, should not be connected in parallel.

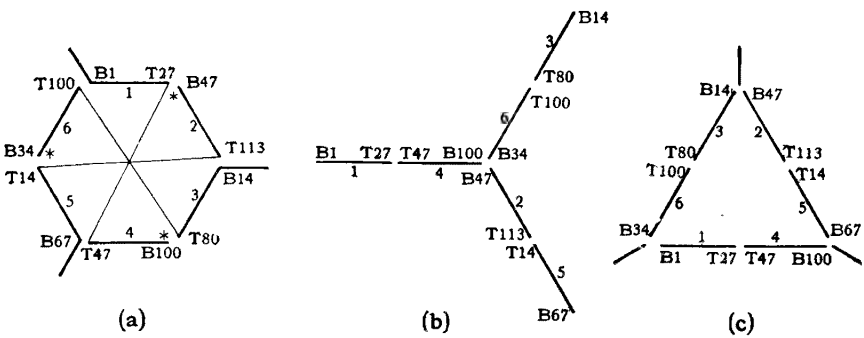


FIG. 88.—Phase relations in the six sections of a wave winding. (*a*) Sections represented as sides of a hexagon, with lengths proportional to the induced voltages and directions showing the phase relations. (*b*) Series-star combination of sections. (*c*) Series-delta combination of sections.

True wave windings are seldom exactly balanced in the several sections, so that they cannot be paralleled in large rotors, where it is often necessary. The numbers of slots must be changed for different numbers of poles, as there are few possible numbers of slots which suit more than one number of poles. The cross connections sometimes are badly distributed around the rotor, which causes unbalancing mechanically.

Modified Wave Winding.—To overcome the objections to a true wave winding given in the preceding paragraph, practically all wave-wound rotors of modern design are modified so that the number of slots is a multiple of the number of phases times the number of poles for equal coil grouping; or so that the number of slots is a multiple of the number of phases times the number of pairs of poles, if unequal coil grouping is permissible. This is done by bending the coil ends at six different places around the

rotor so as to shorten the front pitch by one slot. This is best illustrated by means of a example. For instance, in Fig. 89 is shown a diagram with eight poles for 144 slots. Following one conductor around, we see that lead *A* goes to bottom 1, thence to top 19, bottom 37, top 55, bottom 73, top 91, bottom 109, and top 127. The next conductor would be 127 + 18 = 145, or 1,

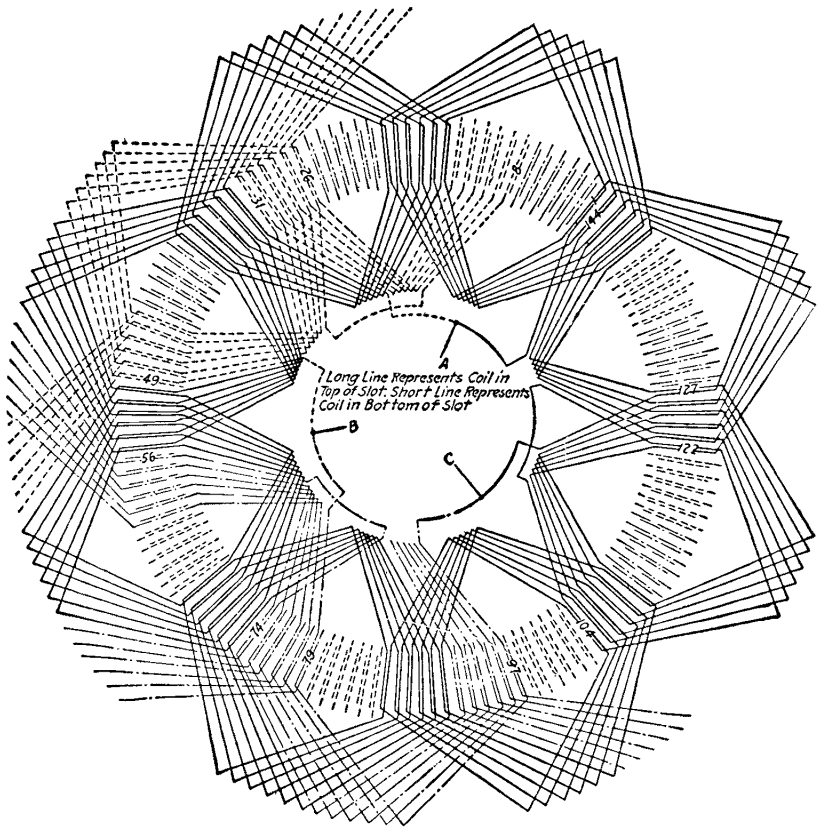


FIG. 89.—Modified wave winding for 8 poles and 144 slots. The sections are connected series delta.

which is the one started from. At this point the end connection is shortened so that the top conductor of slot 127 is connected to the bottom conductor of slot 144. As the other single circuits around the rotor are made, the connectors are shortened at this place until one-sixth the conductors are connected up, completing one-half of one phase. In this case, there are six loops around the rotor in each section.

The last conductor in this section is the top conductor from slot 122, and is one of the leads to be connected by the cross connections. Up to this point only the bottom conductors in slots numbered 1, 144, 143, 142, 141 and 140 have been included; the top conductors in these same slots have the same phase relation and should be so connected to the bottom conductors that current flows in the same direction in the corresponding bottom and top conductor groups. This is done by joining the last conductor included in the first section, *i.e.*, top 122, to the top conductor of slot 104, from which the winding proceeds around the rotor in the opposite direction until it comes to the bottom conductor in slot 127 and completes another section of the winding. In this section we have included the top conductors of slots 1, 144, 143, 142, 141 and 140. These two sections include one-third of the winding or one phase, and are distributed in one-third of the slots, which are evenly distributed and symmetrically placed in all the poles.

The other phases are connected in the same manner, the starts being spaced as near one-third the way around as possible, so as to balance the connections. The connections between phases are checked the same as for primary windings, assuming the current flowing toward the star point in all phases when star connected, and flowing around the delta when delta connected. This check gives three zones per pole, and the current in each zone is in the reverse direction to that in the zones on either side of it; or in other words, the direction of current in the zones is alternately toward and away from the center of the diagram, on the assumption that current flows from the lead to the neutral.

CHAPTER V

SINGLE-PHASE WINDINGS

General Classification of Single-phase Motors.—Single-phase motors may be divided into five general classes, somewhat in the order of the number in use today:

1. Split phase.
2. Repulsion starting and induction running.
3. Capacitor.
4. Shaded pole.
5. Universal.

The reason for these various types is the fact that a straight single-phase winding on the stator would give no starting torque, because there is no rotating field as described in Chapter II; the magnetism simply alternates back and forth, north and south, as in a transformer, without producing any tendency to rotate. Consequently various compromises as covered by these five classifications have to be adopted for getting started. In Classes 1 and 2 these starting means must be taken out of circuit when the motor is up to speed and ready to run under load. A motor with nothing but a straight single-phase winding, if it is started and brought up to speed mechanically by spinning the pulley or some similar outside means, will run and carry its load just as does a polyphase motor. This is because the mechanical rotation of the rotor at practically synchronous speed carries the induced currents around in space until they are at the right position in space and time to produce a rotating field. This was explained in Chapter II.

1. *Split Phase.*—This is the class commonly employed on washing machines and similar domestic applications and, with the possible exception of other types of desk air-ventilating fans, there are more split-phase motors in service than any other type. This type operates on the principle of taking a single-phase power supply with two lines and splitting it into two parallel circuits which are made into a makeshift two-phase circuit by the device

of putting a resistance in one circuit and a reactance or choke coil in the other. This causes the currents in the two branches to be out of phase with each other and to approach the condition of a two-phase circuit where the two phases are exactly 90 electrical degrees apart. This phase splitting can also be accomplished by using a combination of a choke coil in one branch and a condenser or capacitor in the other, in which case the phase angle between the two branches can be made to approach 90 deg. as closely as is desired. However, these auxiliaries add to the cost and it is usually not found necessary to do more than change the number of turns and the resistance in the starting winding in order to get successful operation. Particularly is this true in the case where a mechanical clutch is installed; this lets the rotor core and windings start to rotate independently of the shaft without picking up the torque or load driven by the shaft until sufficient speed and inertia are accumulated to make sure that the load will be started. Such mechanical clutches are actuated by centrifugal devices which can be set so that the clutch does not take hold and pick-up the load until the proper speed is reached. Very many forms and modifications of this split-phase principle have been used. There was an old type, employed for ratings as high as 10 hp., where the rotor winding was double and had a high-resistance squirrel cage for starting and a low-resistance coil winding which was shorted after the motor was up to speed, thus cutting down the slip and providing more efficient operation after the motor was up to speed.

2. *Repulsion Starting and Induction Running.*—This is the well-known type usually credited to Dr. Engelbert Arnold, author of the classic series of German textbooks on the calculation of dynamo-electric machines. This has on the rotor the usual type of direct-current armature winding with a commutator, which is short-circuited after the motor reaches full-load speed. This motor really starts as the equivalent of a series motor and hence is capable of starting loads which are heavy as compared with its running load. It is simple and reliable in its operation which accounts for its well-earned popularity on meat grinders, coffee grinders, shoe-repair machinery and many similar uses. It is built in sizes up to 10 to 15 hp., and even larger when the demand arises. On account of the commutator and windings on the rotor, also the short-circuiting device, this type is somewhat

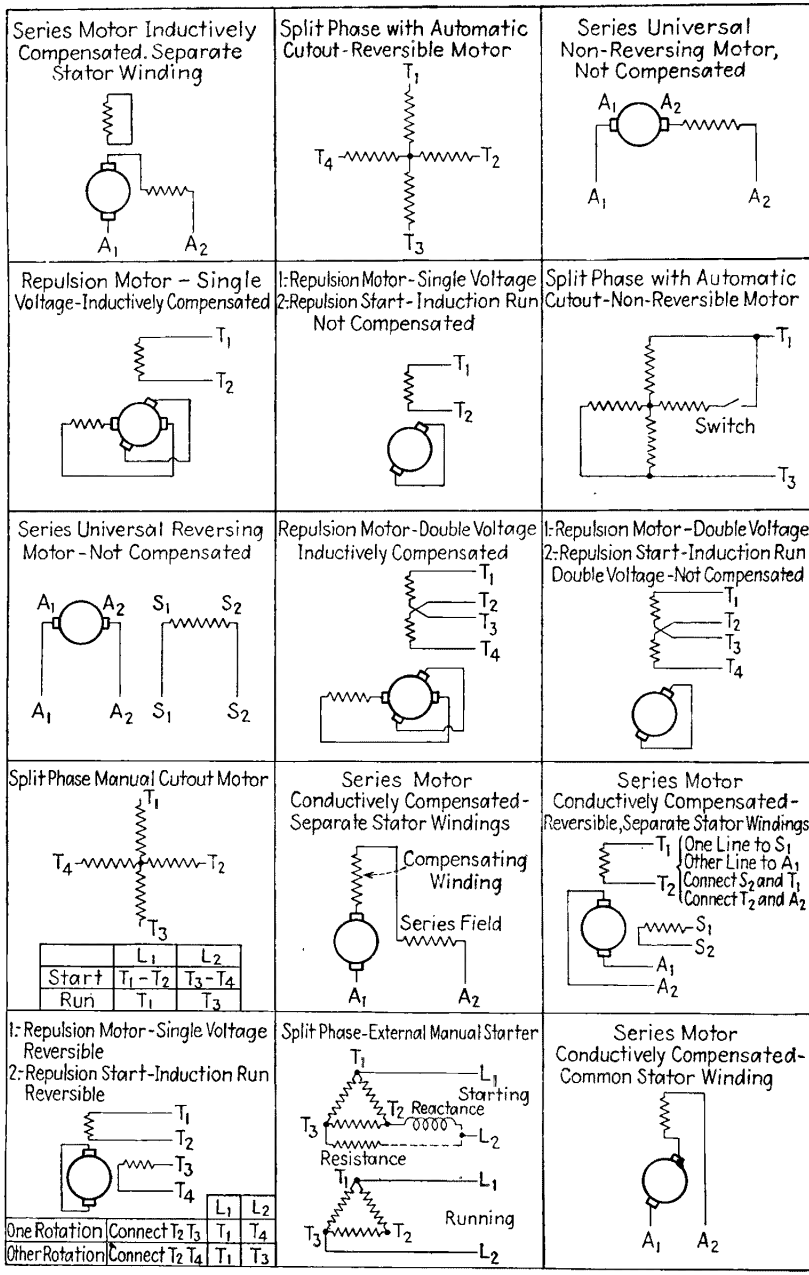


FIG. 90.—Single-phase motor terminal markings. (*National Electrical Manufacturers, Association.*)

more expensive to build and is not so much used in fractional horsepower ratings where other types handle the service very acceptably.

3. *Capacitor Motors.*—These are the latest form of single-phase motors and to all intents, so far as windings and winding problems are concerned, may be considered as two-phase motors started and run on single phase. In some cases, as will be noted from

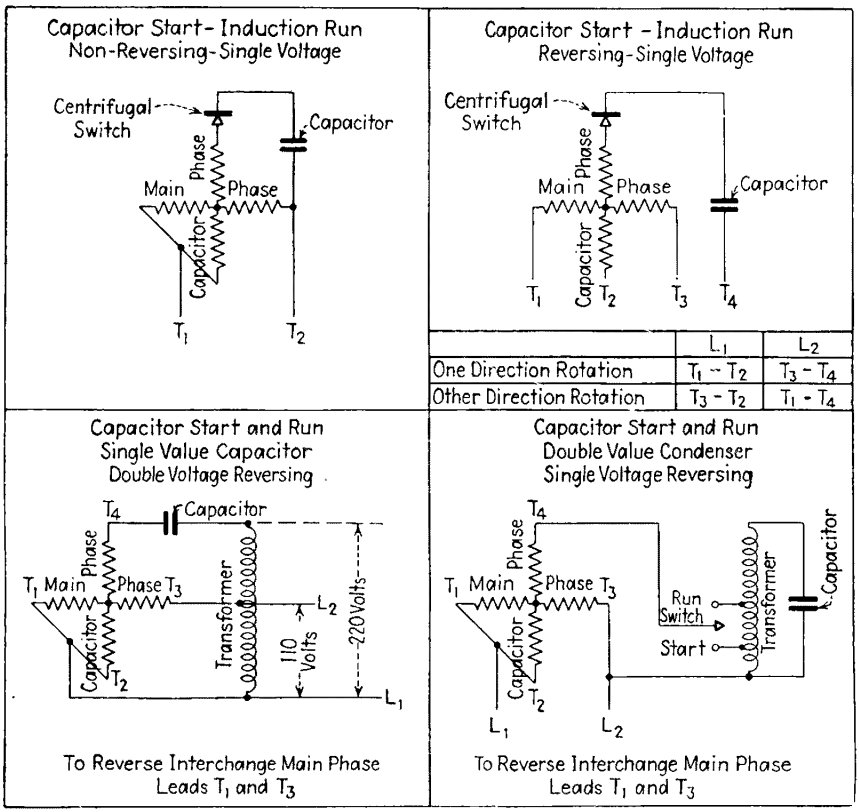


FIG. 91.—Single-phase capacitor motor; schematic diagrams of connection.

the small schematic diagrams of Fig. 91, the capacitor is proportioned for starting duty only and in other cases is for both starting and running duty. The rotor is usually the simplest form of squirrel-cage winding.

4. *Shaded Pole Windings.*—This is a type that is ordinarily employed on quite small ratings only, where great simplicity and reliability are required. The type is really a variation of split-phase operation where the split is produced by a short-

circuited turn of copper enclosing a portion of the iron of one pole, which by induced currents produces the result of the magnetic field enclosed by it, being out of phase with the rest of the pole. This action produces a form of rotating field which allows the motor to start with a limited amount of starting torque. It is ordinarily used for desk ventilating fans only, or similar operations where the starting load is low. It is open to the objection that the short-circuited turn, producing the "shaded

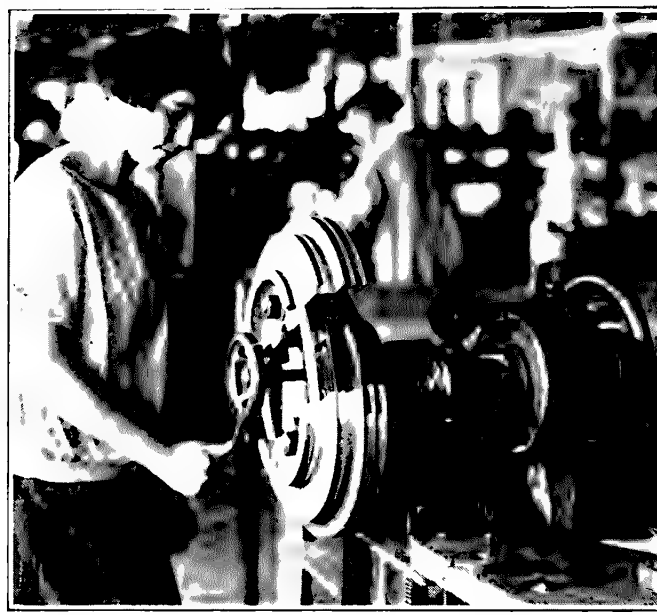


FIG. 92.- Method of winding skeins.

pole," is in circuit at all times and hence has induced in it a useless short-circuited current which produces loss and lower efficiency than would be the case if it could be eliminated. There is another operating peculiarity of this type, which is that it is possible to reverse its direction of rotation, if one wishes to do so, only by turning the whole motor end for end or, if that is impossible owing to the mounting, by taking out the complete stator core and windings and turning it end for end.

5. *Universal.*—The motors usually sold under this classification are series motors with stator field coils and armature windings and commutators equivalent to normal direct-current motors. They are usually employed where high speeds are

required, *e.g.*, on domestic sweepers and vacuum cleaners where 60-cycle speeds above 3600 r.p.m. could not be had with induction motors.

Form of Windings in these Five Types.—Aside from the various devices used for splitting the phase, the fundamental working winding on the stator is straight single-phase. Whether it is made from concentric coils or diamond coils or is skein wound, as described in this chapter, the action is the same. Looking at Fig. 100 on page 90, one can readily see that, if a starting winding or split-phase winding is wound in the vacant slots in the center of the poles, it will occupy the same relation to the

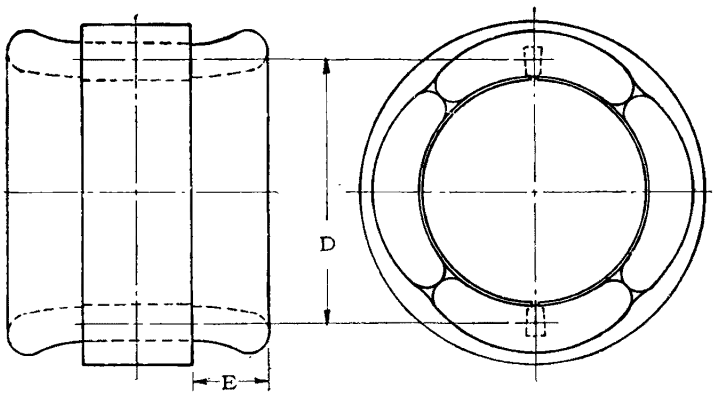


FIG. 93.—Shape of concentric coils for single-phase motors.

running winding as would the two-phase windings in a two-phase motor.

As has been stated the rotor windings are usually of two types, either a simple straight squirrel cage with rotor bars connected to short-circuiting rings at the ends of the core or, as in the case of the repulsion-starting motor, the equivalent of the winding on an ordinary parallel-wound or lap-wound direct-current armature. As noted above, both stator and rotor windings on universal motors are similar to the corresponding direct-current series motors.

Schematic Diagrams Showing Outside Terminal Connections. Because there are so many types and variations of the single-phase windings, it may be helpful to those who use this book to reproduce (Fig. 90) the terminal markings authorized for many years by the National Electrical Manufacturers' Association.

In some of these diagrams reference will be noted to series motors. These will probably be readily understood; if not, consider the armature brushes as short-circuited and bring out both terminals of the stator winding. This reduces the series motor to the well-known repulsion motor, which is what it really is so far as speed-torque characteristics are concerned. The advantage of the short-circuited armature or repulsion type is that it makes no difference how many turns are in the armature coils or what is the corresponding cross section, whereas in the case of a series motor the number of turns in the armature coils

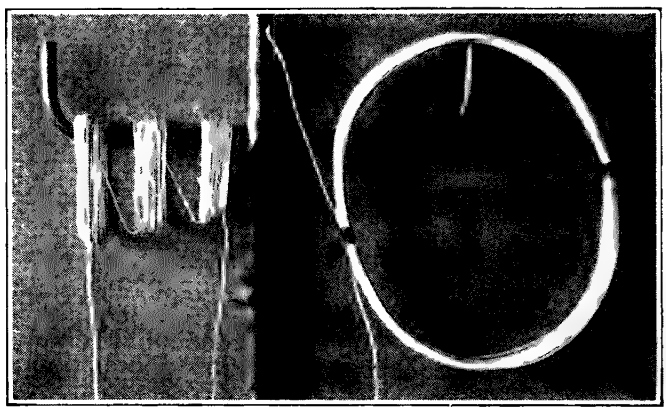


FIG. 94.—Typical mould-wound and skein-wound coils.

and the cross section of the armature must be suitable for the line voltage and current upon which the motor is to be run.

To these diagrams are added some schematic diagrams for capacitor motors which have not as yet been standardized but which will serve to indicate the individual windings and their relation to various outside devices. See Fig. 91.

Stator Windings.—In the case of shaded pole and universal motors the stator winding is similar to a direct-current motor with salient poles.

The other three types are similar and use a winding as shown in Fig. 100, page 90, which shows the running winding. The split phase or starting winding or the capacitor phase would be wound in the vacant slots shown in the center of the poles. Because this type of winding is peculiar to single-phase motors and used by practically all manufacturers, the remainder of this chapter is devoted to its description.

Method of Winding.—There are several methods of winding the primary of this type of motor in common use today. They are:

Skein winding.
Mould winding.
Hand winding.

Skein Winding.—Skein winding as the name implies is a skein of wires. This skein of wires is looped a number of times through the slots to form one pole. That is, the total turns per pole is a multiple of the turns in the skein. The multiple depends upon the number of times the skein is looped through the slots and the number of slots.

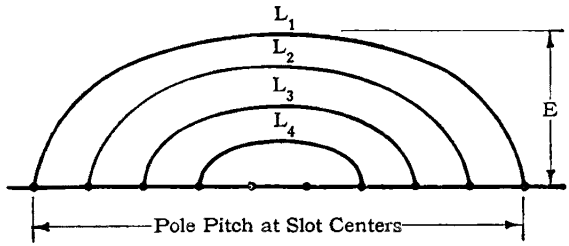


FIG. 95.—Method of determining length of loops.

When rewinding a skein-wound motor the number of times the skein is looped through each slot should be noted and the length of the skein measured and the turns counted.

The skein length may be calculated from the physical dimensions of the primary core by considering the mean diameter D of the winding as the diameter from center to center of slots as indicated in Fig. 93. Using this value of D calculate the pole pitch at slot center,

$$\text{Pole pitch} = \frac{D}{\text{poles}}$$

and then lay this out as a development, indicating the slot centers by dots on this line and drawing the probable mean line of each loop as shown in Fig. 95 by assuming the probable coil extension E as shown in Fig. 93.

Measure the lengths L_1, L_2, L_3 and L_4 and multiply each length by the number of times the skein is looped through the slot for that length. Then add these products together and multiply by 2 for the other side of the core. To this add the total core

width multiplied by the number of times the skein bundle crosses the core. This gives the total skein length. Make a trial skein with the required number of turns and correct if necessary on the remaining skeins. After a little experience, corrections will be found unnecessary.

Another method of determining the skein length or circumference of skein is by trial with a single wire. This wire should be laid in the slots just as the skein of wire would be laid in making proper allowance for the bundle of wires the single wire represents. Then remove this wire and measure, and then wind one skein for trial and correct if necessary.

Distribution of Winding.—The distribution of the windings in the slots varies with the design of the motor and depends upon

Slot No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Main Winding	1 1	2	2	1			1	2	2	1 1	2	2	1			1	2	2	1 1	2	2	1			1	2	2	1 1	2	2	1			1	2	2
Starting Winding				1	1	1	1						1	1	1	1						1	1	1	1						1	1	1	1		

FIG. 96.

Slot No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Main Winding	1	1	1				1	1	1	1	1	1				1	1	1	1	1	1				1	1	1	1	1	1				1	1	1
Starting Winding				2	1 1	2							2	1 1	2							2	1 1	2							2	1 1	2			

FIG. 97.

FIGS. 96 and 97.—Winding distributions for a four-pole 36-slot motor.

the number of slots and number of poles. The distribution most commonly used in a 36-slot four-pole motor is shown in Fig. 96. The numbers in the slots indicate the number of times the loop passes through each slot. Figure 97 shows another distribution for the same number of slots and poles, but this distribution is not often used because the starting winding is too concentrated.

How the Skein Is Wound into the Slots.—After the skein length and distributions have been determined, the next procedure in winding a 36-slot four-pole primary is shown in Fig. 98.

In Fig. 98, *A* illustrates a developed view of the primary core looking at the teeth with the first operation of putting the skein in slots 4 and 7 completed. The short end of the skein should be firmly pressed against the core. The leads of the coil should be on the long end of the skein.

A half twist is next made in the skein as shown in *B* and the long end of the loop is then passed through the core and the sides are threaded into slots 3 and 8 as shown in *C*. The winding

should be pressed firmly against the side of the core. The half twist is repeated as shown in *D* but the twist should be made in the opposite direction to the first twist and throughout the winding the half twists should be alternated in direction so as not to snarl the winding on the last loop. The loop is then laid back

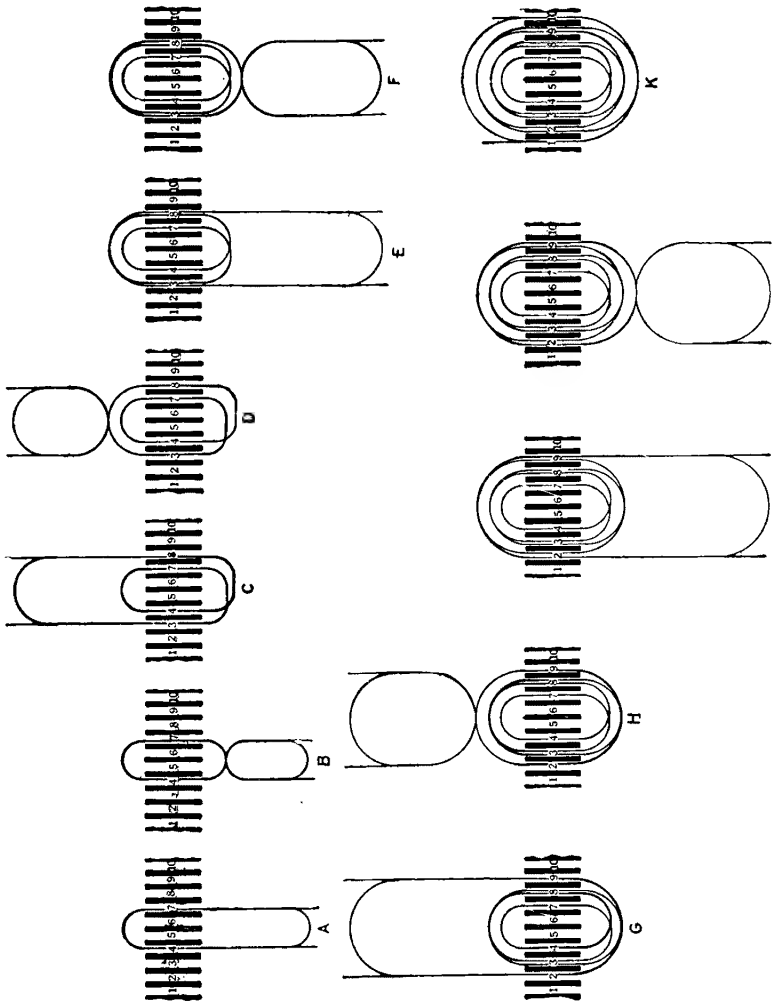


FIG. 98.—Procedure in winding a skein-wound primary, for distribution given in Fig. 96.

in slots 3 and 8 for the second time as in *E*. The half twist shown in *F* is then made and the loop laid in slots 2 and 9 as shown in *G*. Again give a half twist which is shown in *H* and they lie back in slots 2 and 9 for the second time as shown in *I* Another half twist which is shown in *J* is made and the skein is

threaded into slots 1 and 10, *K*, thus completing the winding of one pole. It is very important that the direction of the half

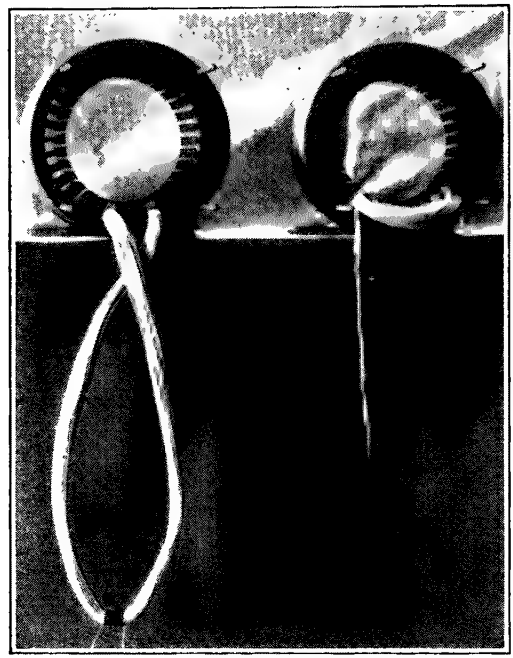


FIG. 99.—Partly wound stator cores. In the stator at the left, the first operation of skein winding has been performed and a half twist taken ready for the next operation. This corresponds to *B* in Fig. 98. The stator at the right has one pole completely wound, corresponding to *K* in Fig. 98.

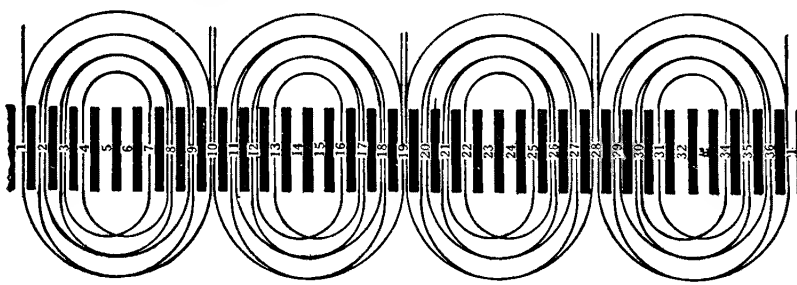


FIG. 100.—Completed main winding with each pole wound as Fig. 98—*A* to *K*.

twists be alternated to obtain a smooth winding and avoid difficulty on the last part of the loop.

The winding of the second and remaining poles is exactly the same as the first. The completed primary winding for a four-pole 36-slot motor is shown in Fig. 100.

For Repulsion-type Motors.—For repulsion-induction and repulsion-starting-induction running motors the primary winding is completed as shown in Fig. 100 and the coils are connected together as shown in Fig. 101. Four leads are brought out so

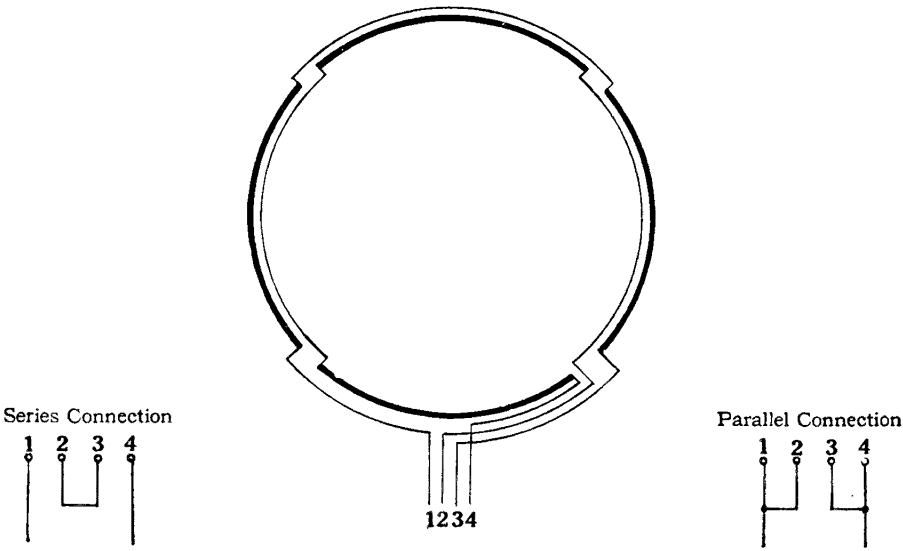


FIG. 101.—Connection diagram for winding in Fig. 100.

that these motors may be connected externally for either 220 or 110 volts.

For Split-phase Motors.—For split-phase motors an additional winding is required on the primary to start the motor. This

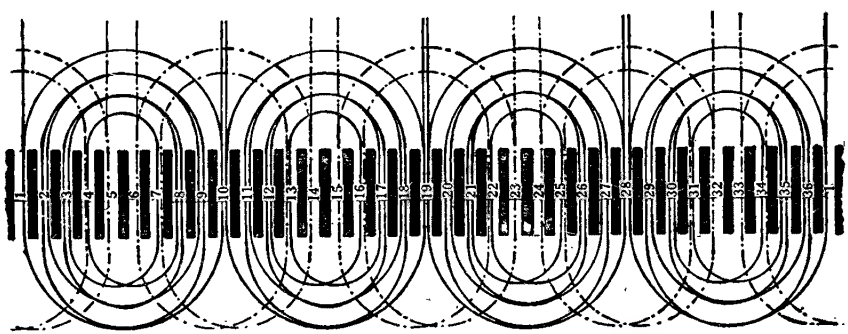


FIG. 102.—Complete diagram for both main and starting windings.

winding is called the starting winding and is connected across the line until approximately two-thirds synchronous speed is reached when its circuit is automatically opened by a centrifugally operated switch. This starting winding is displaced 90 electrical

degrees from the main winding, or in other words, the center of the starting winding is between the pole centers of the main winding. Its distribution and the length of the skein must be determined as explained for the main winding. The starting winding is essentially a resistance winding and consequently it is very important that its resistance in the rewound motor be the same as it was originally, which makes the length of the skein very important. The distribution of both the main and starting winding and the number of times the skein is wound into each slot are shown in Fig. 102.

Connections of Main and Starting Windings.—The main- and starting-winding coils should be connected together, as shown in

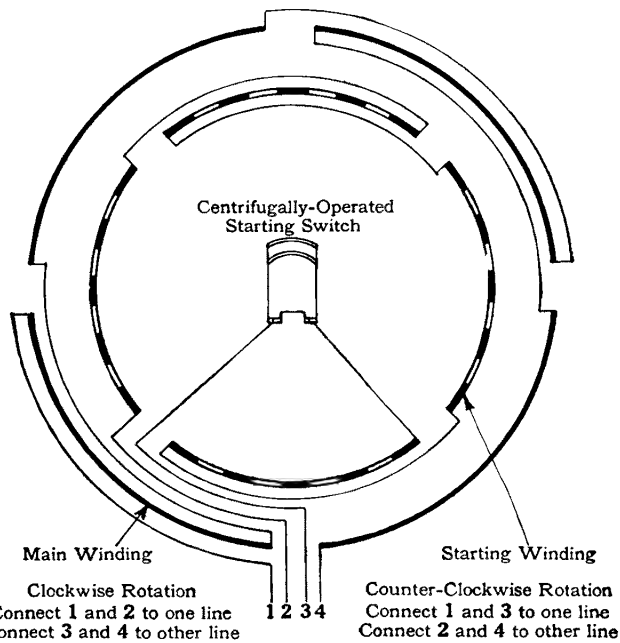


FIG. 103.—Connection diagram of main and starting windings for series connection of four-pole motor.

Fig. 103 or 104. The actual diagram to be used depends upon whether the winding is a series winding or series-parallel winding for the circuit on which the motor is to operate.

It will be noted that in the case selected the poles lap in slots 1, 10, 19 and 28 for the main winding. This is not essential but depends upon circumstances. The winding in the particular

motor selected could have been distributed as indicated in Fig. 97, but in that case the starting winding does not have so advantageous distribution as Fig. 96 and a smaller-size wire would probably be required for the main winding because the actual wires per slot would have been higher for the same effective turns and consequently the copper loss would have been higher. Under some conditions, however, and with other pole and slot com-

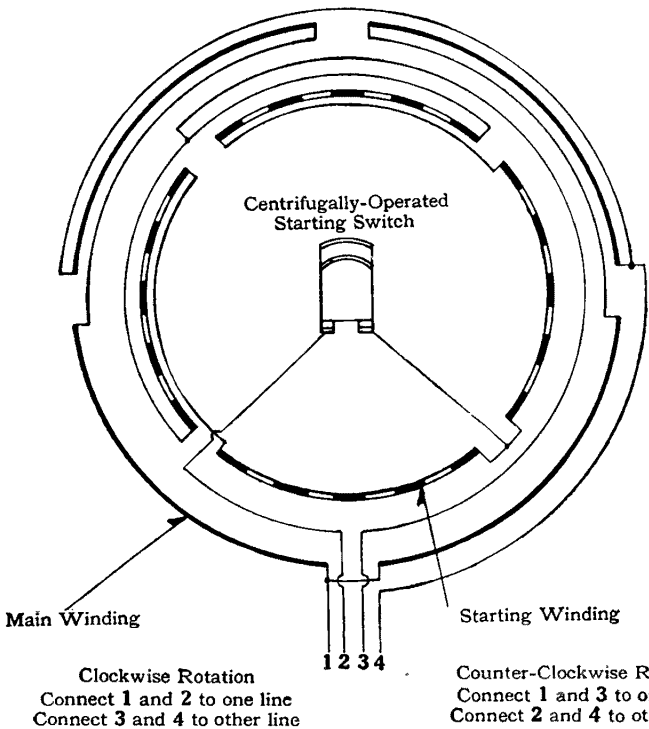


FIG. 104.—Connection diagram of main and starting windings for two-parallel connection of four-pole motor.

binations it is often more advantageous to distribute the winding so that adjacent main poles do not lap.

Mould-winding Method.—The mould-winding method is sometimes employed in winding small motors of the induction type. It takes its name from the fact that the pole coils are first wound on a mould, and then placed in the slots. In most cases one pole set of coils is wound together so that individual coils do not have to be connected together after being placed in the slots. In so far as the final results are concerned the mould type of winding has the same general appearance as the hand-wound

type of winding. Any mould-wound small motor can be hand wound when repairing one of these motors. The winding diagram of both the mould-wound and hand-wound motor is the same and is shown in Fig. 107 for the main winding.

Starting Windings.—The starting windings of motors with both mould-wound and hand-wound main windings are generally skein wound although they too may be mold wound but they

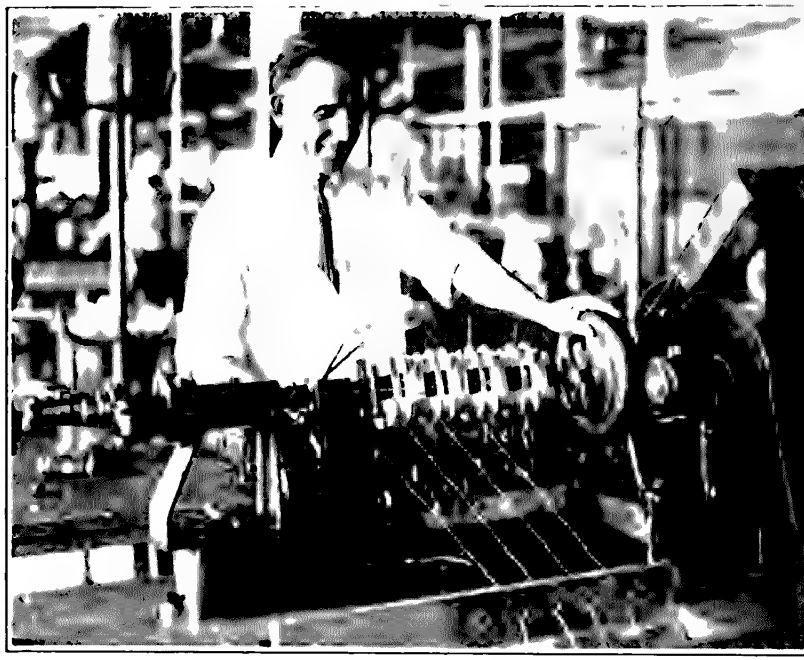


FIG. 105.—Method of winding four mould-wound sets of coils simultaneously for a four-pole motor.

should not be hand wound because the starting-winding resistance is very important and in the case of hand winding would vary too much. In the case where the starting winding is skein wound the procedure is the same as described above, but in case the starting winding is mould wound it would be necessary to measure the entire length of the starting-winding coil of one pole of the motor to be rewound and wind that same amount of wire into the slots by hand for each pole.

The split-phase, cage-wound and single-phase wound-rotor motors most commonly encountered today are generally two, four or six poles.

The four-pole 36-slot winding and connecting diagrams have been given above in connection with the description of the wind-

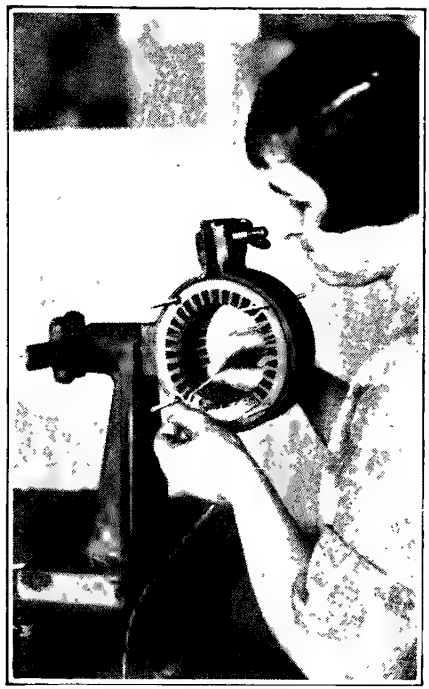


FIG. 106.—Hand winding a four-pole motor.

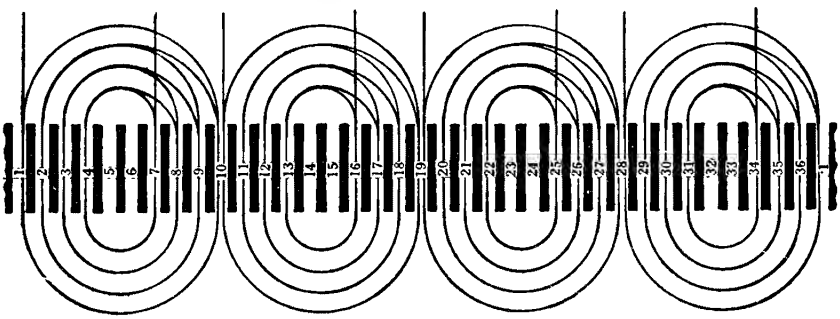


FIG. 107.—Main-winding diagram for mould or hand winding.

Slot No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Main Winding	1 1	2	2	2	1				1	2	2	2	1 1	2	2	2	1				1	2	2	2
Starting Winding			1	2	2	2	1 1	2	2	2	1				1	2	2	2	1 1	2	2	2	1	

FIG. 108.—Winding distribution for a two-pole 24-slot motor.

ing. Two- and six-pole winding distributions and connections are shown in Figs. 108 to 115.

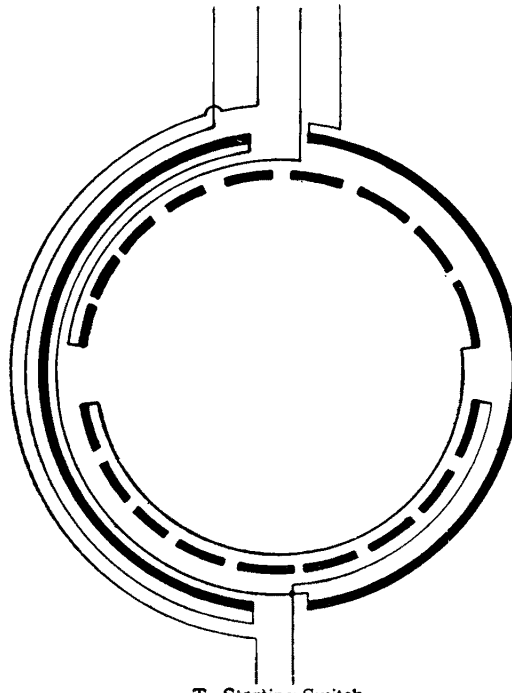


FIG. 109.—Connection diagram of main and starting windings for series connection of a two-pole motor.

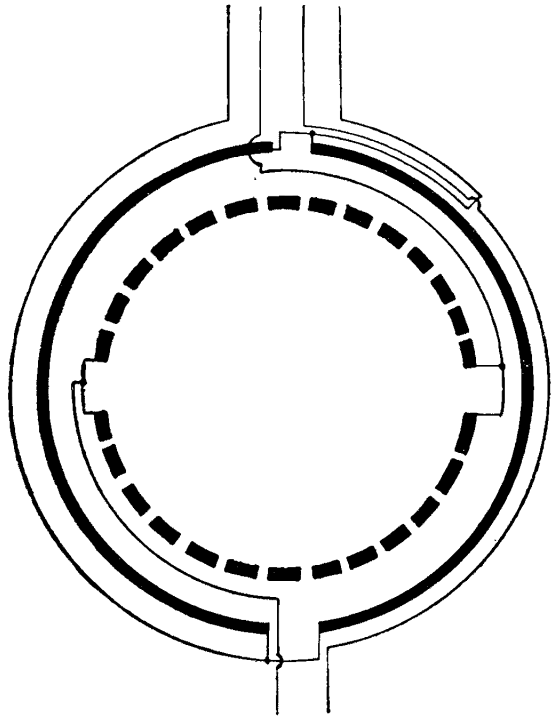


FIG. 110.—Connection diagram of main and starting windings for two-parallel connection of a two-pole motor.

Slot No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Main Winding	1 1	2	1		1	2	1 1	2	1		1	2	1 1	2	1		1	2	1 1	2	1		1	2	1 1	2	1		1	2	1 1	2	1		1	2
Starting Winding			2	1 1	2				2	1 1	2				2	1 1	2				2	1 1	2				2	1 1	2				2	1 1	2	

FIG. 111.—Winding distribution for a six-pole 36-slot motor.

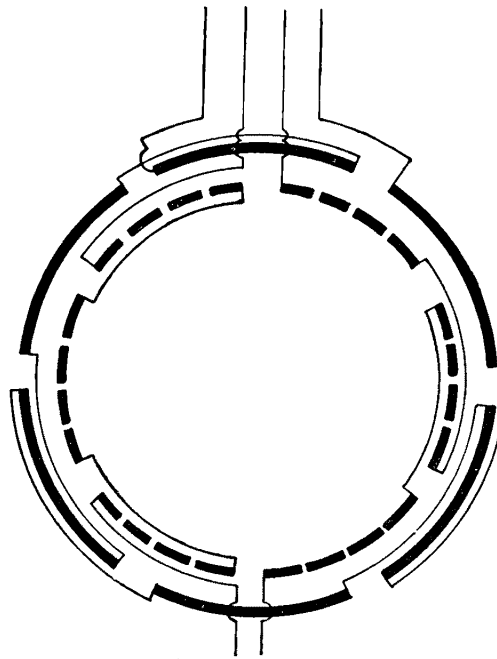


FIG. 112 —Connection diagram of main and starting windings for series connection for a six-pole motor.

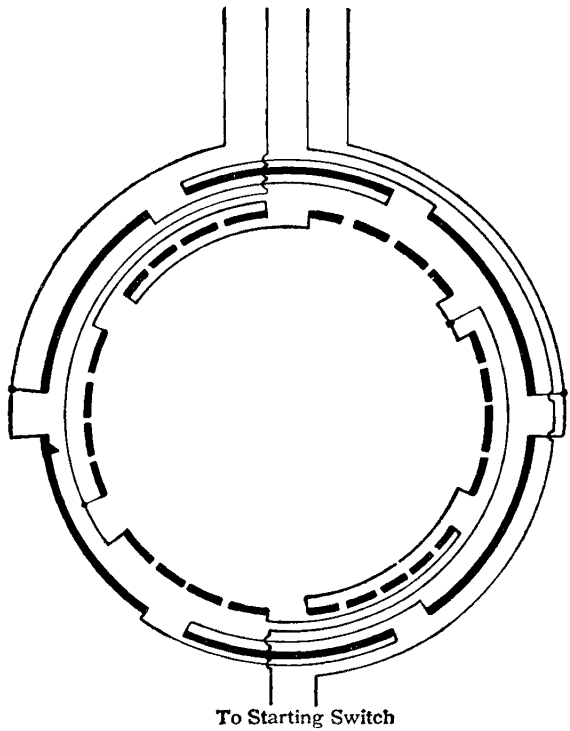


FIG. 113.—Connection diagram of main and starting windings for two-parallel connection of a six-pole motor.

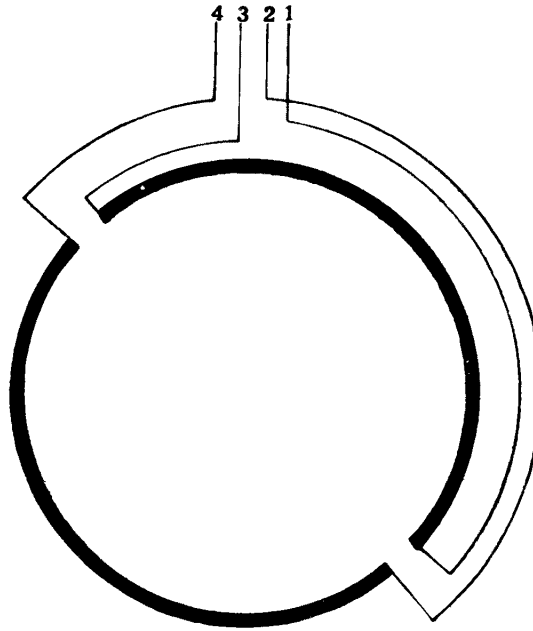


FIG. 114.—Connection diagram for a two-pole repulsion-induction motor.

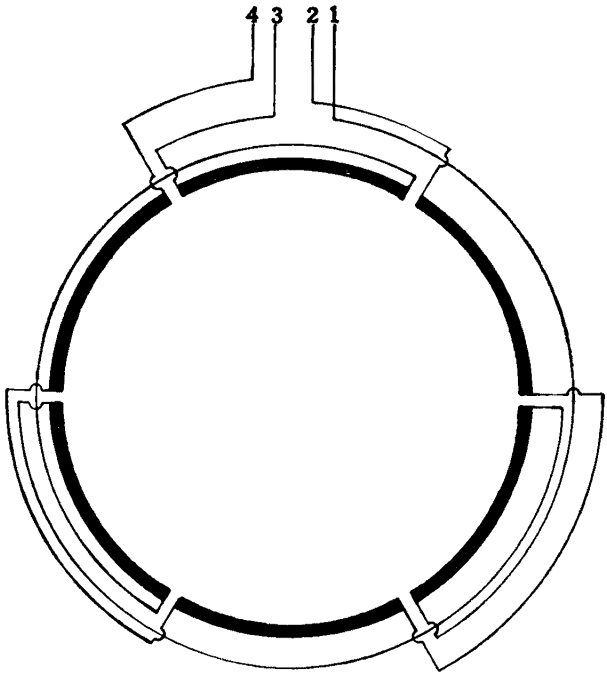


FIG. 115.— Connection diagram for a six-pole repulsion induction motor

CHAPTER VI

CHORDED WINDINGS OR THE EFFECT OF COIL THROW ON THE MAGNETIC FIELD

The effect of changes in frequency, phase, voltage or poles upon the performance of an induction motor and the necessary changes in the windings to preserve normal operation may be considered from the viewpoint of a change in voltage only and worked out by that method. By this is meant, for example, that a three-phase motor may be considered as a two-phase machine of a different voltage, in so far as the magnetic flux in the iron is concerned, also the heating, efficiency, torques, power factor, etc. Likewise a 25-cycle motor may be considered as a 60-cycle machine at a different voltage and operated accordingly.

A change in the number of poles can be looked upon as changing the speed of rotation of the magnetic field. With a given number of conductors this would at once affect the generated voltage or counter e.m.f. It was explained in the second chapter that the counter e.m.f. was practically almost equal to the applied e.m.f., or line voltage. Hence it may be seen that even a change in the number of poles can be considered as a voltage change and the number of wires in the coils correspondingly changed so as to give the same performance under the new conditions.

Since all these changes can be considered as voltage changes and will be so considered in the chapters to follow, it is necessary to investigate closely all the considerations that directly affect the voltage. The first one of these is the effect of winding the coils less than full pitch, or "chording" the coils, as it is most frequently called. The pitch, or span, is expressed in the number of the slots included between the two sides of the coil.

It is common knowledge that this pitch, or throw, must be somewhere near the quotient of the bore periphery of the core divided by the number of poles. For example, if the stator of a given motor had 72 slots and was wound for four poles, an individual coil would be expected to lie in slots 1 and 19 or there-

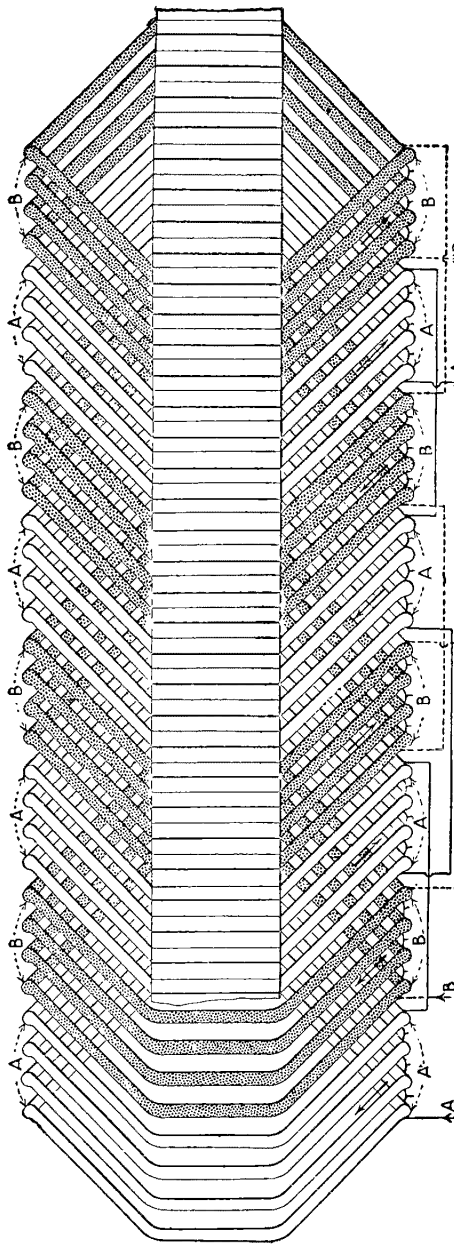


FIG. 116.—Thirty-two-slot four-pole stator with coil throw of 1 and 9 or exactly full pitch.

abouts. The reason for this is that, if there are four poles, the span of each coil must be somewhere near one-quarter of the bore periphery. In this case $72 \div 4 = 18$ slots, and $18 + 1 = 19$, hence the exact pitch for the coils of this winding would be 1 and 19. Similarly, a six-pole coil for the same core would lie in something like slots 1 and 13 and an eight-pole coil in slots 1 and 10. An examination of any induction motor wound in the usual way discloses the fact that the coils are seldom wound full pitch, as in Fig. 116, but always a few slots less, as in Fig. 117. It is the purpose of this chapter to discuss the reasons for winding the coils less than full pitch and the effect upon the voltage of the machine caused by this practice, which gives a fractional-pitch winding.[1]

One of the immediate results of spreading the coil less than full pitch is to place in the same slot coils carrying currents of different phases. This is illustrated in Figs. 116 and 117, which show a two-phase four-pole winding placed in 32 slots. In Fig. 116 the throw of the coil is 1 and 9, or exact pitch, and it can be seen, that all the slots contain coils entirely of the same phase; that is, all slots contain either A or B coils. On the other hand, in Fig. 117, the throw of the coil is one less than full pitch, or it is chorded one slot and wound in slots 1 and 8. As a result, it is seen that in slots 1, 5, 9, 13, etc., the coil lying in the top of the slot is of a different phase from the coil in the bottom of the slot. At first thought this appears to be an interference, but it is really not so, since the values of the currents in the two phases at a given instant are different; and since one is increasing and the other decreasing, the effect on the magnetic circuit is due not only to the amount of current in the two coils, but also to their phase relation. Hence the result of chording is not to make the two phases interfere with each other in any way, but simply to have a tendency to reduce the number of turns in the coils, as will be described. That the resulting magnetic field which rotates is due to the interaction of all the phases in this way was mentioned in Chapter II.

Advantages of Chording the Winding.—There are three main reasons for winding the coils less than full pitch: (1) The length

[1] A longer theoretical discussion of fractional-pitch windings is found in Adams, Cabot and Irving, *Transactions A. I. E. E.*, vol. 26, pp. 1485–1503, 1907; and Jens Bache-Wiig, vol. 27, pp. 1077–1085, 1908.

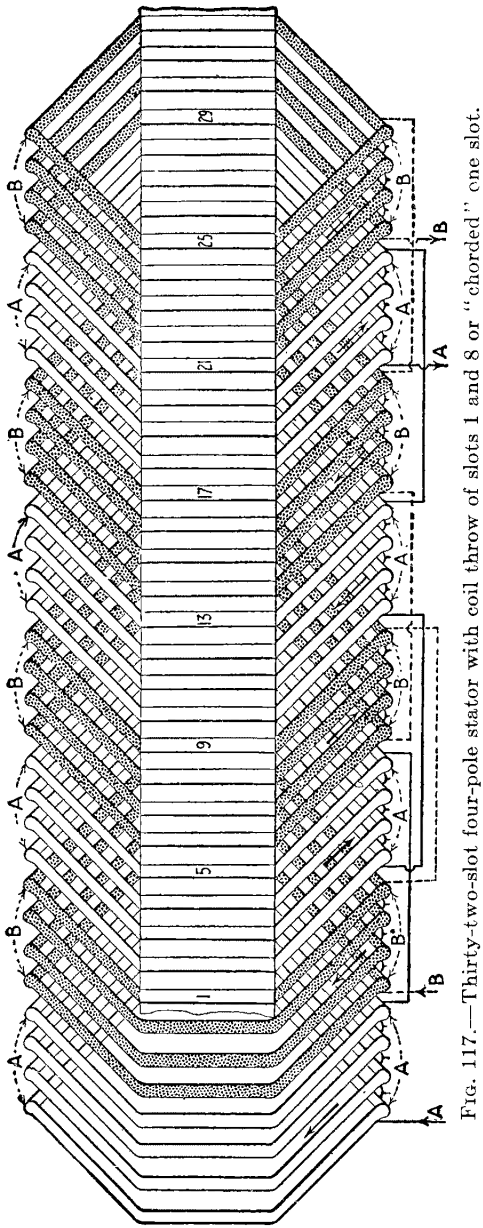


FIG. 117.—Thirty-two-slot four-pole stator with coil throw of slots 1 and 8 or "chorded" one slot.

of the mean turn is reduced; (2) it has the effect of changing the number of turns in the coil; (3) the over-all length of the winding parallel to the shaft is reduced, thus requiring less space in the end brackets which carry the bearings.

Discussing these effects in order, the reduction in the length of the mean turn accomplishes two results: First, less wire is required to form the coils, which is a slight economy; and second, the total resistance of the winding is reduced. This reduction in resistance, in turn, has two beneficial results—the one a reduction in copper loss with a corresponding gain in efficiency and the other a reduction in heating, since the heating is measured by the total losses that must be dissipated. The reduction in cost and the improvement in performance are both of a relatively small order, but they represent the minor details in which a nicely balanced design has an advantage over one more crude. The reason for the shortening of the mean turn can be seen from Fig. 118. The coil *ABCDEF* is wound in slots 1 and 7 and the coil *AGHIJF* is wound in slots 1 and 6. It will be noted that the gain in length by the shorter coil is due not alone to the fact that the chord *AH* is shorter than *AC*, but also to the fact that the point *G* is considerably nearer the core than the point *B*; or in other words, the angle *AGH* is greater than *ABC*.[1]

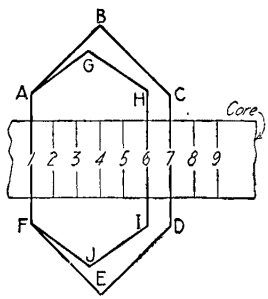


FIG. 118.—"Chording" shortens the length of wire in the coil.

The second effect of chording is that it acts in the same manner as changing the number of turns in series in the coil. Suppose, for example, that a designer of induction motors has made a calculation and finds that if six turns of wire are put in a coil there will be slightly too many turns for the best result, and if five turns are used there will be slightly too few. If there was not the recourse of chording the coil, it would be necessary to decide which was the lesser of the two evils, or else to change the number of slots. The latter might not be possible as it is desirable to have the total number of slots a multiple of the number of phases times the number of poles, and this could

[1] See Gray E. Miller, "Determining the Form of a Diamond Coil," *Electric Journal*, vol. 8, p. 94.

not be shifted in fine adjustments. However, it is possible to chord the coil and by the simple expedient of winding the coils one or more slots less than full pitch, the effect can be produced of putting 5½ or 5¾ turns in a coil, or in fact a very fine adjustment to give exactly the best possible combination. There would of course be six actual physical turns of wire in the coils, but their magnetic effect would be reduced by the chording to 5½ turns or whatever was desired.

The effect of the turns in the coil varies as the sine of half of the angle in electrical degrees which the coil spans. To illustrate, if there are 72 slots in an eight-pole machine, the coils would spread exactly full pitch if they lay in slots 1 and 10; or in other words, if there were eight slots between the two slots in which the two sides of any coil were located. Such a coil would span 180 electrical degrees. One-half of 180 deg. is 90 deg., and the sine of 90 deg. is 1; therefore the effect of the turns in such a coil is 1, or maximum. Suppose, instead, the coil lies in slots 1 and 8. It would then span 140 deg. electrically, since $72 \div 8 = 9$ slots represents 180 deg.; one slot therefore represents 20 deg. and seven slots 140 deg. The sine of half of 140 deg., or 70 deg., is 0.94. Hence it follows that the effect of the turns in this coil is less than that of the full-pitch coil by the ratio of 0.94 to 1.

Changing Poles with Constant Throw.—The foregoing is of interest in the present problem, because it is often possible in making alterations in the winding to change at the same time the span of the coils by one slot, more or less, by springing the coil mechanically, and so improve the performance of the machine under the new conditions. The point becomes of vital importance immediately when changing the number of poles without changing the throw of the coils. Referring again to the 72-slot motor, assume that the coils are wound in slots 1 and 8. For an eight-pole connection these coils will have an effect of 0.94 as explained. If the connections are changed for six poles, the effect is entirely different; $72 \div 6 = 12$ and $180 \div 12 = 15$, or each slot represents 15 electrical degrees. A throw of 1 and 8 covers seven complete slots, or $7 \times 15 = 105$ deg.; the sine of half of 105, or 52.5 deg. = 0.79, which means that when connected for six poles the coils have an effect of only 0.79, as against 0.94 when connected for eight poles.

It is possible to avoid using the sine of half the angle and secure a factor that is sufficiently accurate for all practical purposes by using the expression,

$$\sqrt{\frac{(Number\ of\ slots\ per\ pole)^2 - 2(number\ of\ slots\ dropped)^2}{(Number\ of\ slots\ per\ pole)^2}}$$

Using the same eight-pole example as above, the number of slots per pole is $72 \div 8 = 9$, and the pole pitch is 1 and 10. When the coil is wound 1 and 8, it spans 7 slots and there are $9 - 7 = 2$ slots dropped. The expression then becomes

$$\sqrt{\frac{(9)^2 - 2(2^2)}{(9)^2}} = \sqrt{\frac{73}{81}} = 0.948$$

and similarly for the six-pole,

$$\sqrt{\frac{(12)^2 - 2(5)^2}{12^2}} = \sqrt{\frac{94}{144}} = 0.807$$

which agrees roughly with the other method.

Explanation of Term "Chord Factor."—A coil should in no case be chorded more than half of the pole pitch, as secondary disturbances of the magnetic field are occasioned by chording which become prohibitive at that point. The expression "sine of half the angle spanned by the coil" is given the name "chord factor," and it should be considered in the work of reconnecting. For example, if the poles are changed from 8 to 6, as in the example given, and the chord factor changes from 0.94 to 0.79, the new line voltage should be $0.79 \div 0.94$ times the old, neglecting the effect of other changes that are being made. If nothing else was undergoing change and the normal voltage was 440 in the first place, it should be $440 \times \frac{0.79}{0.94} = 370$ after the change is made; or, expressing it another way, if it was still operated at 440 volts after the change, the motor should be thought of as operating at about 18 per cent. overvoltage.

Since the foregoing is one of the important points in induction-motor winding, it is worth while to consider carefully how this effect is produced. It could be stated briefly by saying that the two sides of the coil, which of course are in series, are not strictly in phase with each other. But this can be seen more clearly

from diagrams. Suppose, for example, that a two-pole motor is considered and that a cross section is taken through the core and windings in a plane at right angles to the shaft, as shown in Fig. 119. The dotted parallel lines in the peculiar twin pattern represent the lines of force, or magnetism of the rotating magnetic field, which is rotating in a clockwise direction, as shown by the arrow outside. The small arrows on the lines of flux indicate the magnetism coming from the stator north pole at the top into the rotor core and out again into the stator at the bottom, forming a south pole.

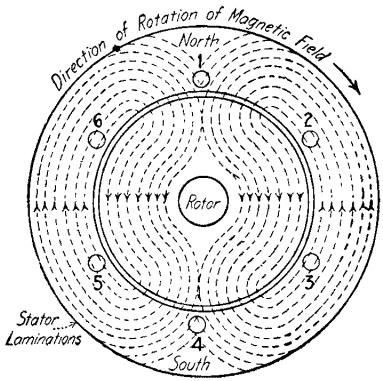


FIG. 119.—Cross section through a two-pole stator showing magnetic lines of force.

Of course this magnetic field is being set up by polyphase alternating currents, but it need only be thought of as shown in the figure and as if excited by direct current. The six small circles, in the stator and near the bore, numbered 1 to 6, represent the conductors of the stator winding. Consider that these six conductors constitute the complete winding. As the magnetic field swings around in a clockwise direction, it cuts these six conductors because without doing so it cannot get from the stator into the rotor and back and at the same time rotate. As the conductors cut this field, each one generates a voltage which in value and direction may be represented by the arrows or vectors of Fig. 120.

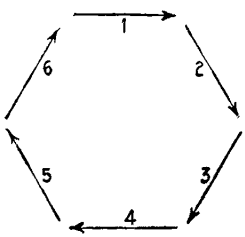


FIG. 120.—Vector diagram showing direction at any instant of voltages generated by conductors in Fig. 119.

The reason these voltages are shown in a hexagon is because they are not all generated at the same time, but in a succession. For example, the north pole sweeps by conductor 1 and a fraction of a second later past 2 and then past 3 and so on around to 6, and this can be represented by the sides of a hexagon which finally closes on itself, as shown in Fig. 120. The reason the arrows for conductors 1 and 4 are shown in the same direction is because the north pole is sweeping past 1 to the right at the same

instant that the south pole is sweeping past 4 to the left, so that the voltages in these two conductors are in the same direction at the same instant. Similarly, 2 and 5 and 3 and 6 are in pairs. Suppose now that 1 and 4 had their ends connected together both at the front and the back of the machine so that they formed a short-circuited turn. The voltage then which would be effective in forcing current around this short circuit would be that generated in 1 plus that in 4 and may be represented by line 1 plus 4, or *KL*, shown in Fig. 121. *KL* then would represent the voltage of a coil wound exactly full pitch or from the center of a north pole to the center of a south pole.

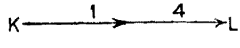


FIG. 121.—Adding voltages generated by conductors 1 and 4, Fig. 119.

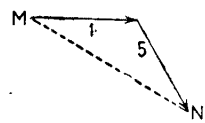


FIG. 122.—Adding voltages of conductors 1 and 5, Fig. 119.

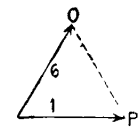


FIG. 123.—Adding voltages of conductors 1 and 6, Fig. 119.

Suppose, instead of 1 and 4, that 1 and 5 had their ends connected so as to form a short-circuited turn. The voltage which would be effective in forcing current around through this short circuit would be *MN*, shown in Fig. 122, which it will be seen is somewhat less than *KL* in Fig. 121. The arrow *MN* is made by adding 1 and 5 which in themselves are just as long as 1 and 4, but instead of lying in a straight line they are at an angle to each other. This angle shows what is meant by the two sides of the coil being out of phase with each other, or still another way to say it would be that the magnetic field is not working on 1 and 5 in exactly the same way at the same instant as it was on 1 and 4. Therefore, when 1 and 5 are short-circuited, giving the voltage *MN*, they represent a coil chorded to two-thirds of full pitch, or they have the effect instead of being two conductors in series, of being only 2×0.866 conductors, or 1.73 conductors. This is because two-thirds pitch would be $\frac{2}{3} \times 180$ deg. = 120 deg. and the sine (0.5 of 120 deg.) = sine 60 deg. = 0.866.

In the same way conductors 1 and 6 could be joined in series to form a short-circuited turn, and the voltage of such a turn would be represented by *OP* in Fig. 123 which is made up of

1 and 6, which are at an angle of 60 deg. with each other. In this case, instead of having the effect of two conductors in series so far as voltage generation is concerned, the effect will be that of only one, since 1 and 6 represent one-third pitch, and ⅓ of 180 deg. = 60 deg. and the sine (0.5 of 60 deg.) = sine 30 deg. = 0.5. Therefore $2 \times 0.5 = 1$. Of course six slots per pole is a small number and it can be seen that with 12 or 15 slots per pole at his disposal the designer can chord to get almost any value required.

Effect of Chording.—It will be noted that in this graphic explanation the conductors were spoken of only as generating counter e.m.f., as explained in the first chapter and never as setting up the field. However, it should be understood that in the magnetizing function of the winding, also, the chording produces the same effect as explained here by means of the generator idea.

The third effect of chording has been mentioned as shortening the coils axially. This is very useful, especially in the case of two-pole and four-pole machines where the coils, if made full pitch, would protrude so far at each end as to require special end brackets. These long end brackets in turn would spread the bearings farther apart and make necessary a larger shaft to keep down the shaft deflection. Hence it is of prime importance to shorten up on the coil ends in this manner. Also, the end windings are mechanically stiffer. There are other effects of chording known to the designer, which are desirable. These are, for example, a reduction in the leakage reactance, thereby giving better torques and possibly better power factor and efficiency. Also, it is very beneficial in reducing magnetic noise to employ chording, depending on the combinations of slot numbers, so that, taken all in all, chording is one of the prime features in studying the effect of winding changes upon the performance of a machine.

Distribution Factor Less Important.—Another winding factor that acts in a similar manner to the chord factor just discussed is the one known as distribution factor. This is not subject to control as is the chording and is relatively much less important, but should be mentioned in passing, as its neglect might occasion trouble if a combination was employed which otherwise was on the ragged edge of failure. This distribution factor has to do

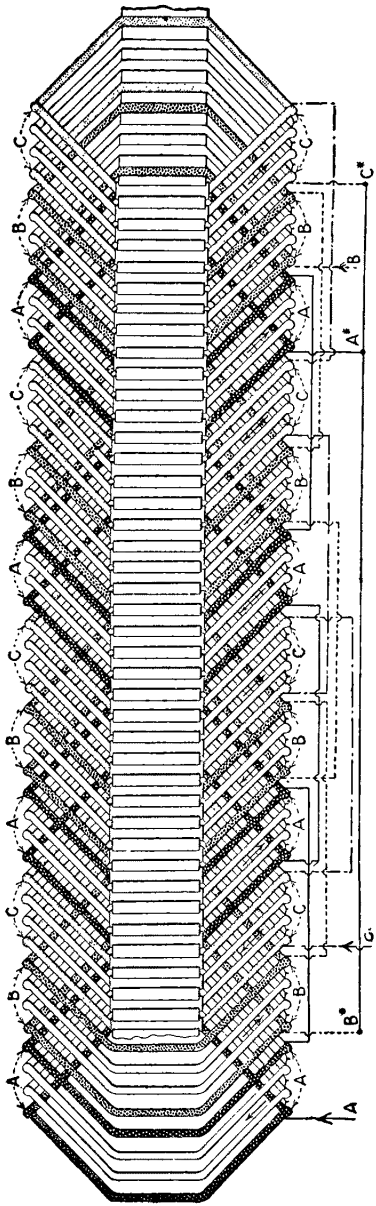


FIG. 124.—Forty-eight slot stator with "phase coils" arranged for three-phase, four poles.

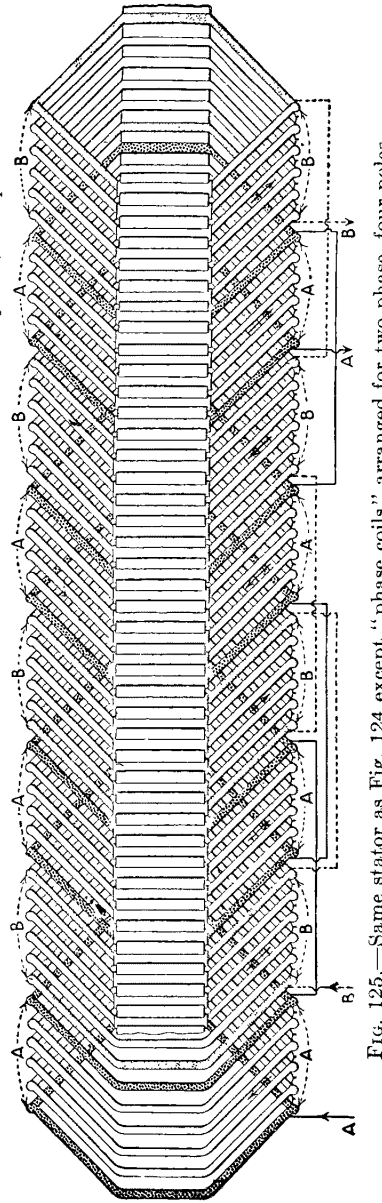


FIG. 125.—Same stator as Fig. 124 except "phase coils" arranged for two-phase, four poles.

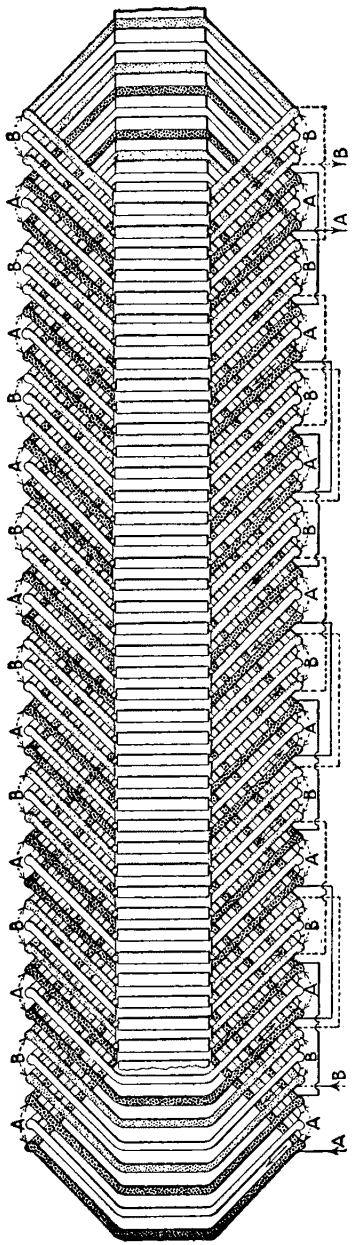


FIG. 126.—Stator as shown in Fig. 124 except with "phase coils" arranged for two-phase, eight poles.

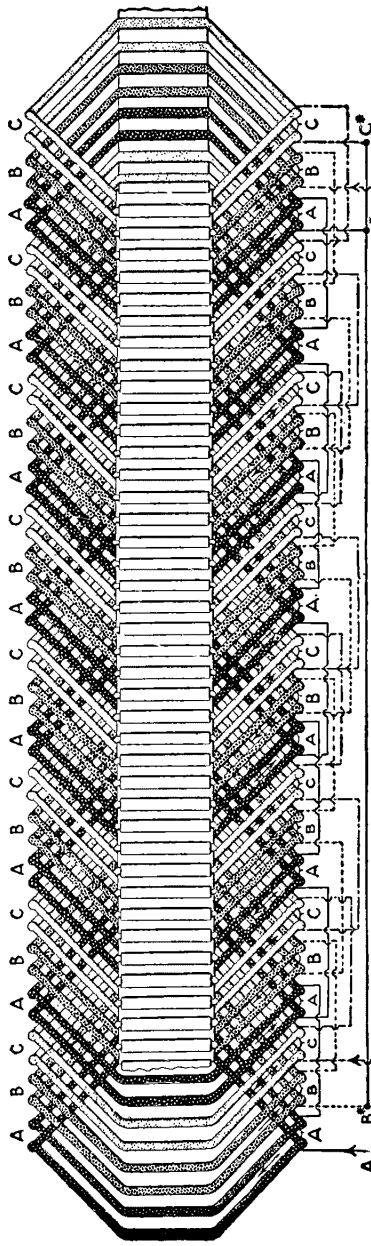


FIG. 127.—Stator as shown in Fig. 124 except with "phase coils" arranged for three-phase, eight poles.

with the fact that the coils in one phase of a two-phase motor are spread over half of the face of a pair of poles and in a three-phase motor are spread over one-third of the face of a pair of poles. This factor varies a trifle with the number of slots per phase and pole, but a fair value for average two-phase windings is 0.905, which is about the ratio of one side of a square inscribed in a circle to one-fourth of the circumference. For a three-phase machine a fair average value is 0.955, which is practically the ratio of one side of a hexagon inscribed in a circle to one-sixth the circumference, or $3 \div 3.14$.

Ordinarily this factor is not troublesome and, if forgotten in changing from two- to three-phase, or vice versa, would not cause any great disturbance. However, in dealing with special machines—as for example, motors wound for two sets of poles—the distribution factor may be more important than the other factors. In such a case the two-phase distribution factor may be as low as 0.707 and the three-phase factor as low as 0.866 because the coils for a four-pole motor, for example, are spread over the pole face of an eight-pole motor. Mention is made of this fact in connection with Fig. 200, Chapter XII.

Phase Insulation Important.—Another general factor is that of "phase insulation." It is the practice of many manufacturers to put heavier insulation on the coils at the ends of the polar groups which are mechanically adjacent to one another and which are also subjected to the voltage between phases, which may be the maximum voltage between supply lines. Such coils are drawn in heavy lines in Fig. 54*b*. By rearranging this diagram for two-phase it appears at once that both the number and location of these so-called "phase coils" are changed, and in changing the number of poles the number and location of the phase coils must also be changed. In fact, whatever reconnection is attempted, the phase coils should be checked and rearranged, since this is comparatively easy and adds considerably to the protection of the machine from breakdowns of insulation.

To illustrate the manner in which the phase coils should be rearranged when changing phases or poles, Figs. 124 to 127 are shown. All four of these figures show the same winding in 48 slots and with a coil throw of 1 and 9. In Fig. 124 the phase coils are arranged for three-phase four poles, in Fig. 125 for two-phase four poles, in Fig. 126 for two-phase eight poles and in

Fig. 127 for three-phase eight poles. It will be noted that, since the throw of the coils remains unchanged, it represents a chord factor corresponding to ⅔ pitch, or 120 deg. for the four-pole winding (since 8 slots = ⅔ of 12), and a chord factor corresponding to 1⅓ or 240 deg. for the eight-pole winding (since 8 slots = 1⅓ of 6). Since the chord factor is equal to the sine of one-half the spread angle and since the sine of 120 deg. = the sine of 60 deg. = 0.866, the effect of the underchording on the four-pole winding is exactly the same as the effect of the overchording on the eight-pole winding.

In all four diagrams the coils having heavier insulation than the others are shown shaded, the different degrees of shading representing the coil having additional insulation in the different phases. In Fig. 124 there are 12 pole-phase groups of four coils each. The two outside coils of each group have heavier insulation, as indicated; this will give 24 phase coils, or one-half the winding is phase coils. The winding Fig. 125 has eight pole-phase groups, with 16 phase coils, or one-third of the total winding is phase coils. In Fig. 126 the winding has 16 pole-phase groups, making it necessary that there be 32 phase coils. The arrangement in Fig. 127 gives 24 pole-phase groups of only two coils per group, hence all the coils must be phase coils with increased insulation.

Plotting Pictures of the Magnetic Field.—In Chapter II there was shown a method of plotting a physical representation of the rotating magnetic field as it varies from point to point around the air gap of an actual machine. The same method may be used to show what effect is produced on its shape by changing the throw of the coil, or chording the winding as it is called. The latter effect is thus investigated for a change of one slot at a time from full pitch to less than half pitch. By full pitch is meant that the span of the coil is exactly the same distance as that from the center of a north pole to the center of an adjoining south pole, and by half pitch that the coil spans or throws only half that distance. Referring to Figs. 18 and 19 of Chapter II the small "stair-step" figures represent cross sections of the magnetic field existing in the motor as the alternating currents in the windings vary in value from instant to instant, and a comparison of the small figures shows that the magnetic field actually travels around the stator bore or "air gap" at a uniform

rate. The number of revolutions that it makes in one minute is equal to 120 times the number of cycles per second of the supply circuit divided by the number of poles in the stator. Expressed in symbols this would be $S = 120\frac{f}{p}$; where S is the speed of rotation in r.p.m., f is the frequency in cycles per second, and p is the number of poles.

In order to make clear the field photographs or diagrams of the present chapter and to obviate the possibility of confusion regarding them, attention is called to the fact that they represent the

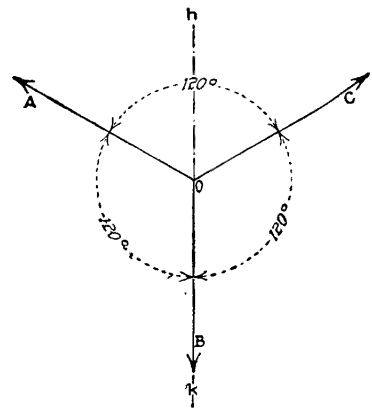


FIG. 128.—Normal relation of the currents in a three-phase motor.

conditions existing in the windings at an instant of time when the current in one of them is at its maximum value. Since we are dealing with three-phase motors, the currents in the windings connected to the other two phases will at that instant both be equal to one-half their maximum values. This may be explained by reference to Figs. 18 and 19 of Chapter II, which represent the values of the currents in the three phases for every 30 deg. of a complete cycle of 360 deg. Suppose these three currents are represented by the three branches, A, B and C of the "Y" illustrated in Fig. 128, each of which is 120 deg. from the other, and that a vertical reference line hk is drawn through the center o. Now assume that the "Y" rotates in a counterclockwise direction about this center while the line hk remains stationary, and that the three branches assume the successive positions represented in the second column of Fig. 129. The values of the

currents at any instant of time will be represented by the length of their horizontal projections upon the line *hk*. If the maximum value of each current is assumed to be 1 ampere, the instantaneous values of the three for each 30 deg. of a complete cycle would be those given in the last three columns of Fig. 129. Projections

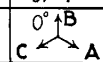
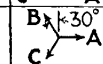
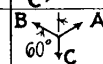
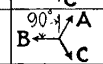
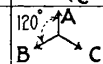
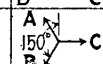
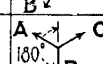
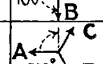
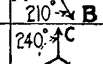
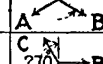
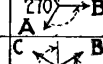
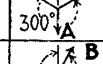
ANGLE, DEG.	POSITION OF BRANCHES OF Y	CURRENT A AMPERES	CURRENT B AMPERES	CURRENT C AMPERES
0		- 0.5	+ 1.0	- 0.5
30		0	+ 0.866	- 0.866
60		+ 0.5	+ 0.5	- 1.0
90		+ 0.866	0	- 0.866
120		+ 1.0	- 0.5	- 0.5
150		+ 0.866	- 0.866	0
180		+ 0.5	- 1.0	+ 0.5
210		0	- 0.866	+ 0.866
240		- 0.5	- 0.5	+ 1.0
270		- 0.866	0	+ 0.866
300		- 1.0	+ 0.5	+ 0.5
330		- 0.866	+ 0.866	0
360	Same as 0 Deg.			

FIG. 129.—Instantaneous values of the currents in a three-phase motor throughout a complete cycle.

that lie above the center *o* are taken to be positive and those that lie below as being negative.

Note carefully at this point that in connecting the winding the *B* phase must be reversed, as explained in Chapter II. For this reason the plus and minus signs in the column of Fig. 129 headed "Current B Amperes" must all be reversed before proceeding to plot the magnetic field. This will be understood by a

reference to Fig. 18 in Chapter II where it will be seen that the plus and minus signs on the B phase are reversed from Fig. 129.

Another fact that should be observed is that in Figs. 18 and 19 the throw of the coils remained constant and stair-step pictures were drawn for 12 different periods of time spaced over one cycle. That is to say, the interval of time between successive pictures in Figs. 19 and 20 is $\frac{1}{12}$ of $\frac{1}{60}$ of a second, or $\frac{1}{720}$ of a second. However, in Figs. 130 and following, now to be considered, the

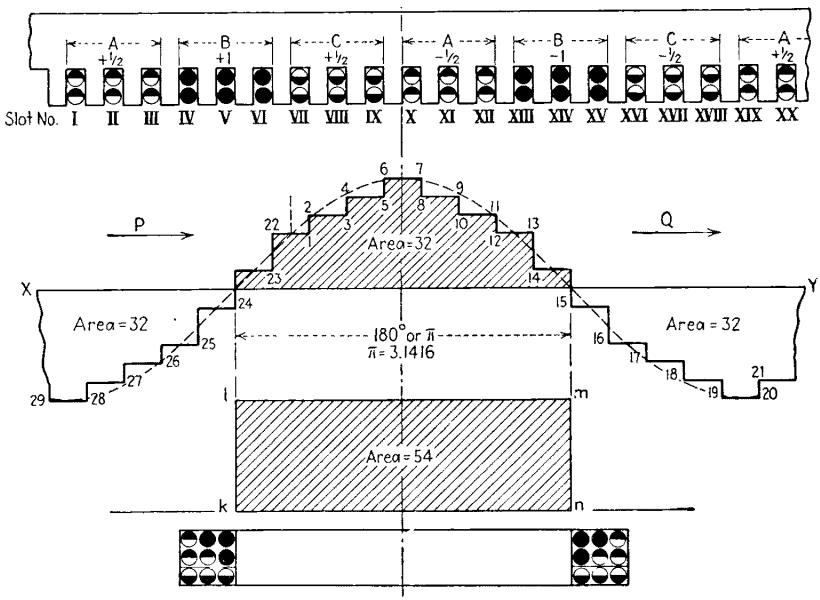


FIG. 130.—Building the magnetic field.

same value of the current is kept in all cases but the throw of the coil is changed one slot at a time from 1 and 10 to 1 and 5. The value of the current which is chosen as most convenient for Figs. 130 and following is the point on the cycle where the current in phases A and C is $\frac{1}{2}$ ampere and the current in phase B is 1 ampere. This is the position marked 0 angle in Fig. 129 remembering to change the sign on the B phase from $+1$ to -1, as explained in Chapter II, otherwise the sum of the three currents would be zero at all points and no magnetic field would result. Another way of explaining this is to say that the leads marked A and C in Figs. 131 to 136 inclusive are terminals but

that the lead marked B is the star connection; hence the values of the B current in Table Fig. 129 must be changed in sign.

Getting from the Current and the Winding to the Magnetic Field.—When polyphase currents flow in the stator winding, a wave or pulse of magnetism, shown by the curved shaded portion of Figs. 130 and following, passes around the rotor and its shape changes with the throw of the coil. It is the object of these paragraphs to explain how the shape of the field can be drawn or visualized. Of course in these figures the stator is assumed to be "developed" or rolled out in a flat line for ease in construction. Consider now the winding in a 54-slot stator core connected three-phase six poles and having the coil throw full pitch, *i.e.*, wound in slots 1 and 10. Such a winding is shown in Fig. 137. Imagine a cross section made through the center of the core at right angles to the shaft. If the resulting cross section of the stator were rolled out flat, the slots and the coil ends would look like the top view in Fig. 130. For ease in explaining there is assumed to be only one conductor in the coil or two in the slot. Because the coils are wound full pitch there is no overlapping of the phases but these follow regularly 3 slots A, 3 slots B, 3 slots C, etc. To distinguish the phases, since the current in phase B is twice that in A or C, the B conductors are shown completely shaded, the conductors in phase A are shaded on their top halves and those of phase C are shaded on their bottom halves. Above the slots are shown the values $+\frac{1}{2}$, $+1$, $+\frac{1}{2}$, then $-\frac{1}{2}$, -1, $-\frac{1}{2}$, showing the current values as they approach or flow away from the observer. Since there are 54 slots and six poles, the conductors in nine slots form one magnetic pole. Since there are 1×6 ampere conductors in the B phase and $\frac{1}{2} \times 6$ in each of the A and C phases, there is a total of $6 + 3 + 3 = 12$ ampere conductors per pole. If these conductors were gathered into one field coil like a direct-current shunt-field coil, shown at the bottom of Fig. 130, they would set up a magnetic field which might be represented by the shaded rectangle as shown, whose area is equal to the 6 ampere conductors multiplied by the span of the nine slots or an area of 54. However, when these same conductors are not concentrated, but are distributed in slots as shown at the top of Fig. 130, the picture of the magnetic pole is no longer a rectangle but a "stair-step" picture as shown in the

center of Fig. 130. Its shape and magnitude are arrived at as follows:

Assume there is no picture in the center of Fig. 130 and that the observer is going to start at some point, as, for example, the vertical line between 22 and 1, and proceed in the direction of the arrows P and Q, exploring the magnetic field as he goes. In passing from the start to 1, slot 7 is crossed, which raises the magnetic potential by $2 \times \frac{1}{2} = 1$ ampere conductor. Accordingly a step up from 1 to 2 is made and another horizontal progression from 2 to 3. Here slot 8 is crossed with $2 \times \frac{1}{2} = 1$ ampere conductor and another rise in potential from 3 to 4 results. From 4 to 5, slot 9 is crossed, raising the line from 5 to 6. From 6 to 7, slot 10 is crossed and here something new happens because the current is now minus in the A phase, so a step down is made from 7 to 8. Similarly steps of 1 ampere conductor are made down from 9 to 10 and from 11 to 12 in passing slots 11 and 12. In going from 12 to 13, slot 13 is crossed and, since it is the B phase, $2 \times 1 = 2$ ampere conductors are passed. This makes the steps from 13 to 14 and the next two steps to 15 and 16 twice as high as the previous ones which involved only 1 ampere conductor. In this way the graph is traced to point 21. Going back to the starting point, which was the start line between 22 and 1, the left-hand branch can be traced by the same reasoning or can be made to correspond to the right-hand branch. That is to say, points 22, 23, 24, 25, 26, etc., correspond to points 13, 14, 15, 16, 17, respectively. This determines the complete graph for a whole cycle but it is not yet determined where the line XY shall be drawn in. This is the base or neutral line and it must be drawn so that all the areas enclosed between it and the graph are equal above and below the base line. In other words, the shaded area marked "Area = 32" above the base line and which represents a north pole, for example, must be equal to the south pole on either side of it. Since in this case the winding is perfectly regular and symmetrical, it is easy to draw in the line XY, as shown, which meets the requirements.

It is interesting to stop at this point and check up on the reason why the same ampere conductors, if concentrated in one field coil, would generate an area equal to 54 and, when distributed, only an area equal to 32. Theoretically, the distributed winding would set up an area bounded by a sine curve, which is the smooth

curve indicated in the center figure of Fig. 130. By geometry the area under a sine curve for 180° or one-half cycle is 2 where the 180° is expressed as π or 3.1416. Also the maximum value of the sine curve is 1; thus, with a base of 3.1416 and a height of 1,

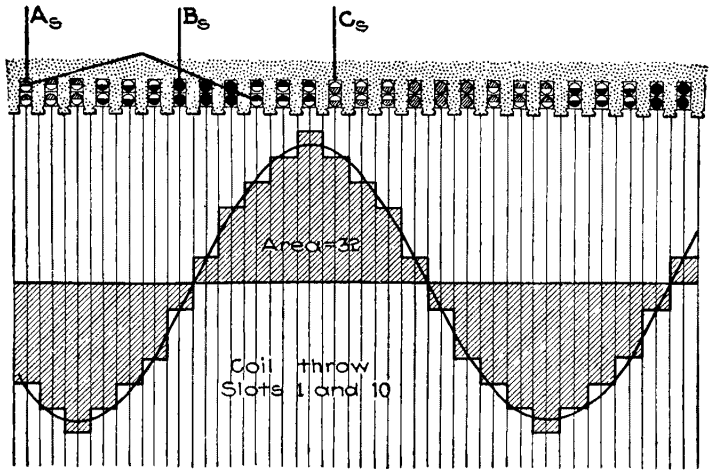


FIG. 131.—Picture of magnetic field set up by winding in Fig. 137.

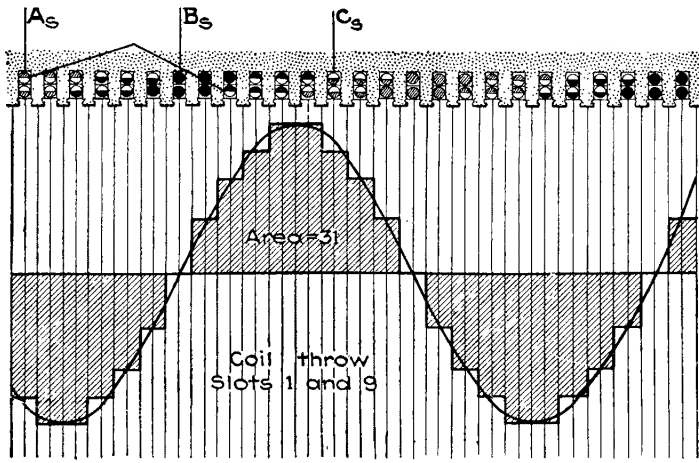


FIG. 132.—Magnetic field if winding in Fig. 137 is chorded to slots 1 and 9.

the resulting rectangle would equal an area of 3.1416 as compared with the value of 2 under the sine curve. Owing to distributing the winding, the rectangular area of 54 is cut by 2/3.1416, making it $\frac{54 \times 2}{3.1416} = 34.3$. Also there is a further reduction

from the fact that distribution is three phase, as explained in Chapter VII. This factor is 0.955. The final resulting area is $34.3 \times 0.955 = 32.75$. This is slightly greater than the 32

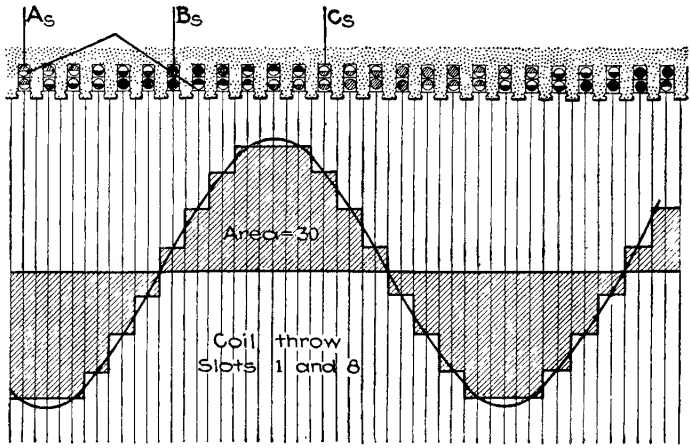


FIG. 133.—Magnetic field if winding in Fig. 137 is chorded to slots 1 and 8.

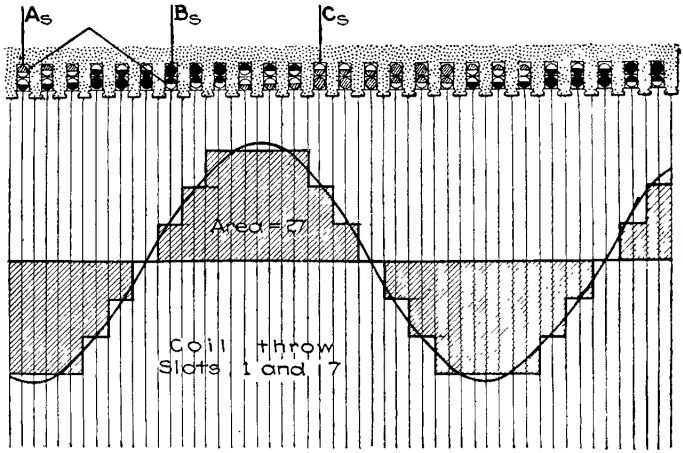


FIG. 134.—Similar to Fig. 133 except chorded to slots 1 and 7.

shown on the graph because the square corners of the graph do not closely enough approximate the area under the sine curve.

Following the same procedure and noting the changes in ampere conductors due to the overlapping of phases, *i.e.*, different phases in the top and bottom of the same slots, the pictures shown in Figs. 131 to 136 inclusive are drawn.

Effect of Chording Shown Graphically.—Figures 131 to 136 inclusive show the magnetic field constructed, as just explained for a 54-slot three-phase six-pole winding when the throw of the coil is changed one slot at a time from slots 1 and 10 as in Fig. 137,

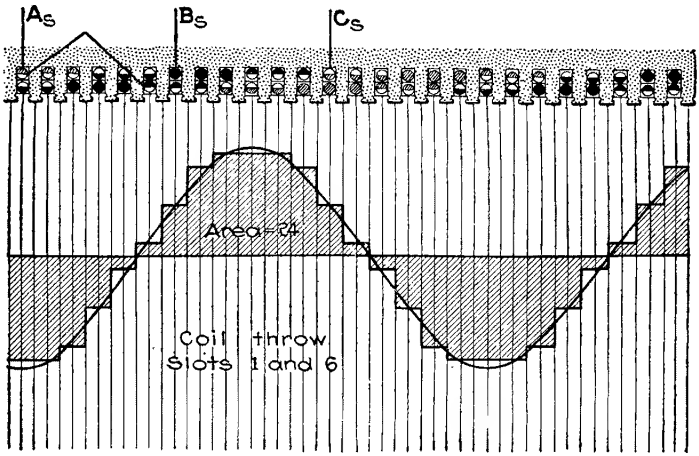


FIG. 135.—Similar to Fig. 134 chorded to 1 and 6.

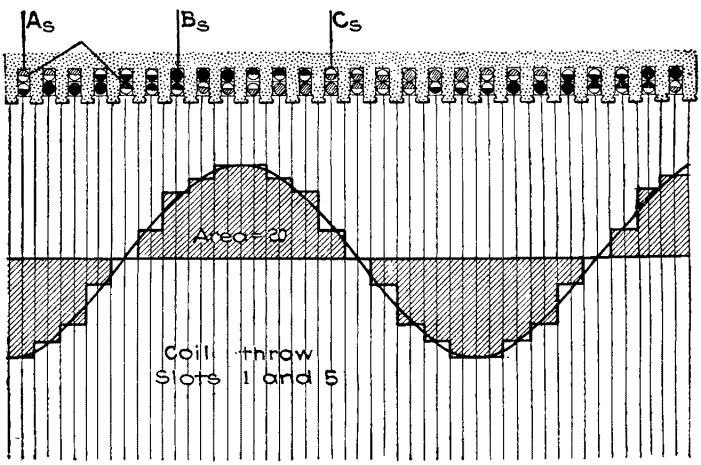


FIG. 136.—Similar to Fig. 135 chorded to 1 and 5 as in Fig. 138.

which is full pitch, or 180 deg., down to slots 1 and 5, as in Fig. 138, or 80 deg.

The same magnetizing current is assumed to flow in the coils in all six cases, although in an actual machine this would not be the case; the magnetizing current would increase with decreased throw of coil due to the attempt of the motor to keep the field at

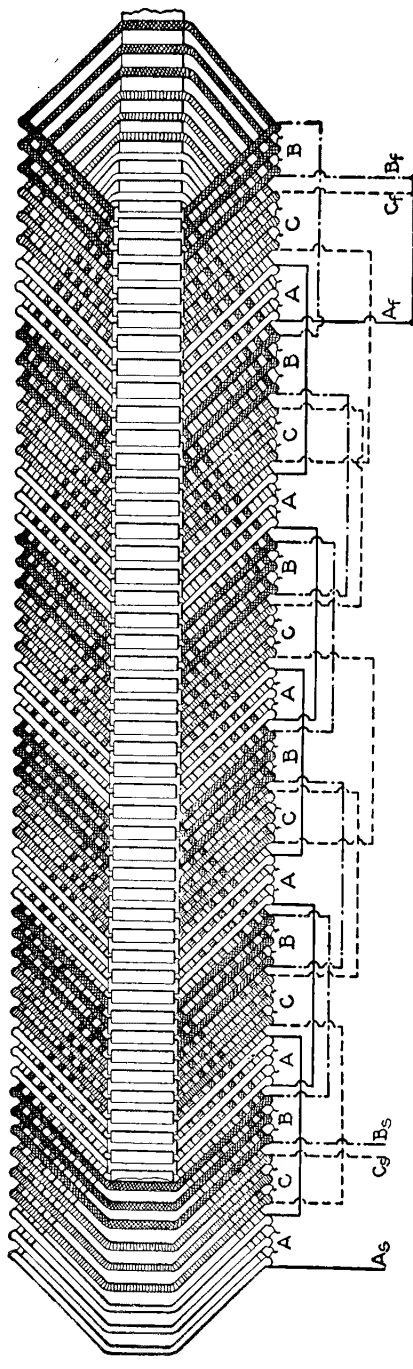


FIG. 137.—Fifty-four slot stator connected for three-phase, six poles, and wound full pitch, in slots 1 and 10.

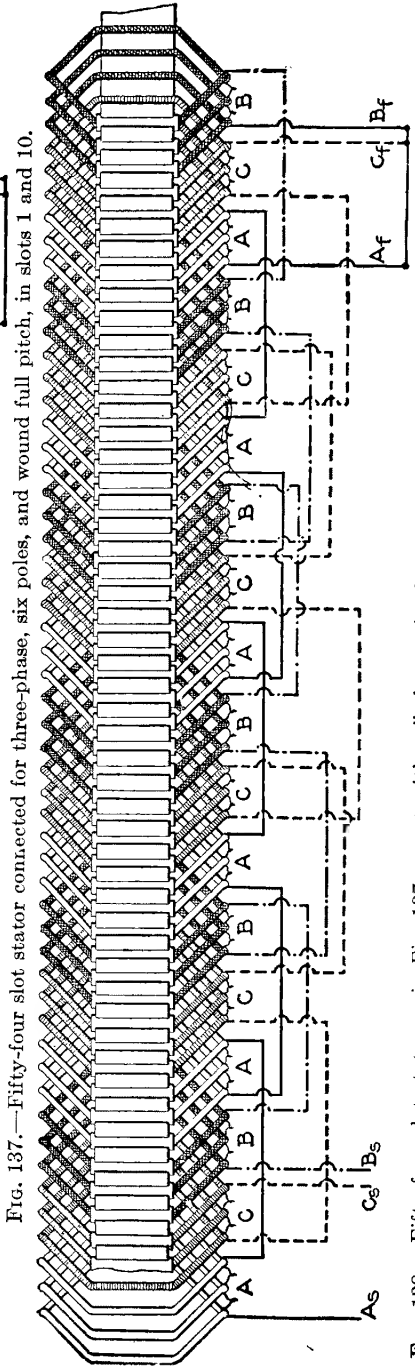


FIG. 138.—Fifty-four slot stator as in Fig. 137 except with coil thrown in slots 1 and 5 to give the magnetic field shown in Fig. 136.

the constant value necessary for the generation of the required back or counter e.m.f. To facilitate comparison, however, this change in current has been disregarded in the figures. The "stair steps" show the magnetic fields as they would look if there were no winding on the rotor, and the smooth curves, having the sine shape, show the fields as they look after being smoothed out by the currents in the rotor winding. It will be noticed that the area of the field for one pole is given in each case and that it varies from 32 for full pitch in Fig. 131, down to 20 in Fig. 136. These areas correspond to what is known as the "chord factor" of the winding. In the earlier part of this chapter it was stated that the chord factor for a chorded winding could be expressed in its effect on the magnetizing or no-load current and in its effect on the generated or counter-electromotive force by the mathematical value of the sine of one-half the electrical angle spanned by the coil. This relation is shown in the following table:

TABLE I.—CHORD FACTORS FOR VARIOUS ANGLES

Figure	Angle spanned by coil = a deg.	Sine ½a, or chord factor	Area of magnetic pole figured from chord factor	Area of magnetic pole graphically from figure
131	180	1.000	32.0	32
132	160	0.985	31.5	31
133	140	0.940	30.1	30
134	120	0.866	27.7	27
135	100	0.766	24.5	24
136	80	0.642	20.5	20

The slight difference between the last two columns in the table is due to the area under the "stair step" curve not being quite the same as the area under the corresponding smooth sine curve. The chord factor as shown in the third column at once indicates two facts: First, that if the winding is chorded more current will have to flow in the windings to produce the same magnetic field strength; and second, that, since the generated or counter-electromotive force in the windings set up by the rotating magnetic field is reduced through chording by the amount indicated by the chord factor, it is necessary to have a stronger magnetic field in the motor if it is to operate at the same voltage when the coil is chorded up. The way this shows up in reconnecting

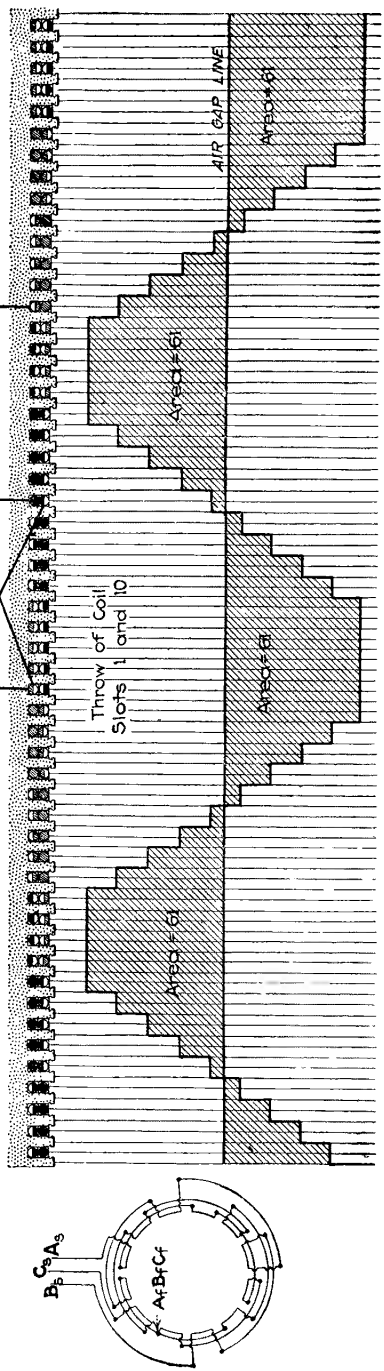


FIG. 139.- Stator shown in Figs. 131 and 137 connected for four poles.

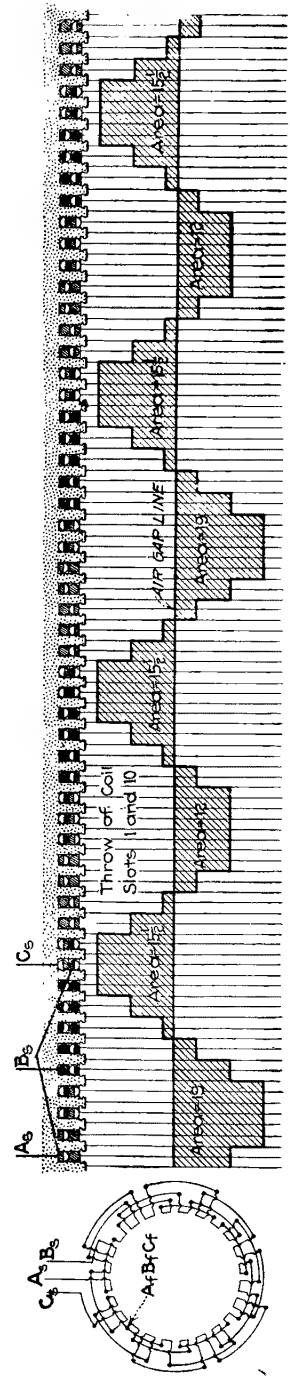


FIG. 140.—Stator shown in Fig. 137 connected for eight poles.

for different numbers of poles, when the reconnection causes chording of the coil, is that the same effect is produced as would be if the motor were connected to a higher voltage. This will be explained fully in a later chapter dealing with the practical application of the principles presented in this chapter to the actual work of reconnecting.

An examination of the shape of the magnetic field indicates that the effect of chording is to flatten the top of the field and make it lower for the same pole span. In Fig. 139 is shown the effect the connecting the winding of Figs. 131 and 137 for four poles instead of six. The mechanical throw of the coils is still 1 and 10, but the pole arc is longer for four poles, hence, the coil is actually chorded to 120 electrical degrees for four poles, although it was full pitch, or 180 deg., when connected for six poles. It will be noted that with the six-pole winding, Fig. 131 the entire area of the poles is $6 \times 32 = 192$, but that for the four-pole winding, Fig. 139, the area is $4 \times 61 = 244$. In the four-pole winding, the speed of the rotating field is 1.5 times that of the six-pole one, and it would therefore seem reasonable that with the same magnetic field density in the air gap and the same currents in the windings, the horsepower when connected as a four-pole machine should be 1.5 times that of the six-pole rating. However, since the coil throw on four poles is only 120 deg. the chord factor is sine of 60 deg. = 0.866 and the rating will be reduced by this fact so that only 1.5×0.866, or about 1.3 the six-pole horsepower can be expected. The total areas of the two fields as previously noted—namely, 244 and 192—have the relation $^{244}/_{192} = 1.27$, which is very close to 1.3, so it follows that a close approximation of the output to be expected from a reconnected motor can be obtained by this simple method of plotting the magnetic fields and comparing the areas. The difference in the saturation of the stator iron would affect this result to some extent, but usually not enough to introduce a serious error.

In Figs. 140, 141 and 142 is shown the effect upon the magnetic field of reconnecting the winding shown in Figs. 131 and 137 for 8, 10 and 12 poles, respectively. The effect of chording becomes more pronounced with each step, and the decreased area of the magnetic field shows that with the decreasing speed the horsepower decreases also until finally in Fig. 142 an impossible con-

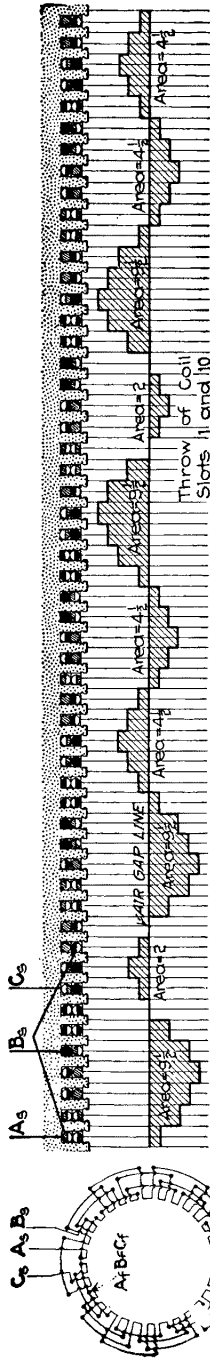


FIG. 141.—Stator of Fig. 137 connected for ten poles.

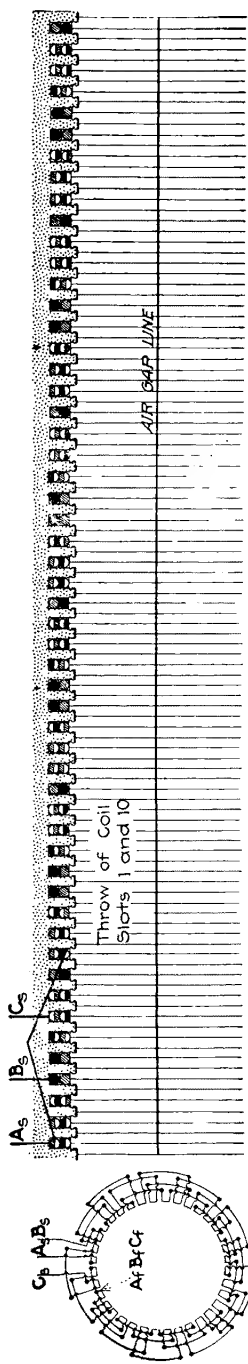


FIG. 142.—Stator of Fig. 137 connected for twelve poles.

dition is reached under which the motor could not run at all, since the throw of the coil is exactly pitch for six poles and therefore substantially becomes dead when connected for 12 poles; or, putting it another way, the throw of the coil is such that when there are 12 poles both sides of any given coil lie in exactly the same polarity. One side is under a north pole and the other, instead of being under a south pole, reaches clear across and lies under the next north pole, so that the counter-electromotive force, which is generated in one side of the coil, is exactly balanced and neutralized by the voltage generated in the opposite side and there is no counter-electromotive force left to oppose the applied electromotive force at the stator terminals, consequently, the current in the stator winding is limited only by the ohmic resistance of this winding, and would cause the circuit-breaker to open, or, if the motor was not properly protected, cause the windings to be destroyed in a very short period. Attention was called to this point in an earlier chapter when speaking of the possibility of connecting some windings as they stand for double or half speed; that is, for half as many poles or twice as many poles. The statement was then made that this should not be attempted if the throw of the coils was exactly pitch on the original winding. Figure 142 explains why this is true and why such a reconnection is not feasible.

In Fig. 143 is shown a very interesting comparison. Figure 143*a* shows the result of reconnecting the six-pole winding of Figs. 131 and 137 for two poles. Ordinarily, this would not be possible because a two-pole motor would require about three times the radial depth of iron behind the slots as is required by a six-pole one; but assuming for illustration that such a reconnection had been attempted, the field would have the appearance shown, and it will be seen that the area of one magnetic pole would be 142. Suppose, on the other hand, that instead of reconnecting, the motor had been rewound with coils having a throw of 180 deg. for two poles or full pitch, as shown in Fig. 143*b*; then the area of the field would be 284 for one pole or just twice the value for the reconnected motor. Since, as has been shown, the comparative areas of the two poles are some measure of the output to be expected, it can be at once concluded from this comparison that the use of a new set of coils would double the output of the motor and that it would be poor economy in such a case to reconnect instead of rewinding.

The comparisons made give a good idea of the effect upon any motor of changing the throw of the coil. The main value of the latter idea is that it is often possible when reconnecting a winding to assist in getting normal conditions in the winding by changing the throw of the coils by a slot or two in a certain direction.

FIG. 143*a*.

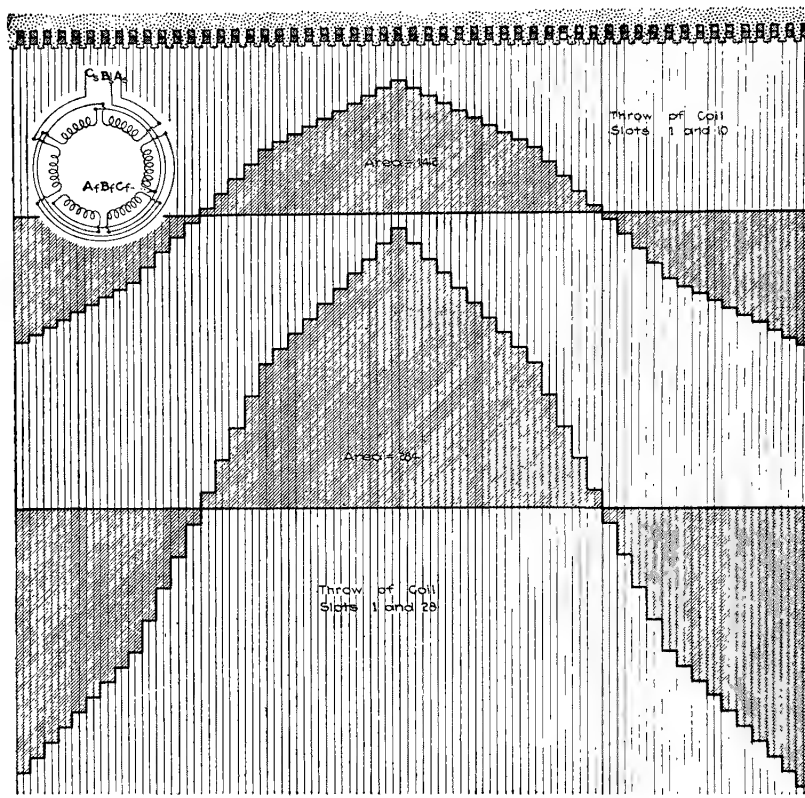


FIG. 143*b*.

FIG. 143*a*.—Stator of Fig. 137 connected for two poles.
FIG. 143*b*.—Stator of Fig. 137 rewound for two poles with coils of correct throw. Note improvement in Fig. 143*b* over Fig. 143*a*.

Table III gives a number of examples of how the chord factor is determined from the throw of the coil. For convenience, Table II gives the sine of half the electrical angle spanned by the coil and hence the chord factor for all throws from half pitch to full pitch. These values correspond to column 9 in Table III.

TABLE II.—TABLE OF CHORD FACTORS FROM 45 TO 90 DEG.

Degrees	Sine or chord factor	Degrees	Sine or chord factor	Degrees	Sine or chord factor
45	0.707	60	0.866	75	0.966
46	0.719	61	0.875	76	0.970
47	0.731	62	0.883	77	0.974
48	0.743	63	0.891	78	0.978
49	0.755	64	0.899	79	0.981
50	0.766	65	0.906	80	0.985
51	0.777	66	0.914	81	0.988
52	0.788	67	0.921	82	0.990
53	0.799	68	0.927	83	0.993
54	0.809	69	0.934	84	0.995
55	0.819	70	0.940	85	0.996
56	0.829	71	0.946	86	0.998
57	0.839	72	0.951	87	0.999
58	0.848	73	0.956	88	0.999
59	0.857	74	0.961	89	0.999

TABLE III.—HOW THE CHORD FACTOR IS FIGURED FROM THE THROW OF THE COIL

(1) Number of slots	(2) Number of poles	(3) Slots per pole	(4) Electrical degrees per slot	(5) Throw of coil	(6) Slots spanned by coil	(7) Electrical degrees spanned by coil	(8) One-half electrical degrees spanned	(9) Sine of half angle or chord factor
Given	Given	(1) ÷ (2)	180° ÷ (3)	Slots coil, sides in	Last figure (5) minus 1	(4) × (6)	(7) ÷ 2	From (8) and sine table
36	4	9	20	1–7	6	120	60	0.866
48	6	8	22½	1–7	6	135	67½	0.924
60	8	7½	24	1–6	5	120	60	0.866
75	10	7½	24	1–7	6	144	72	0.951
96	12	8	22½	1–7	6	135	67½	0.924
126	14	9	20	1–8	7	140	70	0.940
144	16	9	20	1–9	8	160	80	0.985
48	16	3	60	1–3	2	120	60	0.866
20	2	10	18	1–7	6	108	54	0.809
30	4	7½	24	1–7	6	144	72	0.951
120	20	6	30	1–5	4	120	60	0.866
180	30	6	30	1–6	5	150	75	0.966
216	24	9	20	1–8	7	140	70	0.940
252	14	18	10	1–15	14	140	70	0.940

CHAPTER VII

DISTRIBUTION FACTOR AND THE USES THAT ARE MADE OF IT IN WINDING STATORS OF INDUCTION MOTORS

Concentrated Windings.—In a generator or motor the magnetic field which is set up is proportional to the "ampere turns" in the winding. That is to say, the magnetic field is influenced directly by the number of turns of wire surrounding the magnetic circuit and by the number of amperes of electrical current flowing in these turns. In the case of a direct-current machine where the

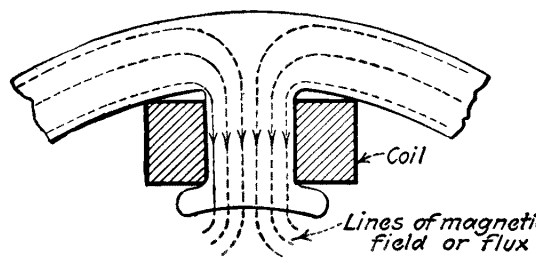


FIG. 144.—Direct-current machine showing concentrated field winding.

field coils have the familiar form shown in Fig. 144, this is very easily calculated because the turns are all concentrated in a group and completely surround the mechanical pole structure in which they are setting up a magnetic field. Such a coil or field winding is called "concentrated."

Distributed Windings and Distribution Factor.—In the case of a "distributed" winding, shown in Fig. 145, which is usually used on induction motors and other alternating-current motors, the case is not so simple. The effective ampere turns or magnetomotive force is not obtained directly by multiplying the number of turns by the amperes flowing, but must be modified by two factors called "chord factor" and "distribution factor." The first of these deals with the "pitch" or "throw" of the coil, which means whether it does or does not surround the whole of the mechanical pole, and the second has to do with the "phase" of

the winding—that is, whether it is single phase, two phase or three phase—and in how many groups or bands per pair of poles each phase winding is distributed or arranged.

How Current Bands Vary When Winding Connected for Different Number of Phases.—What is meant by "current bands" is groups of adjacent conductors in which the current is of the same phase and flowing in the same direction. This can be understood from consideration of Fig. 146, diagrams *A* to *F* inclusive, which represent a portion of a typical induction-motor winding and the current conditions which exist in such a winding when variously connected for two phase or three phase, with standard connection or consequent-pole connection.

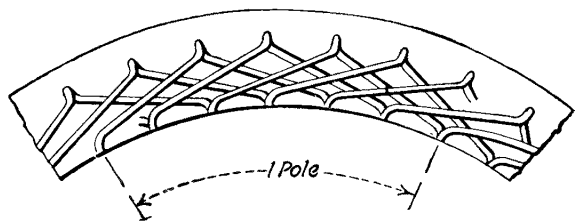


FIG. 145.—Alternating-current machine showing distributed field winding.

Diagram *A* shows 12 slots of the stator core of an induction motor and the 12 coils that lie therein. These coils have a throw of slots 1 and 7, which in this case is assumed to be full pitch. The portion of the winding shown covers 360 electrical degrees, *i.e.*, from the center of one north pole to the center of the next north pole. The coils as shown are unconnected, the terminal wires for the beginning and ending of each coil being brought out and marked 1*b*, 1*e*, 2*b*, 2*e*, and so on.

In diagram *B* are represented the current conditions that would exist in the winding of diagram *A* if the coils were connected to form two pole-phase groups of a single-phase winding. The top line marked "Way coils are connected" is a schematic diagram showing the coil-to-coil connections and the group connections for such a connection. The small coil marked 1 represents the coil lying in slots 1 and 7 of diagram *A*, having the wire marked 1*b* in diagram *A* brought out for a lead, *A*, and having the wire marked 1*e* connected to 2*b* on coil 2, and so on. In this manner coils 1 to 6 inclusive are connected in series to form a pole-phase group, and coils 7 to 12 inclusive are connected

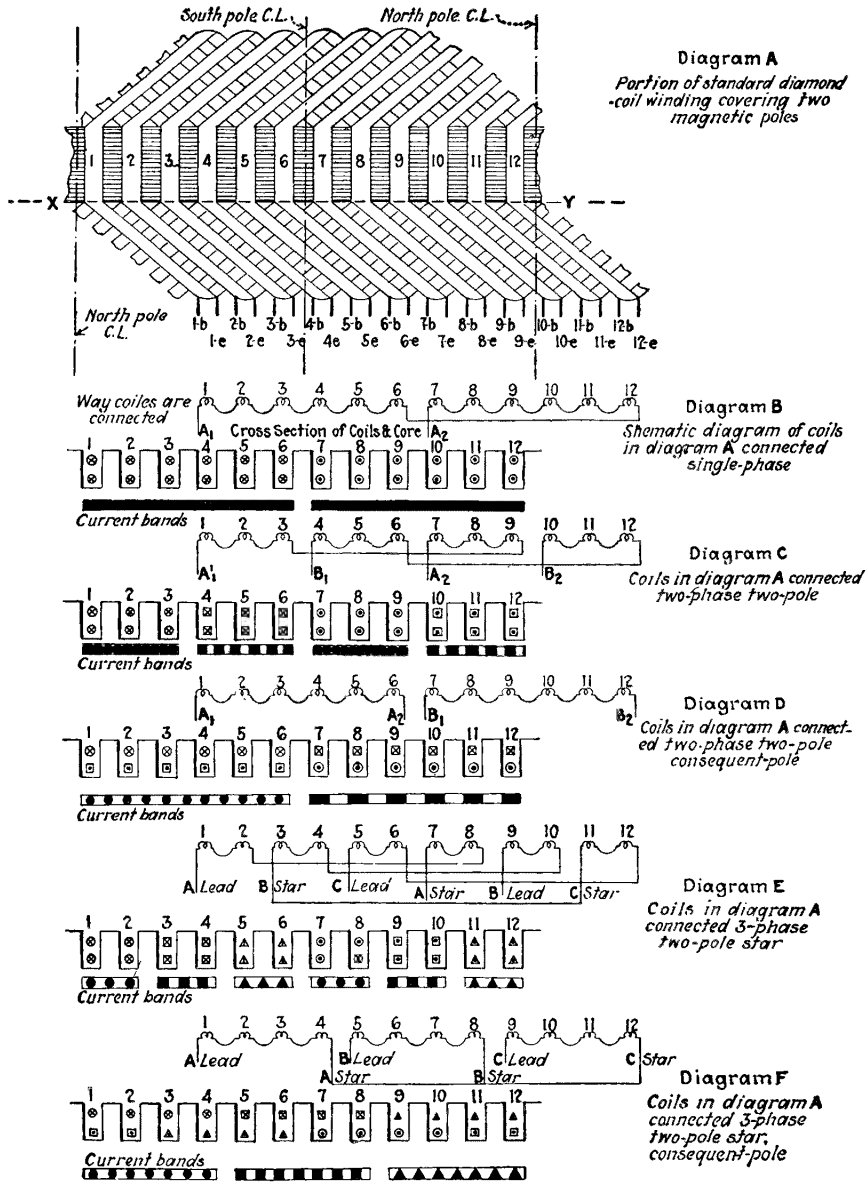


FIG. 146. - Portions of typical induction-motor windings and current conditions when variously connected. In these diagrams marked *A* to *E* are shown the current conditions which exist when an induction-motor winding is variously connected for two phase or three phase with standard connection or consequent-pole connection.

in series to form a second pole-phase group. These two groups are then properly connected to form an adjacent north and south pole in the winding. The next horizontal picture in diagram *B* marked "Cross section of coils and core" shows the direction of the current in each slot and is shown as if a cross section of all coils were taken along the line *XY* shown in diagram *A*. Conductors in which current is assumed to be flowing away from

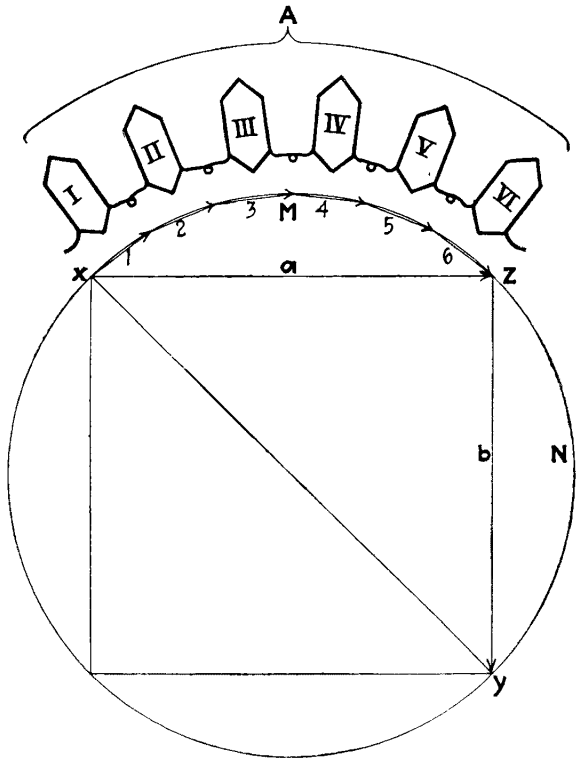


FIG. 147.—Volts generated in each coil and in a pole-phase group of six coils in a two-phase winding.

the observer are marked *X* and conductors in which the current in assumed as flowing toward the observer are marked with a circle and dot in the center. The slots are marked 1 to 12, inclusive. A consideration of the currents in all twelve coils shows that they come in groups or bands of twelve conductors in one direction and then twelve in the opposite direction. Therefore, the current bands are as shown by the wide black lines at the bottom of diagram *B*, and also as shown there are two

current bands per pair of poles. In Fig. 149, as discussed below, is shown the geometrical derivation for the distribution factor of such a winding.

Diagram *C* of Fig. 146 shows the corresponding conditions which would exist if the coils of diagram *A* were connected for two-phase, two-pole as schematically indicated at the top of diagram *C*. An examination of the current bands at the bottom of diagram *C* shows that there are four current bands per pair of poles. For convenience, in the current bands *A* phase current is shown by circles and *B* phase by squares. Figure 147 shows how the distribution factor for such a connection is derived.

Again, diagram *D* of Fig. 146 assumes that the coils of a diagram *A* have been connected for two-phase, two-pole, but with a "consequent-pole" arrangement, that is to say, with all pole-phase groups, for example, connected to produce north poles and letting the south poles result from the magnetic flux finding its way back in between the north poles as best it can. There are only two current bands per pair of poles as shown at the bottom of diagram *D*. In this case current bands for the top coils in the slot alone are shown. The distribution factor of such a winding may be derived from Fig. 147, as explained below, and is the same as for the single-phase winding of diagram *B*. Such a connection might be found on the slow-speed connection of a two-speed winding.

Similarly, diagram *E* of Fig. 146 shows the conditions if the coils of diagram *A* are connected for a standard three-phase two-pole star connection as shown by the schematic diagram at the top. For convenience, in the current-band diagram at the bottom of diagram *E*, the *A* phase current is represented by circles, the *B* phase by squares, and the *C* phase by triangles. It will be seen that there are six current bands per pair of poles, and hence such a connection really has as good a distribution factor as a six-phase winding. The manner of deriving the distribution factor for this winding is explained in connection with Fig. 148, p. 142.

Finally, diagram *F* of Fig. 146 shows the conditions where the coils of diagram *A* are connected for three-phase two-pole star, but with a consequent-pole arrangement. It will be seen that there are three current bands per pair of poles. The distribution factor for this connection can be obtained from Fig. 148, as

explained below. It will be noted that this connection, which is a real three-phase connection, has a distribution factor of only 0.827, whereas the usual three-phase arrangement of diagram *E* which, in turn, is really six-phase, has a distribution factor of 0.955. This is the explanation of why six-phase rotary converters are greatly to be preferred over three-phase, although requiring the increased complication of six sets of brushes and six collector rings as against two sets of brushes and three collector rings on a straight three-phase rotary.

Geometric Method of Finding Distribution Factor.—With this preliminary physical conception of how the distribution factor is affected by the arrangement of the coil and group connections, a ready transition can be made to the geometric method of determining the value of the distribution factor as explained below in connection with Figs. 147, 148 and 149.

In calculating the windings for an induction motor, the number of turns required in the coils is found from this expression:

Conductors[1] *per phase* = (45,000,000 × *volts per phase*) ÷ (*cycles* × *magnetic flux per pole* × K_1 × K_2). In this formula:

Conductors per phase = (*number of wires per slot which are in series* × *number of slots*) ÷ *number of phases.*

Cycles = the frequency of the supply circuit as expressed in cycles per second, that is, 60 or 25, or whatever the circuit may be.

Volts per phase = line volts in the case of a two-phase winding or a delta-connected three-phase winding and line volts divided by 1.73 in the case of a star-connected three-phase winding.

Magnetic flux per pole = number of magnetic lines of force set up in each pole by the magnetizing current.

Chord Factor.—K_2 is the so-called "chord factor" depending upon the pitch or throw of the coil. "Full pitch" would mean a throw from the center of one north pole to the center of an adjacent south pole and would be called 180 electrical degrees. A "chorded winding" or "fractional-pitch" winding would have a span or throw of each individual coil something less than 180 electrical degrees, and the corresponding chord factor or K_2 would be the *sine of one-half of this span angle.* In the case of full pitch or 180 deg., the *sine of one-half of* 180 *deg.* or *sine of* 90 *deg.* = 1, and hence the turns of a winding are most effective

[1] See p. 273.

in inducing or generating voltage when the throw of the coil is full pitch. In actual practice this is usually not done because chording one or two slots does not seriously affect the voltage generated and results in some saving of wire due to the physically shortened length of the average turn of wire in the coil.

Distribution Factor.—K_1 is a different kind of factor and depends upon the phase of the winding: that is, whether it is a two-phase, three-phase or single-phase winding. It is usually called the distribution factor. For most of the present-day distributed windings used in induction motors it can be placed equal to 0.905 for two-phase windings and 0.955 for three-phase windings and practical calculations based on these values will be accurate within close limits. In the case of single-phase windings, as explained below, the distribution factor becomes more involved and must be worked out for each individual case.

In Fig. 144, is suggested a cross section of one pole and a part of the field yoke of a direct-current machine. In Fig. 145 is indicated a part of one end of the stator winding of an induction motor without any cross connections or leads. Consider first Fig. 144. It is readily seen that each turn of wire in the field coil surrounds all the magnetic lines of force which thread through the coil and make up one magnetic pole of the field circuit. On the other hand, in Fig. 145, it is seen that not only does each coil not surround all the magnetic-field flux of one pole, but that there is, in addition, the complication that the magnetic field, which acts as if it were set up by direct current, relies on the combination of two or three phases of alternating currents in the windings to produce this effect of a rotating direct current for magnetization.

It is the difference in the action of two-phase windings from three-phase windings that gives rise to the difference between the values of 0.905 and 0.955 cited above, and the physical reason for this difference can best be seen from geometric figures.

Graphic Solution.—In Fig. 147 is represented a half square erected in a semicircle, the diameter of the circle becoming one diagonal of this inscribed square. This figure represents diagrammatically the voltages set up or induced in each individual coil in a two-phase winding, which would be the small chords subtending the small arcs 1, 2, 3, 4, 5, 6. The coils themselves are physically pictured as I, II, III, IV, V, VI; also the voltage

in one pole-phase group is represented by the sides of the square *a* or *b*, where *a* would be the *A* phase and *b* the *B* phase. It also represents in the diameter of the circle *XY* the voltage induced by one rotating magnetic or "direct-current" pole, which is the result of the combined action or interaction of the two-phase windings *a* and *b*.

Arithmetic Formula.—The physical interpretation of the diagram in Fig. 147 is that, if the small coils 1, 2, 3 and so on, represent six individual coils in series forming one pole-phase group of a two-phase winding, each coil will generate or have induced in it a voltage represented by the short chord or straight line subtending the arc representing this coil. The six coils in series would generate a total voltage equal to the sum of the six chords or practically equal to the quadrant of the circle from *X* to *Z*. This quadrant represents in its length the voltage that would be generated or induced in the pole-phase group if all six of the coils were bunched into one coil as in the case of the direct current machine shown in Fig. 144. However, these six coils are not bunched but are spread over an arc in the bore of the motor representing one *pole pitch*. For this reason there is a phase difference in time represented by the small angle between the chords 1 and 2, which means that because the main magnetic field in the motor is rotating, it does not generate voltage in coil 1, for example, at exactly the same instant that it does in coil 2, but a fraction of a second sooner or later. Since this is true, we cannot add together end to end in one straight line the six small voltages generated by the six coils, but must add them geometrically, as shown in Fig. 147, so that, instead of having a voltage across the six in series, or the pole-phase group, practically equivalent to the arc *XMZ*, the combined voltage is really the chord of this arc or the straight line *XZ*. Theoretically, the number of coils in the pole-phase group would have to be infinite before the arithmetical sum of their voltages would be equal to the quadrant of the circle, but practically in most commercial motors this assumption can be made without serious error.

Two-phase Distribution Factor.—Now since the arc *XMZ* represents in its length what the combined voltage of the six coils would be if bunched or concentrated into one coil, and since the chord or straight line *XZ* represents by its length what the real voltage across the six is (since they are not quite in phase),

the actual effective number of turns or what is called the distribution factor can be represented by the expression:

Distribution factor = (*length of the chord XZ*) ÷ [*length of small chords* (1 + 2 + 3 + 4 + 5 + 6)].

And, since in practically all commercial windings there is a considerable number of coils in each pole-phase group, the sum of the chords (1 + 2 + 3 + 4 + 5 + 6) can be set equal to the arc *XMZ*, as stated above, and the expression for distribution factor becomes:

Distribution factor = (*length of the chord XZ*) ÷ (*length of the arc XMZ*).

Since these quantities make up the quadrant of a circle, the chord *XZ* = the radius of the circle ÷ 0.707 and the arc *XMZ* = 3.14 × the radius of the circle ÷ 2.

The expression above then becomes:

Distribution factor = (*radius* × 2) ÷ (0.707 × 3.14 × *radius*) *or* 2 ÷ (0.707 × 3.14) = 0.90.

When this value is worked out for a number of practical cases involving different numbers of coils per pole-phase group, it is found that the value 0.905 given in the general paragraph above is a satisfactory one to be used in the calculations for ordinary commercial machines.

Distribution Factor for Consequent-pole Winding on a Two-phase Motor.—Figure 147 can be used, also, to derive the distribution factor for a consequent-pole winding such as that illustrated in diagram *D* of Fig. 146. In this case the diameter *XY* represents the voltage induced in the winding for one pair of poles, that is to say the winding A_1-A_2, for example. This voltage may be considered as made up of the two voltages or chords *XZ* and *ZY*. In return, the voltage *XZ* may be considered the voltage set up by the physical winding making a north pole, and the voltage or chord *ZY*, representing the consequent pole or resulting south pole which has no winding but which is obliged to be present in order for the magnetic flux sent out by the winding to get back into the core. In other words, the direction of the current in the windings is such as to cause only one polarity, but since one polarity cannot exist without the other, the other polarity comes "in consequence" and hence is called a "consequent" pole. Referring again to Fig. 147, if the sum of all the voltages in all the individual coils is equivalent to the arc

$XMZNY$ and the resulting voltage at the terminals A_1-A_2 of the winding is represented by the diameter XY, the distribution factor is the expression:

$XY \div (arc\ XMZNY) = (2 \times radius) \div (3.1416 \times radius) = 2 \div 3.1416 = 0.636.$

This is the same as the single-phase distribution factor in Fig. 149. Another conception of this factor of 0.636 would be to say that it is the product of 0.905 and 0.707, the first being the relation of the chord XZ to the arc XMZ, and the second being the relation of the diameter XY to the sum of the chord XZ, plus the chord ZY. In other words, the factor 0.905 is because the winding is two phase, and the factor 0.707 is because it is connected for consequent poles, and the product of $0.905 \times 0.707 = 0.636$, which makes it no better than the single-phase winding shown in Fig. 149.

Three-phase Distribution Factor.—Considering Fig. 148, there is illustrated a similar condition for a three-phase motor. In this case the construction is that of a hexagon inscribed in a circle. Again, the coils are physically pictured as I, II, III, IV, V, VI and the voltages induced in each of these coils, or the magneto-motive force set up by each, as the small chords 1, 2, 3, 4, 5, 6. If these coils were all concentrated into one large coil, their effect would equal the sum of the small arcs 1, 2, 3, 4, 5, 6. Since this is not the case and they are distributed as shown, the effect of the pole-phase group is the chord XZ or a. Then, similarly to the two-phase case, the expression for the distribution becomes: (*length of the chord* XZ) $\div$ (*length of the arc* XMZ). Since the chord a in this case is one side of an inscribed hexagon, and since the side of a hexagon is equal to the radius of the circle in which it is inscribed, this expression becomes: (*radius of a circle*) $\div$ (*one-sixth of the circumference*) $= (6 \times radius) \div (2 \times 3.14 \times radius) = 3 \div 3.14 = 0.955$ which it is proper to use in all practical calculations for ordinary commercial machines.

Distribution Factor for Three-phase Consequent-pole Winding.—From Fig. 148 may also be derived the distribution factor for a three-phase consequent-pole winding such as that illustrated in diagram *F* of Fig. 146. In this case the chord *tz* represents the voltage induced at the terminals of one phase of the winding, as for example between the *A* lead and the *A* star. As in the case of the similar two-phase consequent-pole winding described

under Fig. 147 and diagram *D* of Fig. 146, this voltage *tz* is made up of the pieces *tx* and *xz* combined. Of these two pieces we may consider *xz* as actually caused by the ampere turns in the winding and *tx* as resulting from the consequent pole similar to the two-phase case above. Hence the distribution factor for this winding is the length of the chord *tz* divided by the length of the

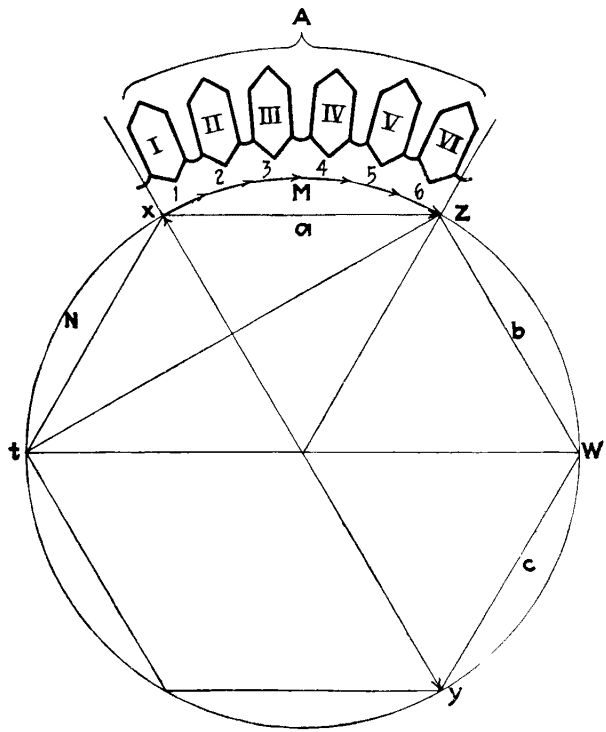


FIG. 148.—Volts generated in each coil and in a pole-phase group of six coils in a three-phase winding.

arc *tnxmz* or approximately 0.827. This factor can be considered as made up of 0.955 × 0.866. The first of these is due to its being a three-phase winding, as already explained above, and is the relation of the chord *xz* to the arc *tmz* and the second factor or 0.866 being due to the consequent-pole connection. This factor 0.866 is the relation of the chord *tz* to the sum of the chords *tx* + *xz*.

Distribution Factor for Fully Distributed Single-phase Winding.—The case of a single-phase winding, if completely dis-

tributed as shown for a two- or three-phase winding, is shown in Fig. 149. Here the illustration shows neither a square, nor a hexagon, nor a triangle inscribed in a circle, but simply the diameter of the circle itself, *xy*, is used to represent the electrical or magnetic effect of the pole-phase group made up of the coils I, II, III, IV, V, VI. As in the previous cases, if the six coils

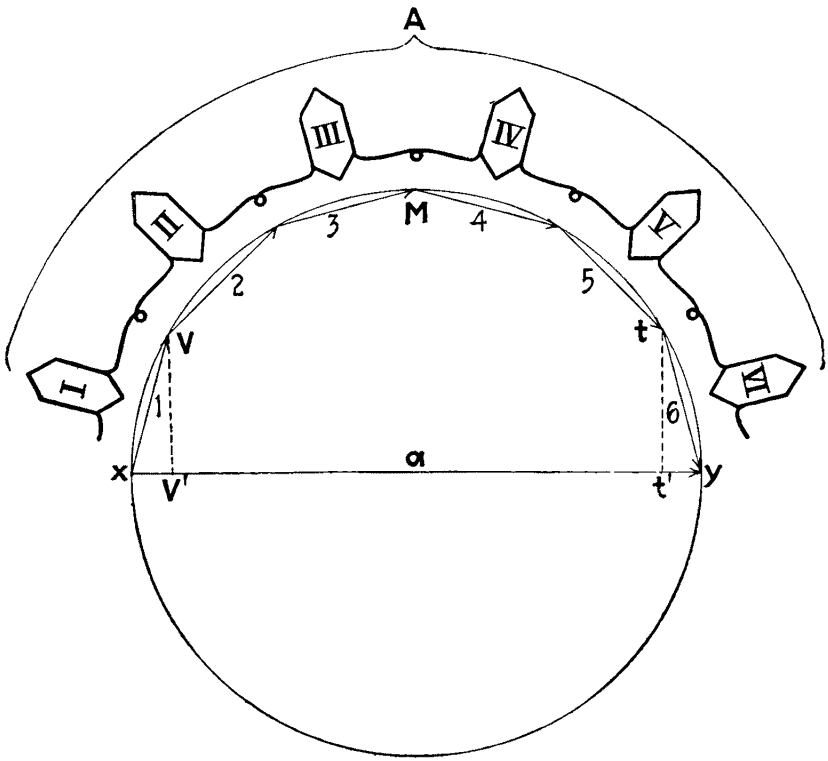


FIG. 149.—Volts generated in each coil and in a pole-phase group of six coils in a single-phase winding.

were concentrated into one field coil, their total effect would be measured by the sum of the chords $1 + 2 + 3 + 4 + 5 + 6$. Since they are distributed and hence each one has a little different phase from the others, the combined effect of the pole-phase group is *xy* and the expression for the distribution factor becomes:

(*Diameter of a circle*) ÷ (*one-half the circumference*) = $D \div (3.14 \times \textit{radius}) = 2 \div 3.14 = 0.636$.

This factor shows that there is a very inefficient use of copper because it means that something less than two-thirds of the

actual number of turns in the winding is actually effective in producing the magnetic field or in inducing voltage from that field. From the figure it will be seen that the two end voltages of the groups I and VI are so far out of phase as to be of little value, and that if they were left out of the winding altogether the resulting pole-phase effect v^1t^1 would be almost as great as xy from all six coils. Also in such a case the distribution factor would be represented by the expression:

(*Line* v^1t^1) $\div$ (*arc vmt*) which approximates 0.8 and is a much better figure than 0.636 with all six coils in.

Why Single-phase Windings Are Often Concentric-coil Wound.—This explains why the majority of single-phase motors

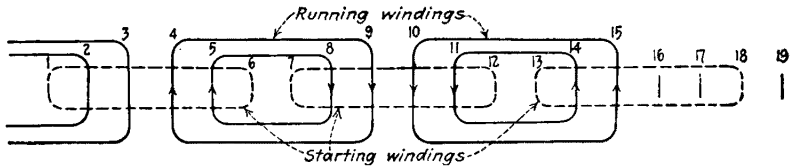


FIG. 150.—Single-phase split-phase winding with concentric coils

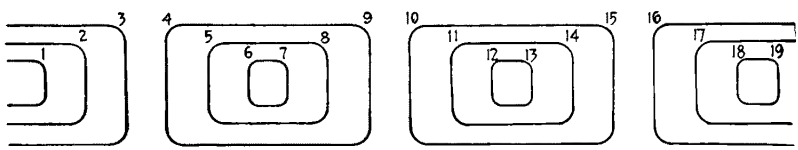


FIG. 151.—Single-phase concentric-coil winding having all slots filled with the running winding.

are wound with concentric coils and the middle of the pole is vacant, or if a distributed winding is used about one-third of the winding is omitted in a suitable manner. When the motor is of the split-phase starting type, the slots left vacant by the omission of a part of the main running winding (for reasons just given) are utilized for the phase splitting or starting winding and are mechanically in the right position to make the starting winding 90 space degrees out of phase with the running winding, which makes the mechanical relation of the starting to the running winding similar to the two windings of a two-phase, polyphase motor. Such an arrangement is shown in Fig. 150, where the full lines represent coils in the main or running winding, and the dotted lines represent coils in the starting winding, which are cut out of circuit by the starting switch as soon as full speed is reached by the rotor.

A winding such as that shown in Fig. 150 may have a distribution factor of 0.8 or higher, when considering the running winding alone, whereas the winding of Fig. 151, where all the slots are filled with the running or operating coils, may have a distribution factor as low as 0.636 for the reasons explained in connection with Fig. 149. In the case of Fig. 151, starting is accomplished by a commutator and brushes, or in some way other than by the split-phase principle. In this winding,

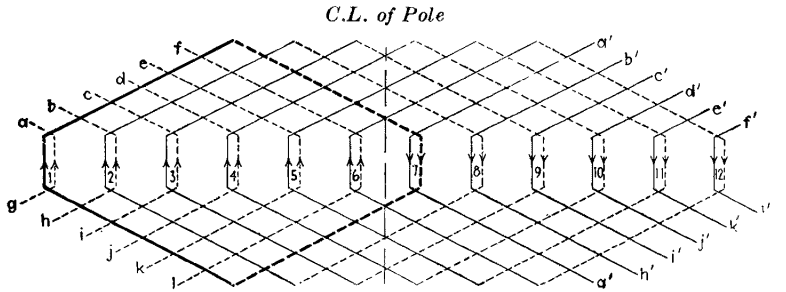


FIG. 152.—Single-phase winding for two poles in twelve slots using a completely distributed winding with full-pitch coils.

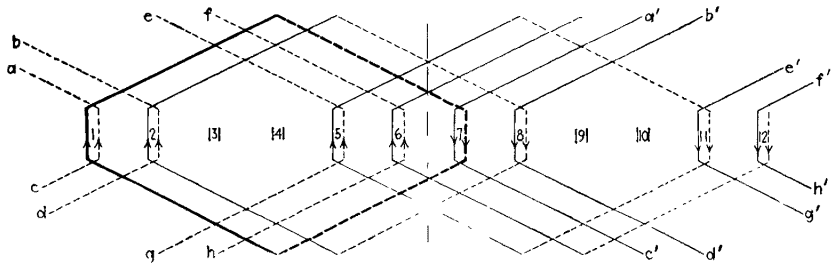


FIG. 153.—Single-phase winding for two poles in twelve slots with one-third of coils omitted to improve the distribution factor of the winding.

although the two middle slots are not needed for a starting winding, it would be an advantage to leave them vacant, as in this way perhaps 20 per cent of the copper in the stator winding can be saved and the electrical or magnetic effect of the winding would be very little less, being something of the nature of the length of the line v^1t^1 in Fig. 149 as compared with the whole line xy.

Standard diamond coils of the type usually found in polyphase motors may also be used for single phase. Such a winding is shown in Fig. 152 for a two-pole motor with 12 slots, having all slots filled and using all the coils for the running winding

In this figure the sides of the coils in the tops of the slots are shown in full lines, and the sides in the bottoms of the slots are shown in dotted lines. The distribution factor of such a winding would be low, being approximately 0.636 as explained in connection with Fig. 149. Here again, as in Fig. 151, it would be an advantage to omit the coils in the center of the poles and leave them vacant. Such a winding is shown in Fig. 153. With such an arrangement, if a split-phase starting winding was required on the same stator for some other job, the starting coils could be placed in slots 3 and 9 and 4 and 10.

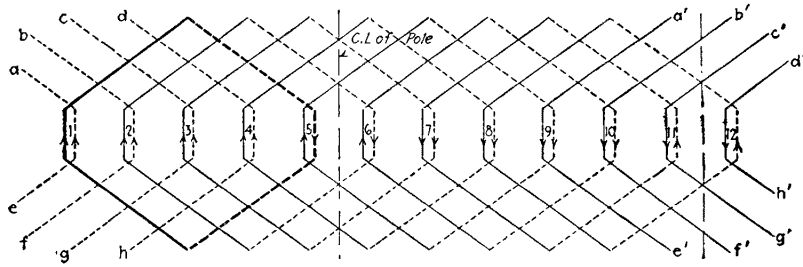


FIG. 154.—Single-phase winding for two poles in twelve slots similar to Fig. 152 except that it has short pitch or chorded coils.

Distinction between Chord Factor and Distribution Factor on Single-phase Windings.—One other feature of interest in single-phase distributed windings is the difference between chord factor and distribution factor. In Fig. 152 is shown a distributed winding whose coils are wound full pitch or whose sides lie in slots 1 and 7. It will be noted from the arrowheads on the coil sides showing the direction of the current at a given instant, that the center line of one pole lies between slots 6 and 7, and that the arrowheads for both tops and bottoms of slots are in exact symmetry on either side of this center line. Comparing Fig. 154, which shows a similar winding except that the coils are chorded and wound in slots 1 and 5, it will be seen that the center line of a pole now lies between slots 5 and 6, and another between slots 11 and 12. The arrows in slots 5 and 6 and 11 and 12 are in opposite directions, and hence the effect of the winding in these slots cancels out. If the shape of the magnetic field form in the two cases of Fig. 152 and Fig. 154 is plotted out by the well-known stair-step method,[1] it will be found that the

[1] See p. 118.

shape of the resulting field from the winding in Fig. 154 is flatter on top and not so close to a sine-curve shape as would be the field resulting from the winding shown in Fig. 152. From this comparison can be drawn directly the fact that the chord factor, which depends on the throw of the coil, is different for the windings shown in Figs. 152 and 154, but that the distribution factors are the same. On the other hand, comparing Figs. 152 and 153, the chord factors of these two windings are the same because the coils have the same throw but the distribution factors are different, as explained in connection with Fig. 149.

Practical Application of Distribution Factor.—From the foregoing discussion it may be seen that the distribution factor is a measure of how efficiently the turns in the winding are used. In other words, a direct-current field coil, as illustrated in Fig. 144, makes 100 per cent use of the turns, whereas a single-phase winding as shown in diagram *B* of Fig. 146 only uses them 63.6 per cent efficiently. Again the effectiveness of the turns is a measure of the possible output, either in the form of electrical energy as in a generator, or mechanical energy as in a motor, from a given weight of copper, so that all things being considered, that connection should be used which will give the highest possible distribution factor.

CHAPTER VIII

EFFECT OF VOLTAGE ON WINDINGS AND POSSIBILITY OF CONNECTING A WINDING FOR MORE THAN ONE VOLTAGE

Changing the winding connections of induction motors to accommodate a changed voltage supply is more often considered and accomplished than any other winding change. As was suggested in an earlier chapter, this may arise from the purchase of a used motor, a change in power supply from an isolated plant to central-station power, the remodeling of an old distributing system or in other similar ways. It was stated in Chapter VI that other changes, whether of phase or frequency or speed, could be considered as voltage changes and so worked out. This chapter outlines the considerations involved in the simplest form of voltage changes, thus establishing a basis for the solution of changes in the other characteristics.

In changes of voltage there are two main conditions that have to be met if the operation of the motor is to be kept normal. The first is to determine whether the insulation on the winding is proper for the new voltage that is to be used, and the second is how to adjust the number of turns in series in the winding, so that there will be substantially the same voltage per turn or per coil in the winding as existed under the original voltage. It is assumed that there is to be no change in the frequency of the supply circuit, the throw of the coils, the number of poles in the winding, the horsepower output or the number of phases.

Checking Insulation for New Voltage.—In considering the insulation alone, if the new voltage is to be lower than the old, no further attention need be given this point other than to determine that the insulation is mechanically in good condition and clean and dry. If the new voltage is higher than the old, the amount of insulation must be considered, and if there is any question as to this, it should be settled by the insulation tests described in the following, before proceeding with the actual work of reconnection. This may sometimes save work that

would otherwise be lost by discovering too late that the insulation is inadequate for the new conditions.

In many cases suitable facilities are not available for making either of the insulation tests described, and it is well to have some general information on the standard practice followed by good manufacturers with regard to insulation. There is an old saying among insulation engineers that "a winding that will stand any insulation test at all will stand 1000 volts." Like most general statements, this is not strictly true, perhaps, but it brings out the fact that the insulation for all voltages up to 750 volts is practically the same and is determined more by mechanical strength than by strictly electrical considerations. This means that usually a 110- or a 220-volt machine will be all right on 440 or 550 volts provided the number of turns in the winding is suitable for the higher voltage.

Sometimes the insulation for 550 volts is increased over that for 440, but most 440-volt insulation will stand 550 volts if in good condition and the operating temperature of the machine is reasonably cool. Voltages between 550 and 2200 are seldom met with commercially, and the caution which needs to be observed is that machines wound for 550 volts or below should not be operated on 2200 volts even if the number of turns in the coils could be properly arranged. However, there is no reason why machines built for a higher voltage should not be operated on a lower. The only handicap in such a case would be that the temperature would be somewhat higher, owing to the insulation being heavier than would be required for a machine normally wound for the lower voltage. In order to indicate the limits on different classes of insulation, the following shows broadly the classification followed by many manufacturers: Class 1, up to and including 500 volts; Class 2, from 500 to 1200 volts; Class 3, from 1200 to 3500 volts; Class 4, from 3500 to 6000 volts; Class 5, from 6600 to 8000 volts. Very few induction motors are built at voltages higher than 6600.

The general statement may be made regarding these classes that any machine of a higher-voltage class may be operated on a lower voltage, but no machine in a lower class should be operated on a higher voltage than its own class.

Insulation Tests.—Where a reference to classification will not settle this matter or there are a number of units involved and the

possibility of reconnection is serious, tests should be made. The insulation of electric machines may be tested in two ways—one measures its ability actually to withstand the voltage strains that occur between the parts of the winding and the ground, and the second determines the condition of the insulation as to dryness and cleanliness. The first is called a test for dielectric strength and is performed by applying for one minute, between the winding and the ground, an alternating voltage equal to twice the normal voltage of the circuit to which the apparatus is to be connected, plus 1000 volts.[1]

The second test is called a test for insulation resistance and is usually made by applying a direct-current voltage of 500 volts

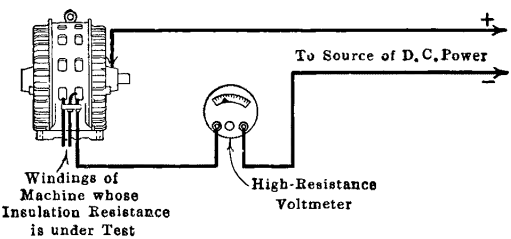


FIG. 155.—Test for insulation resistance.

between the conductors in the winding and the ground, having a direct-current voltmeter of high internal resistance in series with the insulation. Since the insulation is in series with the circuit, there will be practically no current flowing, but the direct-current voltmeter will show a slight deflection and the insulation resistance is measured thereby. The arrangement of this test is shown in Fig. 155.

Then the insulation resistance R of the winding under test is given by the following equation:

$$R = \frac{r(E - e)}{e}$$

where r = internal resistance of the voltmeter, which must be known and is usually given by the maker.
E = direct-current voltage which is used for the test.
e = reading of the voltmeter.

[1] Standardization Rules of the American Institute of Electrical Engineers.

For example, suppose the values for the test are, $E = 545$ volts, $e = 5$ volts and $r = 55{,}000$ ohms. Then $R = \frac{(545 - 5)55{,}000}{5}$ = 5,940,000 ohms, which would indicate that the insulation was in good condition.

This test is of secondary importance as compared with the test for breakdown under high-voltage alternating current, since the insulation resistance can be considerably increased by baking, but this gives no real increase in the actual ability to withstand voltage strains. Commenting on these two tests, the standardization rules of the American Institute of Electrical Engineers say: "The insulation resistance of a machine at its operating temperature should be not less than that given by the following formula:

$$\textit{Insulation resistance in megohms} = \frac{\textit{Normal terminal voltage}}{\textit{Rated capacity in kv.-a.} + 1000}"$$

A megohm is 1,000,000 ohms and the symbol kv.-a. or kilovolt-amperes is the voltage of the machine times the full-load current times 1.73, if three-phase, or times 2 if two-phase. A general rule is that machines up to 1000 volts should show somewhere near a megohm. The institute rules say further: "It should be noted that the insulation resistance of machinery is of doubtful significance by comparison with the dielectric strength. The insulation resistance is subject to wide variation with temperature, humidity and cleanliness of the parts. When the insulation resistance falls below that corresponding to the foregoing rule, it can, in most cases of good design and where no defect exists, be brought up to the required standard by cleaning and drying out the machine. The insulation resistance test may therefore afford a useful indication as to whether the machine is in suitable condition for the application of the dielectric test."

These two tests indicate a method of settling any doubt as to whether the insulation on a machine is suitable for a new voltage higher than the old. The method of procedure would be to see that the windings were clean and dry and free from grounds, the latter point to be determined in the usual way with a 110-volt lighting circuit or by "ringing out" with a magneto. If the winding shows clear of grounds the insulation resistance should be measured with any convenient source of direct-current supply,

preferably 500 volts. If the insulation resistance is up to or beyond the value specified by the A. I. E. E. formula, the winding may be given the further dielectric or breakdown test for one minute with high-voltage alternating current provided a suitable small testing transformer is available. In making this test great care should be used in handling the high voltage to guard against personal injury and also a suitable circuit-breaker should be in circuit which will open if the insulation breaks down.

Volts per Turn.—Assuming that the question of the adequacy of the insulation is settled, the second main consideration in all voltage changes may be taken up. This is the question of

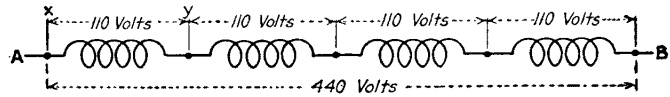


FIG. 156.—Four 110-volt coils connected in series across 440 volts.

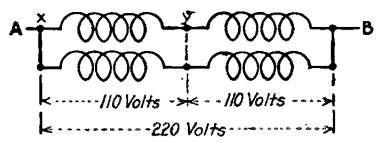


FIG. 157.—Same coils connected two in series in two parallels across 220 volts.

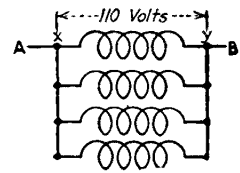


FIG. 158.—Same coils connected four in parallel across 110 volts.

rearranging the coils or coil groups in the windings so that the voltage on each coil under the new conditions may be substantially the same as under the original. In this regard an induction motor is similar to a transformer. It is designed originally for a certain voltage across each coil or group of coils. These coils or groups may be arranged in series or in various parallels to accommodate different line voltages, and so long as the voltage across each coil remains at the figure originally calculated, the operation of the motor will be normal in all respects. This can be shown graphically as in Figs. 156 to 158. In these figures *A*-*B* represents one phase of a two-phase four-pole winding. It will be seen that the voltage across one pole-phase group, or *X*-*Y*, is 110 volts at all times. When the motor is connected for 440 volts, Fig. 156, all four pole-phase groups are in series. When the line is 220 volts, there are two parallels with two pole-phase groups in series in each parallel, Fig. 157.

When the line voltage is 110 volts, all four pole-phase groups are in parallel and each group is across the line, Fig. 158, since each group has within itself the proper number of turns for

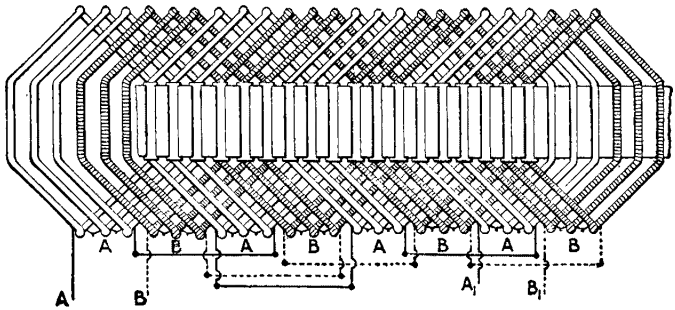


FIG. 159.—Four-pole, series.

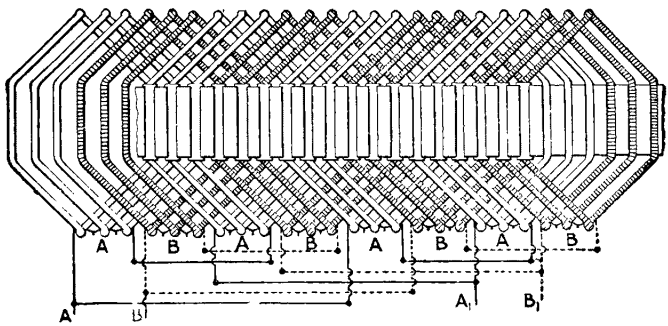


FIG. 160.—Four-pole, two parallels.

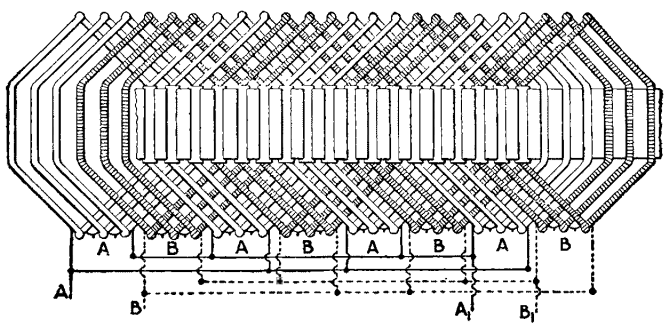


FIG. 161.—Four-pole, four parallels.

FIGS. 159–161.—Different groupings of a two-phase, four-pole winding.

110 volts. Figures 159, 160 and 161 show a 24-coil four-pole two-phase winding connected in series, two parallels and four parallels, respectively, as shown schematically in Figs. 156, 157

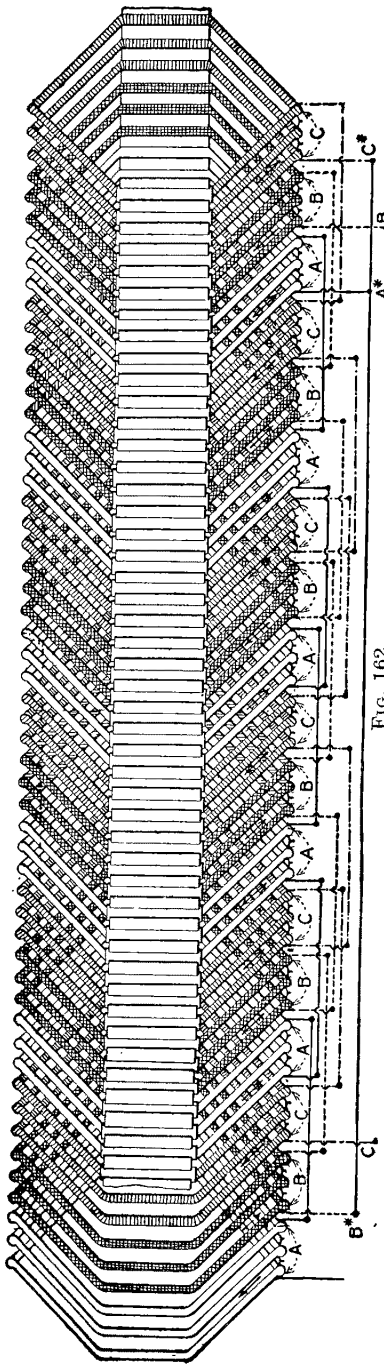


FIG. 162.

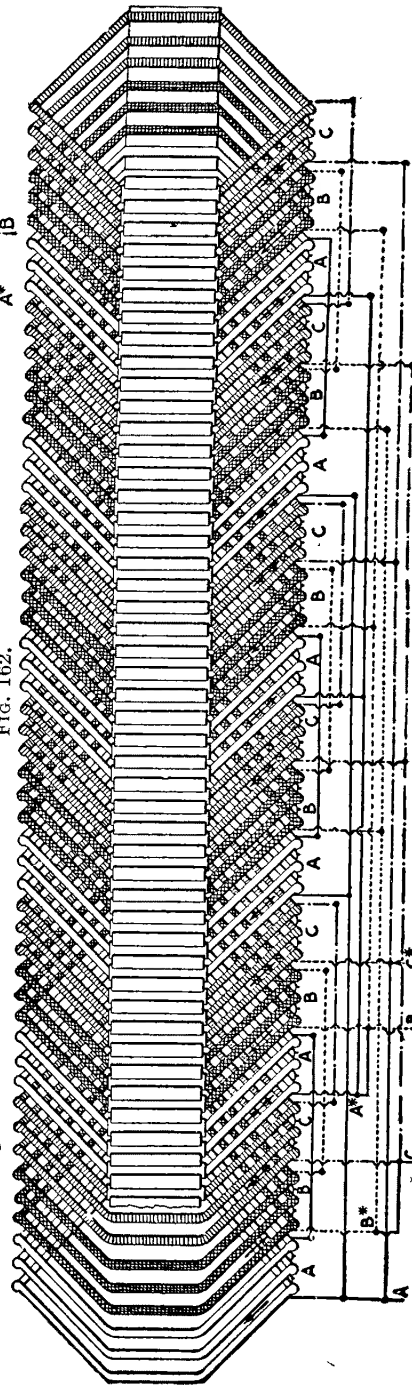


FIG. 163.

and 158, respectively. If the connection, Fig. 159, is to operate on 440 volts, for 220 volts the winding will be connected as in Fig. 160 and for 110 volts as in Fig. 161.

The foregoing is very simple and is all that need be borne in mind for changes of this nature. One caution needs to be observed, and that is to handle the pole-phase groups as units and not attempt to split them in the middle again—to make eight parallels, for example, for a 55-volt connection. Such attempts result in improper connections as will be pointed out in Chapter XII, Fig. 204. If the number of poles is divisible by three or five or seven, corresponding numbers of parallels may be made, which is often convenient.

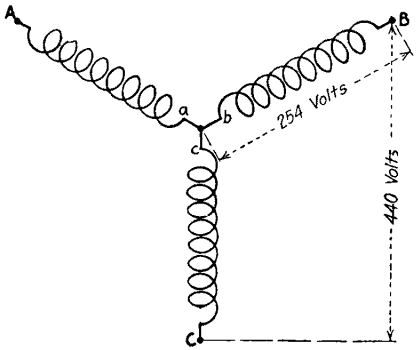


FIG. 164.

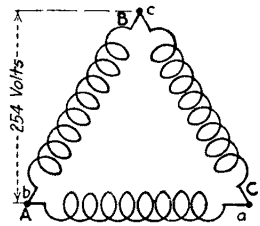


FIG. 165.

FIGS. 164 and 165.—Equivalent voltage for star and delta connection.

For example, if a three-phase six-pole 2200-volt motor is to be reconnected for 440 volts, it may be connected three parallel delta, Fig. 163; it would give 423 volts if it had been connected series star, as in Fig. 162, on 2200 volts. The quotient of $\frac{2200}{3 \times 1.73} = 423$; the 3 comes from the three parallels and the 1.73 is due to changing from star to delta. The latter change is one of the advantages or points of greater flexibility of three-phase over two-phase. This is illustrated in Figs. 164 and 165. The "star" diagram shows the winding connected for a line voltage of 440. The voltage which then exists between any lead and the star point is 254 volts, as shown on the *B* phase. Since this is true, the winding can be connected in delta as shown in Fig. 165, and operated on a line voltage of 254. This change is sometimes made to operate a 440-volt motor on 220 volts, but since 254 volts is normal, the delta-connected winding will com-

pare with the star winding as though operated on 220/254 of normal voltage, or 87 per cent. Many motors have sufficient margin to stand this reduction, but the copper heating will be four-thirds as great and the starting and maximum torques only three-fourth as great as on the winding connected in star and run on 440 volts.

Changes of this nature can be summed up in convenient form as in Tables IV and V for three-phase and two-phase motors respectively. *If a motor connected originally as shown in any horizontal column has a voltage of* 100, *its voltage when reconnected, as indicated in any vertical column is shown at the intersection of the two columns.*

TABLE IV.—COMPARISON OF MOTOR VOLTAGES WITH VARIOUS THREE-PHASE CONNECTIONS

	Series star	2 parallel star	3 parallel star	4 parallel star	5 parallel star	6 parallel star	series delta	2 parallel delta	3 parallel delta	4 parallel delta	5 parallel delta	6 parallel delta
Series star	100	50	33	25	20	17	58	29	19	15	12	10
2 parallel star	200	100	67	50	40	33	116	58	39	29	23	19
3 parallel star	300	150	100	75	60	50	173	87	58	43	35	29
4 parallel star	400	200	133	100	80	67	232	116	77	58	46	39
5 parallel star	500	250	167	125	100	83	289	144	96	72	58	48
6 parallel star	600	300	200	150	120	100	346	173	115	87	69	58
Series delta	173	86	58	43	35	29	100	50	33	25	20	17
2 parallel delta	346	173	115	87	69	58	200	100	67	50	40	33
3 parallel delta	519	259	173	130	104	87	300	150	100	75	60	50
4 parallel delta	692	346	231	173	138	115	400	200	133	100	80	67
5 parallel delta	865	433	288	216	173	144	500	250	167	125	100	83
6 parallel delta	1038	519	346	260	208	173	600	300	200	150	120	100

TABLE V.—COMPARISON OF MOTOR VOLTAGES WITH VARIOUS TWO-PHASE CONNECTIONS

	Series	2 parallel	3 parallel	4 parallel	5 parallel	6 parallel
Series	100	50	33	25	20	17
2 parallel	200	100	67	50	40	33
3 parallel	300	150	100	75	60	50
4 parallel	400	200	133	100	80	67
5 parallel	500	250	167	125	100	83
6 parallel	600	300	200	150	120	100

General Tables Covering All Voltage Connections.—The figures in the tables should be considered as percentages or comparative values rather than actual voltages. For example, in the case just cited, of the 2200-volt motor to be reconnected for 440 volts, assume that an inspection of the existing winding connection shows it to be series star. Since 440 is 20 per cent of 2200, the problem resolves itself into how a series-star connection may be changed so that the resulting voltage will be 20 per cent of its value. Looking at Table IV, locate the horizontal line reading "series star," or the existing connection. Since 20 per cent is required, read along the same horizontal line till the figure 20 is reached. This is found under the vertical heading "5 parallel star." In other words, if the number of poles is divisible by 5, the winding can be put in five parallels and operated on 440 volts, since $2200 \div 5 = 440$. Since six poles were assumed the number of poles is not divisible by 5 and a five-parallel connection is not possible. A further search across the table shows the figure 19 under the vertical heading "3 parallel delta"; 19 per cent of 2200 is 418, which is 95 per cent of 440. This varies from the figure 423 previously mentioned, for the reason that the table is made to the nearest whole number and $\frac{100}{3 \times 1.73} = 19.2$ per cent. It will be accurate enough to reconnect in 3-parallel delta and operate on 440 volts. Similar problems can thus be solved by inspection, making such a table a very convenient reference. In Chapter IX this table will be elaborated and combined with changes in phase also, thus covering a large percentage of the possible changes in windings at a glance.

Effect on Motor Performance of Unbalanced Voltages.—In a normal three-phase circuit the three voltages are not only equal but they are symmetrically spaced in a proper three-phase relation. Considering Fig. 166*a* the three lines *A*-*B*-*C* represent a three-phase circuit. With a voltmeter the voltage is read between the three lines taken in pairs, *i.e.*, between *A* and *B*, *B* and *C* and *A* and *C*. If it is a normal three-phase circuit, not only will the three voltages be equal but they will actually be spaced in time one-third of a cycle apart. If they are represented graphically, they can be drawn as an equilateral triangle (delta) as in Fig. 166*b* or they can be considered as the result of the star system of Fig. 166*c*. In either case the three voltages are in the

proper three-phase relation, which means that the angles of the triangle *ABC* in Fig. 166*b* are 60 deg. or that the arrows or vectors *OA*, *OB* and *OC* in Fig. 166*c* are 120 deg. apart.

It is possible for things to happen to such a three-phase circuit so that the three voltages are no longer exactly equal or that they are not spaced exactly in a three-phase angular relation. Such a condition might result from too heavy a load taken from two transformers in vee or "open delta," as it is called; or a heavy single-phase railway or lighting load may be drawn from one phase of a three-phase distribution line; or it may be caused by cutting out and leaving dead, damaged coils in one phase of a large three-phase motor. When this happens and the three

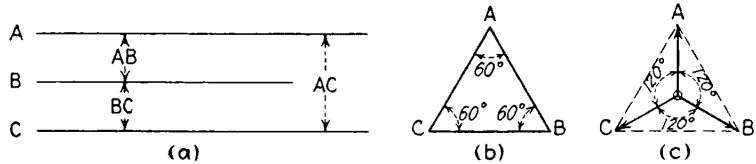


FIG. 166.—Normal three-phase voltage relations.

voltages no longer are exactly equal or when the phase angles between them are wrong, the question immediately arises, what effect will this have on the performance of a motor run from such a circuit? The answer can immediately be made that it will reduce the starting and maximum torques of the motor and increase its heating. The other questions that naturally follow are, how bad will this condition be and is it safe to operate the motor?

The answers to these questions concern some fairly involved theoretical conditions but they can be drawn graphically and made so simple that they will be readily understood by anyone who can follow the material of this book.

Speaking very simply, the result, such an unbalanced voltage produces in the motor, is to set up the usual magnetic field rotating in the right direction to produce useful torque and in addition to set up another and smaller magnetic field rotating in the wrong direction and directly opposing the useful torque. Of course, if it is a 60-cycle motor, the field which is rotating in the opposite direction from the rotor produces double frequency (nearly) in the rotor iron and this produces higher iron loss and heating.

Referring to Fig. 167, suppose that a circuit, which if normally balanced would produce a field whose three voltages could be represented by Fig. 167*a*, becomes for some reason unbalanced. If the normal voltage such as *OA* in *a* is 110 volts, the result of the unbalance will be a field, such as *b*, of perhaps 90 volts rotating in the same direction as *a* and a smaller field, such as *c*, of perhaps 15 volts rotating in the wrong direction or opposed to *a*. The problem then is to take the unbalanced voltage readings and from them to determine graphically what to expect of the motor. The assumption is that by a test on a normal balanced circuit it is known how much starting torque and pull out or maximum

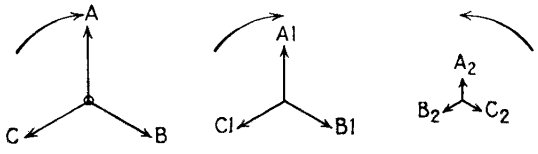


FIG. 167.

torque the motor has. Then if it can be determined graphically what voltage corresponds to the opposite rotating fields of Figs. 167*b* and 167*c*, the rule can be applied that the torque varies as the square of the voltage and the result can be closely approximated. For instance, in the case cited above, if the balanced motor had a starting torque of 100 pounds on 110 volts, starting torque on 90 volts would be $(90/110)^2 = 81/121$ times 100 pounds or about 67 pounds. The reader raises the question that the opposing torque due to Fig. 167*c* should be subtracted, and so it should. However, this is very small for two reasons: the voltage is very low and $(15/110)^2 = 225/12100 \times$ 100 pounds = 1.9 pounds, or less than 2 per cent. But it is not really that much because the speed-torque curve due to the oppositely rotating field is "upside down," so to speak. Referring to Fig. 168, there is shown on the right-hand side of the vertical line *YOY* the speed-torque curve *A* of the motor under discussion when running on a balanced 90-volt three-phase circuit. To the left of *YOY* is shown dotted curve *B*, which is the torque that the oppositely rotating field would produce at 90 volts. Since the voltage is only 15 and the torque decreases as the square of the voltage, the curve *C* which results has torques in the ratio of $15^2 = 225$ to $90^2 = 8100$ as compared with curve *B*.

The combination of the two curves would be, then, the torques of the curve A reduced by the very small torques between curve C and the line YOY. As worked out above, near the origin O the back torque is 1.9 pounds as compared with the forward torque of 67 pounds. It will be readily seen that the back or negative torque is low not only because of the low voltage but also because it is on that part of the curve which is found only when the motor is driven backward.

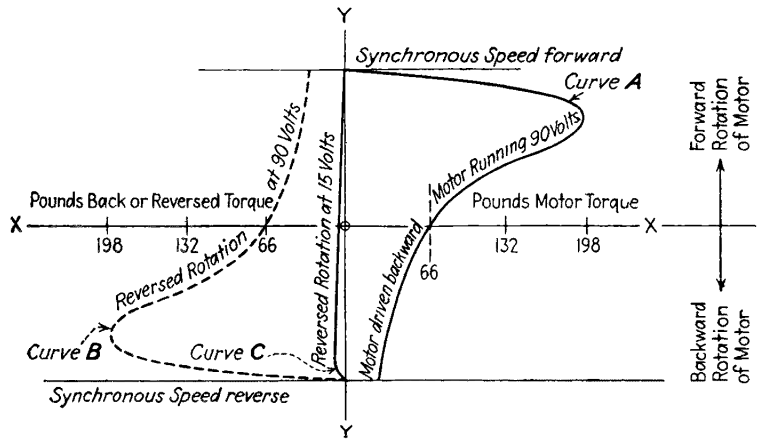


FIG. 168.

How to Resolve Three Unbalanced Three-phase Voltages into Two Balanced Systems.—From the foregoing it is not difficult to conceive of the two magnetic fields rotating in opposite directions. The reader is ready for the next step of taking three unequal voltage readings and converting them into two balanced voltage systems with opposite rotations as compared with the normal direction of rotation of the motor. It might be mentioned here that these two oppositely rotating fields are what the transmission engineers call "positive-sequence" and "negative-sequence" voltages when they are using symmetrical components to solve line-to-ground or line-to-line faults on a transmission system. But one does not need to be scared off by this reference to what is usually considered fairly difficult mathematics; the graphical construction is very simple and easy and after a little practice it becomes an interesting diversion to assume various conditions of unbalance and then work them out. It is still more satisfactory if one can run a brake test on the motor, taking

its speed-torque curve with an unbalanced condition and then with a balanced voltage equivalent to the so-called "positive-sequence" voltage in order to see how it checks.

Suppose the voltages have been checked on a set of three-phase lines as in Fig. 166 and that they are found to be $AB = 104$, $BC = 78.5$ and $AC = 56.5$. This is an extreme case of what might happen on a circuit supposed to be normally 110 volts on all three phases.

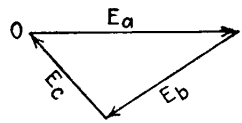


FIG. 169.

The first thing to do is to cut three pieces of toothpick to lengths corresponding to these three voltage values and to see if they can be put together to form a triangle. Immediately it follows that they can as in Fig. 169, where $E_a = 104$, $E_b = 78.5$ and $E_c = 56.5$.

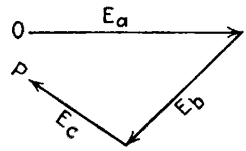


FIG. 170.

Right here the question arises whether *any* such set of irregular three-phase voltages would form a triangle or, if they were really plotted with their correct phase angles, whether there might not be one corner of the triangle which would not close, as in Fig. 170. If so, what would such a condition mean? The answer is that, if the circuit is connected in star and the neutral is not grounded, the triangle will always close. If the neutral is grounded or if the circuit is connected in delta, the triangle may not close and this produces a so-called "zero-sequence" voltage, which is explained later.

The three voltages of Fig. 169 can be rearranged in star as shown in Fig. 171 by moving each arrow or vector parallel to itself without rotating it until the three ends are connected at the point O. The fact that E_a in Fig. 171 seems longer than E_a in Fig. 169 is purely an optical illusion. They are actually constructed the same length. E_b and E_c are also equal in the two figures. The problem now is to take the three arrows of Fig. 171 and reduce them to two balanced systems as in Fig. 167*b* and *c*. The construction for the positive-sequence or useful voltage, or the one that produces a magnetic field rotating in the right

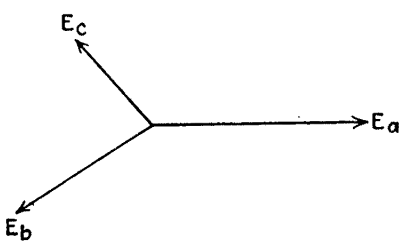


FIG. 171.

direction to give useful torque, is to start from the point O and run along the vector E_a to its end; from this point lay off a vector aE_b, Fig. 172, which is obtained by rotating the vector E_b counterclockwise around the point O by 120 deg., or one-third of a complete circle; this locates the point aE_b. From this point, travel to a^2E_c over a vector obtained by rotating the vector E_c of Fig. 171 counterclockwise through 240 deg., or two-thirds of a complete circle. This construction is easily understood from Fig. 172. A protractor may be needed to get the angles 120 deg. and 240 deg. A simpler method is to construct with a compass an equilateral triangle, in which all the angles are 120 deg.; this can

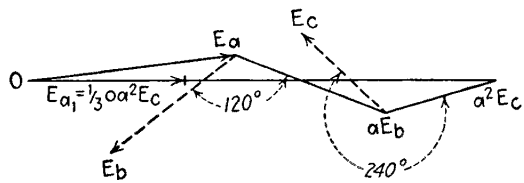


FIG. 172.—Construction for positive-sequence voltages.

be used as a protractor. Finally join the original point O with the point a^2E_c; the resulting long line is the sum of the phase A, phase B and phase C voltages of a positive balanced sequence system similar to Fig. 167*a*. In other words, draw a figure like Fig. 167*a* and make the lengths of the vectors or arrows equal to one-third of the line Oa^2E_c of Fig. 172. Since the original vector Ea of Fig. 171 was made equal to 104 volts, it follows by scaling Fig. 172 that each one of the balanced positive-sequence voltages which were made equal to one-third of the length of Oa^2E_c or Ea_1 of Fig. 172 is 77 volts. Hence the conclusion is immediately reached that, if a normal 110-volt three-phase circuit is so unbalanced that the voltages read 104, 78.5 and 56.5, the forward torque on the motor is that which would be produced by a balanced three-phase circuit where all three voltages are 77. This gives the system corresponding to Fig. 167*b*.

The negative-sequence system corresponding to Fig. 167*c* is constructed according to Fig. 173. As in Fig. 172 a start is made from O, traveling the length of E_a; the next step to a^2E_b is over a vector obtained by rotating E_b of Fig. 171 counterclockwise through 240 deg., or two-thirds of a complete circle. From a^2E_b proceed to aE_c over a vector obtained by rotating vector E_c of Fig. 171 counterclockwise through 120 deg. As before, join

the initial point O with the final point aE_c and this line is the sum of the three voltages of the negative-sequence system. In other words take one-third of OaE_c or E_{a2} of Fig. 173 and use it for the lengths of the three vectors in Fig. 167*c*. By scaling this carefully to the same scale as Figs. 171 and 172, it is found to be 28.4 volts. Referring again to Fig. 168, the "upside-down" curve C is that which is due to 28.4 volts, whereas the curve corresponding to curve A is that due to 77 volts, as demonstrated in Fig. 172. Of course, one should have a curve taken on the motor on a balanced three-phase 110-volt circuit in order to get actual values for 28.4 and 77 volts, but at least this will give an idea of what to expect. 28.4 volts would be too low to run the motor but, by taking the square of 28.4 as compared with the square of 110 and using the 110-volt torque values, such a curve can be plotted.

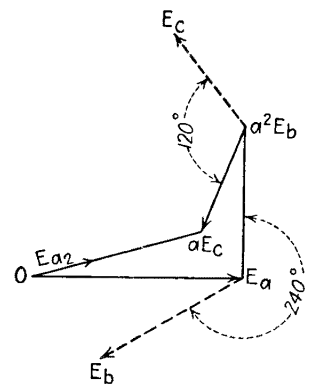


FIG. 173.—Negative sequence.

An actual brake test was run on a motor with the unbalanced voltages of 104-78.5 and 56.5. A curve was then run on three balanced voltages of 77-77-77 volts. The torques obtained were found to be less than those of the unbalanced test by just the amount of the field which would be due to 28.4 volts rotating in the wrong direction. This completely checked and verified this simple construction and theory.

James A. Wasmund of the University of Pittsburgh has just completed a thesis for a master's degree in which he ran a number of tests similar to this under various conditions of unbalance. It is his conclusion that, for any amount of unbalance normally met with on usual commercial circuits, the speed-torque curve of a motor can be nearly enough calculated by using the positive-sequence voltage only and neglecting the negative-sequence voltage, as it is too small to be of serious consequence. For example, on a circuit reading 108-98-98 volts, the motor behaved almost exactly as if operated on 101-101-101 volts and he could neglect a backward component due to 7-7-7 volts. Also a circuit of 104-96.5-90 acted like 97-97-97 volts and 103-80-83 was the same as 88-88-88. This was a beautiful check on the symmetrical

component theory and reduces it to a practical tool easily used by any operating engineer.

There remains only the point of what happens if the triangle does not close as in Fig. 170. In that case the closing line *OP* would represent a voltage circulating current around inside the delta of a delta-connected circuit or causing current to flow to ground in a grounded star circuit. It should also be divided by 3 and considered as $\frac{1}{3}$ in each phase, with the exception that all three of these "zero-sequence" components are in phase or parallel to one another and not in a three-phase relation as in Figs. 167*b* and *c*.

CHAPTER IX

HOW THE NUMBER OF PHASES AFFECTS THE WINDINGS AND THE RESULT OF CHANGING VOLTAGE AND PHASE AT THE SAME TIME

It was shown in Chapter VIII that changes in voltage of the supply circuit can be taken care of with comparative ease and simplicity by the proper changes in connection of the motor windings, provided that the maximum number of turns which can be placed in series in the coils is equal to or greater than the number required under the new conditions. For example, a 220-volt motor may be reconnected for 440 volts, provided that windings can be so arranged that there will be twice as many turns in series between the terminals of each phase as there were with the original connection. These changes, when possible, offer no particular difficulty.

On the other hand, changes in the number of phases of the supply circuit are usually difficult to accommodate by changes in the motor connections and many times when they can be accomplished are attended with a loss in capacity of the motor or a serious reduction in the excellence of the motor's performance as regards torque, heating, power factor and efficiency.

Changes in Phase.—By far the commonest change of this nature is changing from two-phase to three-phase and vice versa. Of the two changes, that from two-phase to three-phase can more often be taken care of for the reason that a normal two-phase motor has approximately 25 per cent more turns in series in its windings than a three-phase motor of the same characteristics. Thus it is usually possible to cut out 20 per cent of the turns in a two-phase winding, leaving them dead, and have left the proper number of turns for the corresponding three-phase winding. However, in going from three-phase to two-phase a corresponding increase of 25 per cent of the total number of turns in series is required; and, if the three-phase winding as it stood had all the turns in series, any further increase is not possible and a set of new two-phase coils will be required.

There are three methods of reconnecting from two-phase to three-phase, which are here given in the order of their desirability: (1) Twenty per cent of the coils are cut out and left dead and the motor operated on 80 per cent of the two-phase turns; (2) the number of coils is not changed, and the coils are reconnected according to the proper diagram; (3) a "T" or Scott two-phase to three-phase connection is used.

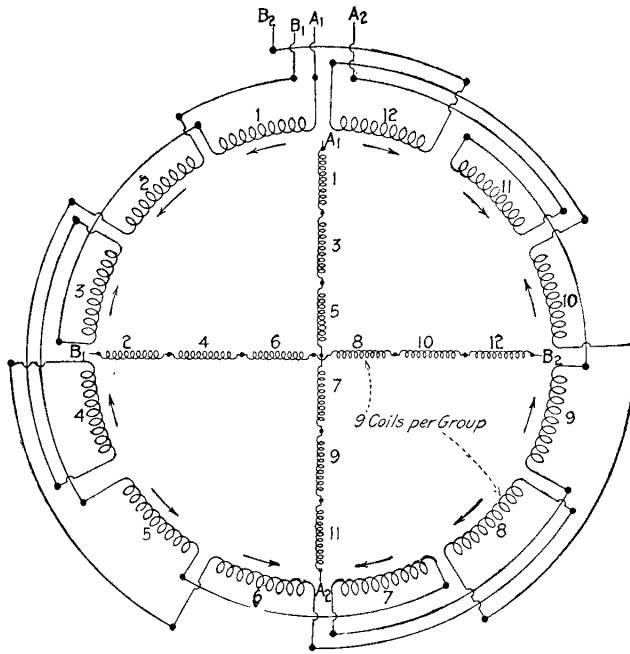


FIG. 174.—Normal two-phase, six-pole series connection, nine coils per group.

None of these is ideal, and in general it is a good investment to rewind the motor with proper three-phase coils. In the first method it must be borne in mind that the full-load current of a three-phase motor is 2/1.73 or about 115 per cent of the current in a two-phase motor. This means that for the same heating the horsepower output when reconnected for three-phase can only be in the neighborhood of 87 per cent of what it was on two-phase. This loss of 13 per cent of the horsepower when capitalized in the proper manner will be found to pay a high rate of interest on the money that would be invested in a new set of coils for normal three-phase operation which would give the same horsepower output as the original two-phase winding.

Another way of arriving at the foregoing conclusion is as follows: If one-sixth of the two-phase coils are to be cut out of circuit and left dead, as shown in Fig. 175, the amount of active copper is reduced by the same percentage; and it might be expected that the horsepower output would be similarly reduced, which is the case. This method of reconnecting from two-phase to three-phase is shown in Figs. 174 and 175. Figure 174 shows a

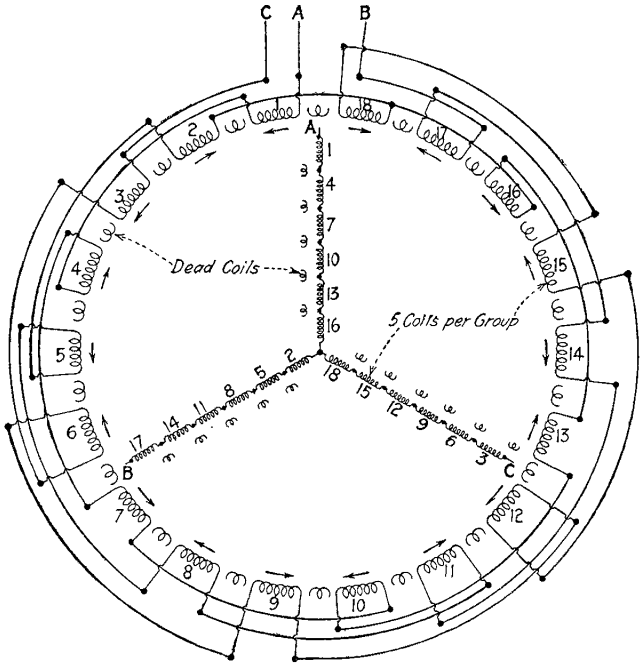


FIG. 175.—Winding of Fig. 174 reconnected for three-phase by leaving "dead" coils.

winding with 108 coils connected in series for two-phase and six poles. There are $2 \times 6 = 12$ pole-phase groups and $108 \div 12 = 9$ coils in each group. As already stated, if this winding is to be reconnected for three-phase, six poles, there should be only 80 per cent as many coils in series in the winding, or $0.80 \times 108 = 86.4$ coils.

Since there are to be $3 \times 6 = 18$ pole-phase groups in the new connection, there should be the same number of coils in each group; the nearest integer is 5, and $5 \times 18 = 90$ coils, which will be used instead of 86.4, which is theoretically correct. This leaves $108 - 90 = 18$ coils dead, or 1 dead coil in each group, as

shown in Fig. 175. Since 90/108 = 0.833, then 83.3 per cent of the coils are active instead of 80 per cent, and this will have the effect of operating a three-phase motor on 80/83.3, or 96 per cent of normal voltage, as compared with the two-phase motor. The starting and maximum torques of the three-phase motor will be about $(80/83.3)^2 = 92$ per cent of their value on a two-phase connection; but this is sufficiently close for all practical purposes, especially as the horsepower rating will have to be reduced 13 per cent, as stated above, if the original maximum heating in the stator coils is not to be exceeded. A comparison of Figs. 174 and 175 indicates that the position of the coils, which are specially insulated to stand the voltage between phases, will have to be changed. This was mentioned in the Chapter VI under Phase Insulation.

A consideration of the fact that there are 18 dead coils in the three-phase winding which are active on the two-phase connection suggests at once that if the reconnection was attempted from three-phase to two-phase there might in many cases be insufficient coils to put in series for the two-phase connection. If the coils are regrouped for two-phase and run on the same voltage, the motor shows all the signs of a machine operating on 25 per cent overvoltage and may even overheat when running light and not connected to any load whatever. On the other hand, if a two-phase winding is regrouped and operated three-phase on the same voltage without cutting out any coils, as explained in connection with Fig. 175, the three-phase motor shows all the effects of a motor operating on 80 per cent of normal voltage; that is, the starting and maximum torques will be considerably reduced and the heating increased. These two latter conditions are covered by the second method of reconnecting listed in the foregoing—namely, changing the grouping and connections properly, but neglecting the change in the total number of coils in series.

The third method occasionally employed is that of making a "T" connection of the two-phase windings or a Scott connection inside the motor and operating the resulting winding on a three-phase circuit. This should not be confused with the use of Scott-connected transformers for changing from two-phase to three-phase or vice versa. The latter may be an excellent solution in many cases where there are several motors affected by the change

in phase. Let us assume, for example, that a user of motors has 15 machines of various sizes from 1 to 50 hp., which have been operating from his own steam-driven plant at two-phase, 220 volts. He decides to purchase power from a neighboring distribution system at three-phase, 220 volts. It is a matter of considerable expense to rewind all the motors for three-phase and if simply reconnected the losses on the rated capacity are as previously suggested. In addition, it is desired to hold the old

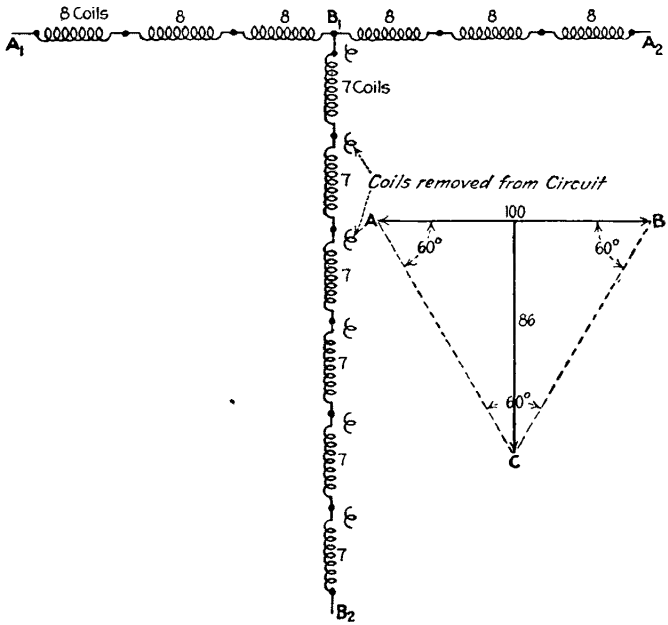


FIG. 176.—Schematic diagram of "Tee" connection.

generating plant as a stand-by, in case of interruption to the purchased service. All these results can be secured by putting in transformers equivalent to 50 or 60 per cent of the capacity of motors installed and by means of a Scott connection on the transformers operate the two-phase motors from the three-phase supply in a perfectly normal manner. This is one very neat solution for a problem in reconnecting induction motors which does not involve any reconnection whatever.

On the other hand, assume that in the same plant the generating system has broken down and, in the emergency, power can be purchased from the same neighboring power line at three-

phase. There is no time to secure transformers, and there is no time to secure three-phase coils for the motors—it then becomes essential to make some kind of reconnection so that the two-phase motors will operate on three-phases. One of the possibilities in such a case is a Scott connection inside the motor winding itself. This is shown in Chapter XII and Fig. 176.

Table VI shows comparative performances of a two-phase motor reconnected for operation on three-phase by a "T" con-

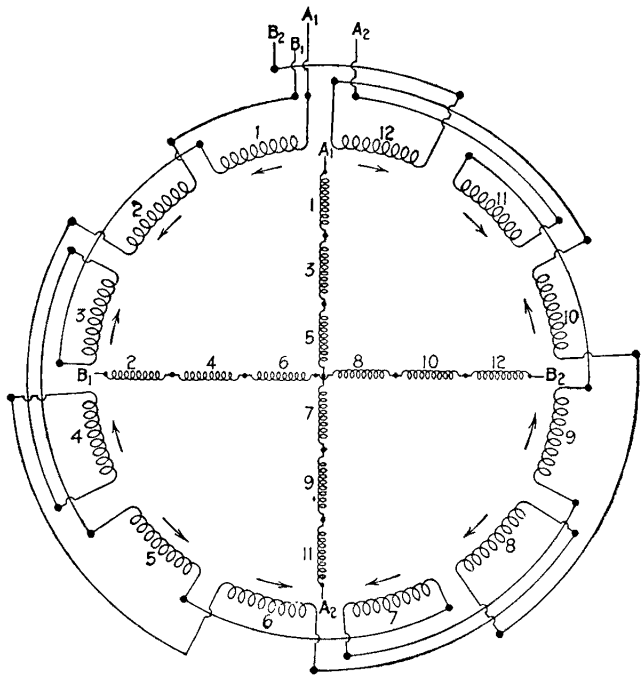


FIG. 177.—Normal two-phase, six-pole, series connection, eight coils per group.

nection and the performance of the same motor when supplied with new three-phase coils and connected in a normal three-phase manner.

In order to make this connection clear, Fig. 177 shows the windings on the motor connected for two-phase, and Fig. 178 the motor as reconnected with a "T" connection, corresponding to the schematic diagram, Fig. 176.

The principle of the Scott connection is well understood and explains the reason for omitting the coils in one leg, as indicated.

TABLE VI.—COMPARISON OF A TWO-PHASE MOTOR CONNECTED "T" TO OPERATE ON THREE-PHASE WITH NORMAL THREE-PHASE WINDING

	Normal two-phase winding	Three-phase "T" connection	Normal three-phase winding
Full-load efficiency	88.0	86.9	88.5
Full-load power factor	89.0	84.8	90.0
Starting torque	1.75	1.20	1.94
Maximum torque	3.3	3.17	3.3
Deg. C. Rise at Full Load:			
Stator copper	22.5	32.0	21.0
Stator iron	20.0	32.5	19.0
Rotor copper	22.0	30.0	22.0

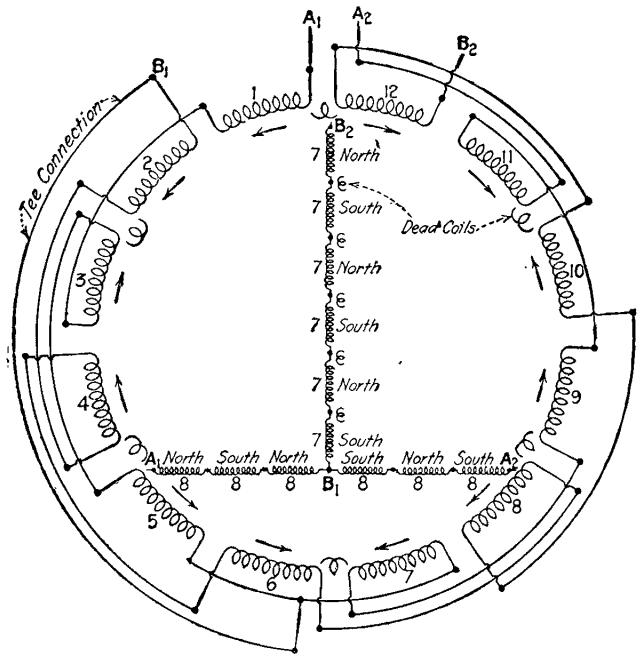


FIG. 178.—Winding of Fig. 177 reconnected "Tee" for three-phase, so-called "top to top" connection.

It may be of interest, however, to consider what would happen if these coils were not omitted. This is indicated in the voltage diagram, Fig. 179; BD represents the voltage generated in the phase B_1B_2 of Fig. 176, by the rotation of the magnetic field and AC the voltage generated in the phase A_1A_2. The result is

three perfectly balanced voltages, AB, BC and CA, which correspond to the voltage of the line in the three phases and allow normal operation. If the coils had not been cut out of the B phase, as shown in Fig. 176, the voltage generated in that phase by the rotating magnetic field would have been the same in value as that in the A phase and would be represented by DE in Fig. 179. The voltages AE and EC would then be represented by 111, while CA would be 100. This would be equivalent to having one alternating-current generator representing the lines with balanced voltages of 100 each, or AB, BC and CA connected in parallel with another, alternating-current generator representing

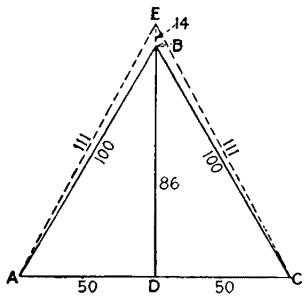


FIG. 179.—Voltage diagram for Fig. 178.

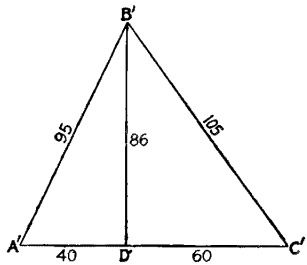


FIG. 180.—Effect on voltages of uneven air gap.

the motor windings and having unbalanced voltages, AE, EC and CA of 111, 111 and 100, respectively. The result of this would be a component BE equal to 14, which would spend itself driving useless wattless currents through the motor windings in an effort to balance properly the voltages and make them equal to AB, BC and CA. The immediate result of this useless current would be to increase the heating of the machine and decrease its torque and efficiency and power factor.

It is characteristic of an induction motor that it always makes this attempt to balance by circulation of wattless current any eccentricities existing either in its own windings or in the circuit to which it is connected. At times when such eccentricities exist in the stator winding, there will be wattless currents flowing in the rotor winding trying to correct them through the medium, always, of the rotating magnetic field. At other times when a power circuit of relatively large power is somewhat unbalanced and is connected to an induction motor, the motor will take upon

itself the burden of correcting the dissymmetry of the entire line with disastrous results to the motor from overheat due to excessive corrective currents, although the motor may have been running idle at the time and developing no actual power. This explains why the coils are cut out of one phase, as shown in Fig. 176.

Poor Results of the "T" Connection.—The reason for the comparatively poor results on the "T" connection, as shown in

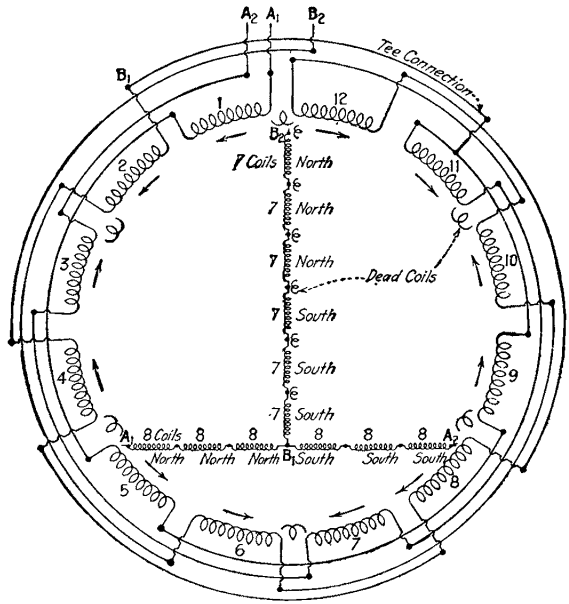


FIG. 181.—Preferable connection to Fig. 178 so-called "top to bottom" connection.

Table IV is that the motor was connected as shown in Fig. 178. The result of this connection, if the air gap was not absolutely the same all around the rotor, would be to make AD and DC in Fig. 179 unequal; and a voltage diagram, as shown in Fig. 180, might result. When the voltage triangle $A'B'C'$ of Fig. 180 is connected in parallel with the symmetrical line triangle represented by ABC in Fig. 179, the result is that corrective current will flow and these corrective currents pull down the performance, as shown in the table. A much better connection is the one shown in Fig. 181, since this will have a tendency to keep the

point D in Fig. 179 in the middle of the side AC and not let it be moved to one side, as in Fig. 180.

A comparison of Figs. 178 and 181 shows that in Fig. 178 the half legs A_1B_1 and B_1A_2 of the A phase, represented by AD and DC, Fig. 179, each contain both north and south polar groups, while in Fig. 181 the half leg A_1B_1, represented by AD, Fig. 179, contains only north poles and B_1A_2 only south poles. The result of this is that, if the rotor is displaced slightly in the stator bore from any cause, when the motor is connected as in Fig. 178, it may narrow the air gap opposite the B_1A_2 and widen it opposite to A_1B_1, which means the field will be stronger opposite B_1A_2. Consequently, the voltage generated in this section will be greater, as represented by $D'C'$ in Fig. 180. However, when connected as in Fig. 181, no matter if the rotor is near the stator at some point, it cannot affect any north pole without affecting the corresponding south pole, since all the lines of force that start out from a north pole must return through a south pole. Since the legs A_1B_1 and B_1A_2 are so arranged that one has all the north poles and the other all the south poles, this means that they will be affected exactly alike by any displacement of the rotor, and the voltage in the two sections will be maintained equal as represented by the lines AD and DC in Fig. 179. Therefore, in connecting a two-phase motor in "T" for operation on three-phase a diagram similar to Fig. 181 should be used, in which case the three-phase results will be much more favorable than shown in the table.

The statement has been made above that the winding of a normal two-phase motor has approximately 25 per cent more turns in series than the corresponding three-phase motor. This is, of course, true only if the turns are all in series in either case and the three-phase motor is arranged for connection in series star. If the three-phase motor under consideration is connected delta instead of star, it should be thought of as a star-connected motor at a corresponding voltage before reducing it to terms of a two-phase winding. For example, if a motor is connected series delta for operation on 220 volts, it should be reconnected series star and operated on $1.73 \times 220 = 381$ volts; or connected for two-phase, it would be suitable for approximately 80 per cent of 381 volts, or 305 volts. It will thus be seen that a delta-connected three-phase motor when reconnected for two-phase has

about 38 per cent more turns in series than are actually required, and this condition will have to be balanced up by some one of the various schemes suggested.

In general, manufacturers prefer a star to a delta connection, for the reason that the delta connection requires 1.73 times as many turns for the same operating voltage and these turns are of a correspondingly smaller sized wire. The greater number of turns of smaller wire is an objectionable condition for several reasons, among which may be mentioned that more space is occupied in the slots by insulation, leaving less for copper; the coils mechanically are less rigid and self-supporting; the smaller sized wire costs more per pound and the same number of pounds are required; and it is more expensive to wind a coil with a greater number of turns. For these reasons, if there is no other good reason for the contrary, a three-phase winding is apt to be arranged for star connection.

It often happens that in changing the winding of a motor to accommodate a change in the number of phases, it is necessary to arrange for a change in the operating voltage at the same time; as for example, changing a three-phase 440-volt winding to operate on two-phase 220 volts. Reference was made above to the fact that on a given winding the normal three-phase voltage would be 125 per cent of the normal two-phase voltage. Expressing the same condition in another way, if two motors that are otherwise identical are made to operate on the same voltage except that one is two-phase and the other is three-phase, the three-phase winding will have only about 80 per cent of the number of turns in series that are necessary in the two-phase winding. The foregoing is on the assumption that the three-phase winding is star-connected, which is usually the case. This fact permits one very convenient reconnection of this nature; namely, the one where a two-phase 440-volt winding is to be reconnected for three-phase 550 volts or vice versa.

Since 440 is 80 per cent of 550, the number of turns in series is exactly right for either the two-phase or the three-phase combination, and the only thing that has to be done is to regroup the coils for the proper number of pole-phase groups, which in a three-phase motor is 50 per cent greater than in a two-phase, and to shift the so-called "phase coils" or coils with heavier insulation to their proper positions at the beginning and end of

each pole-phase group. Other combinations of change of phase and voltage are met with, and it is useful to make up a table such as Table VII, which indicates at a glance the possible changes between two- and three-phase, star and delta, series, 2, 3, 4, 5 and 6 parallels.

Phase Changes and Voltage Changes Combined.—This table is a combination of the two given in Chapter VIII under voltage changes and shows the combination of phases as well. The manner of using this table has been explained under voltage changes, but further examples will be given here showing the way to apply it, since it gives a ready answer to practically any questions that may be asked regarding the possibility of changing windings when a change of voltage or phase or a combination of the two is involved. It will be noticed that the table as arranged is really given in percentages. That is to say, the original connection on the motor is called 100 or assumed to be good for a normal voltage of 100, and then if the winding is assumed as reconnected in some other way, the normal voltage on which the reconnected motor should be operated is shown at the intersection of the horizontal and vertical columns.

Take, for example, a motor which was originally connected three-phase two-parallel delta. Following across this horizontal line, the number 100 is found under the vertical heading that also reads "three-phase 2-parallel delta," or, in other words, when a motor is normally connected for three-phase two-parallel delta and is operated as three-phase two-parallel delta, it is being operated at 100 per cent, or exactly as the designer intended it should be operated. Suppose, however, that the winding is reconnected two-phase series, the question at once arises upon what voltage the motor should be operated to give normal operation. Following the same horizontal column, "three-phase 2 parallel delta" (since that is the original connection) across until it intersects the vertical column marked "two-phase series," the number 280 appears at the intersection of the two columns. In other words, if the three-phase two-parallel delta-connected winding is regrouped and reconnected two-phase series it must be operated on a voltage 280 per cent of the original voltage for which it was designed.

The reason these values are given in percentages instead of actual voltages is to make the table more flexible and of wider

TABLE VII.—COMPARISON OF MOTOR VOLTAGES WITH VARIOUS CONNECTIONS AND PHASES

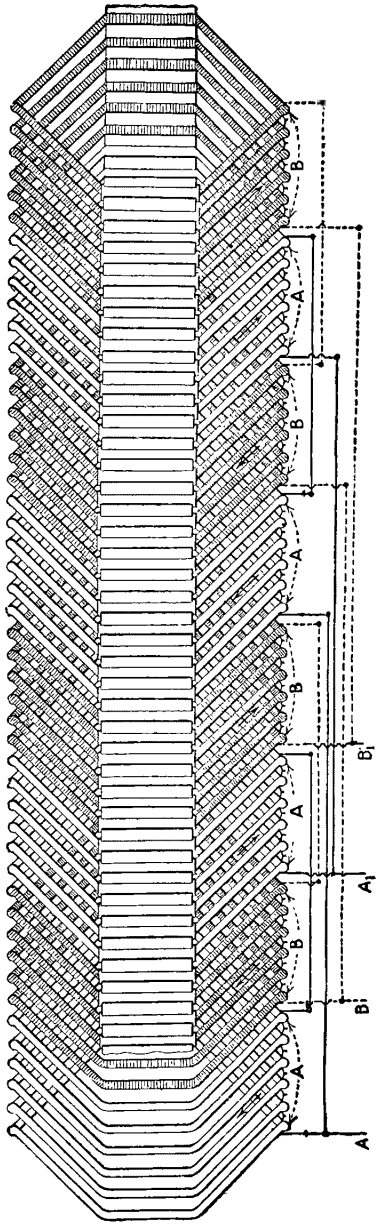
If a motor connected originally as shown in any horizontal column had a normal voltage of 100, its voltage when reconnected, as indicated in any vertical column, is shown at the intersection of these two columns.

Form of connection	Three-phase series star	Three-phase 2-parallel star	Three-phase 3-parallel star	Three-phase 4-parallel star	Three-phase 5-parallel star	Three-phase 6-parallel star	Three-phase series-delta	Three-phase 2-parallel delta	Three-phase 3-parallel delta	Three-phase 4-parallel delta	Three-phase 5-parallel delta	Three-phase 6-parallel delta	Two-phase series	Two-phase 2 parallels	Two-phase 3 parallels	Two-phase 4 parallels	Two-phase 5 parallels	Two-phase 6 parallels
Three-phase series star	100	50	33	25	20	17	58	29	19	15	12	10	81	41	27	20	16	14
Three-phase 2-parallel star	200	100	67	50	40	33	116	58	39	29	23	19	162	81	54	40	32	27
Three-phase 3-parallel star	300	150	100	75	60	50	173	87	58	43	35	29	243	122	81	60	48	41
Three-phase 4-parallel star	400	200	133	100	80	67	232	116	77	58	46	39	324	163	108	80	64	54
Three-phase 5-parallel star	500	250	167	125	100	83	289	144	96	72	58	48	405	203	135	100	80	68
Three-phase 6-parallel star	600	300	200	150	120	100	346	173	115	87	69	58	486	243	162	120	96	81
Three-phase series delta	173	86	58	43	35	29	100	50	33	25	20	17	140	70	47	35	28	23
Three-phase 2-parallel delta	346	173	115	87	69	58	200	100	67	50	40	33	280	140	94	70	56	47
Three-phase 3-parallel delta	519	259	173	130	104	87	300	150	100	75	60	50	420	210	141	105	84	70
Three-phase 4-parallel delta	692	346	231	173	138	115	400	200	133	100	80	67	560	280	188	140	112	93
Three-phase 5-parallel delta	865	433	288	216	173	144	500	250	167	125	100	83	700	350	233	175	140	117
Three-phase 6-parallel delta	1,038	519	346	260	208	173	600	300	200	150	120	100	840	420	280	210	168	140
Two-phase series	125	63	42	31	25	21	72	37	24	18	15	12	100	50	33	25	20	17
Two-phase 2-parallels	250	125	84	63	50	42	144	73	49	37	29	24	200	100	67	50	40	33
Two-phase 3-parallels	375	188	125	94	75	63	216	111	73	55	44	37	300	150	100	75	60	50
Two-phase 4-parallels	500	250	167	125	100	84	288	148	97	73	58	49	400	200	133	100	80	67
Two-phase 5-parallels	625	313	208	156	125	105	360	165	122	91	73	61	500	250	167	125	100	84
Two-phase 6-parallels	750	375	250	188	150	125	433	217	144	108	87	72	600	300	200	150	120	100

application. The percentages, however, can be very simply changed to voltages by using them as a multiplier. Applying this to the case just used as an example, assume that the voltage on which the original motor operated was 220. This then represents the 100 per cent which was called "three-phase 2-parallel delta." When changed to two-phase series, it has been shown that a voltage of 280 per cent would be required. From this it follows at once that the new operating voltage for the motor when reconnected two-phase series must be 280 per cent of 220 volts, or $2.8 \times 220 = 616$ volts.

As another example of applying the table take a case where a four-pole motor connected two-phase two parallels, as in Fig. 182, and operated on 220 volts is to be changed, if possible, for operation on a three-phase 550-volt circuit and it is desired to know what particular kind of a three-phase connection on the winding will give normal operation when the motor is run on 550 volts. In this case the horizontal line "two-phase 2 parallels" represents 100 per cent. If the original voltage was 220 and that was 100 per cent, the new voltage 550 must be 250 per cent, since it is 2.5 times 220. To find the proper form of three-phase connection, follow the horizontal column "two-phase 2 parallels" (since that was the original connection) across until it shows the value 250 under some vertical column which is headed "three-phase." This is seen to be the first vertical column, marked "three-phase series star." From this the conclusion is at once correctly drawn that, if a motor is connected two-phase two parallels and run on 220 volts and it is reconnected to three-phase series star, it will be suitable for operating normally on a three-phase 550-volt circuit. It is assumed, of course, in this problem that the number of poles and the frequency and horsepower remain the same on the new circuit as on the old, the only difference being that the old circuit was two-phase 220 volts and the new circuit three-phase 550 volts. The changed connection is shown in Fig. 183.

To illustrate further the use of the table, assume that an eight-pole motor is connected series star, as in Fig. 184, and operated on a three-phase 2200-volt circuit, what form of reconnection would make it suitable for operation on a two-phase 440-volt circuit? In this case "series star" is 100 per cent in the horizontal column and 100 per cent equals 2200 volts. The desired voltage is 440, which equals one-fifth or 20 per cent of 2200.


A
B
A
B
A
B

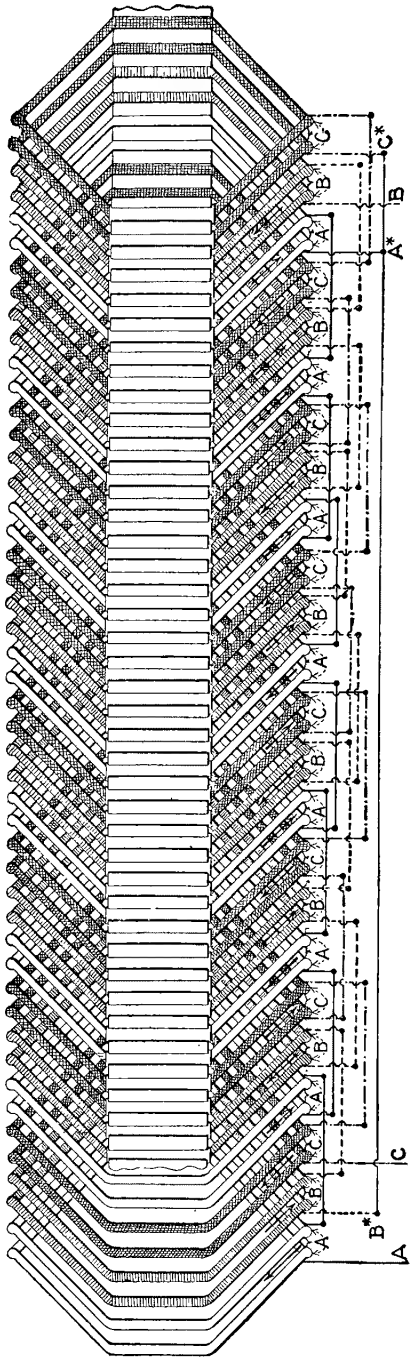


FIG. 184.—Three-phase, eight-pole, series star connection.

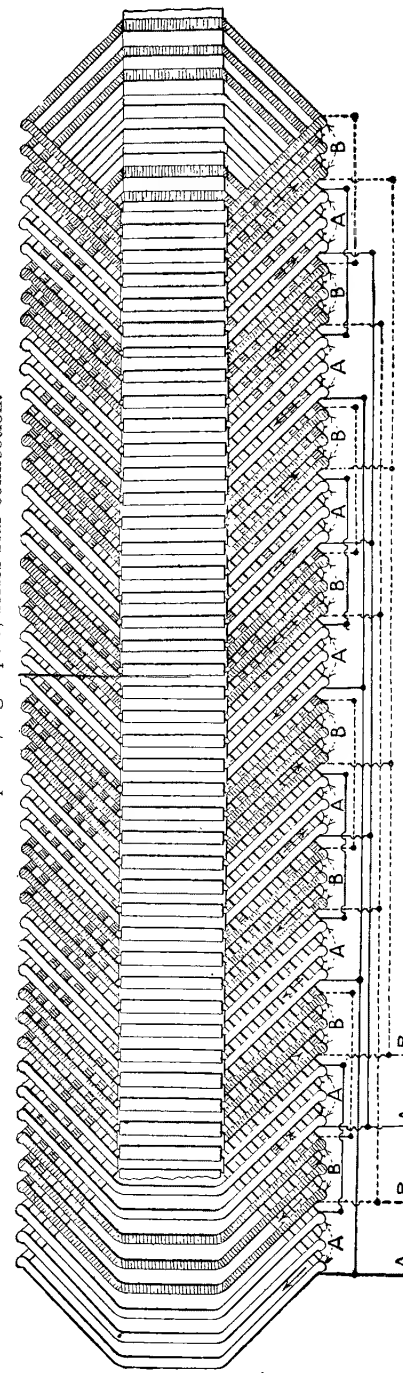


FIG. 185.—Same winding as Fig. 184 reconnected for two-phase, eight-pole, four parallels.

Following the "three-phase series star" horizontal column across to the value 20, it is found first under "three-phase 5 parallels," but this is discarded since a two-phase connection is wanted; furthermore, an eight-pole winding cannot be connected in five parallels. The value 20 is seen the second time under the vertical column marked "two-phase, 4 parallels." If the number of poles is divisible by 4, as in this case, the winding can be put in four parallels; therefore the conclusion is reached that this is the desired connection, or, in other words, if a three-phase motor is connected series star and operated on 2200 volts and is reconnected to two-phase four parallels, it will be suitable for operation on 440 volts. This connection is shown in Fig. 185. Again, assume that the motor has only six poles, as in Fig. 162, and it is to be changed from three-phase 2200 volts to three-phase 440 volts. In this case 2200 volts is again 100 per cent and 440 volts is 20 per cent. Following the horizontal column marked "three-phase series star," the value 20 is found under "three-phase 5 parallel star," meaning that, if the winding could be put in 5 parallels, it would be good for 440 volts, since $2200/5 = 440$. However, a six-pole winding cannot be connected in 5 parallels and the horizontal column is followed farther. There is not another 20 under the three-phase vertical columns, but there is a 19, which is nearly right, under "three-phase 3 parallel delta." Since a six-pole winding can be arranged in three parallel delta, as in Fig. 163, this is the connection desired, and the normal operating voltage will be 19 per cent of $2200 = 418$, which is near enough to operate satisfactorily on a 440-volt circuit.

From these scattered examples it can be seen that the table is of wide application and answers two types of questions. The first of these is what will be the new operating normal voltage if a winding is reconnected in a certain way, and the second is, what will be the form of the connection to get a new operating voltage which is desired. Indirectly, the table answers the question of whether it is at all possible to get the desired combination of changes without new coils, and if not exactly possible, what degree of approximation may be obtained by means of the working combination utilized.

In the case of wound-rotor machines it may be noted that changing either the phase or voltage of the stator has no effect on the rotor winding as long as the table shows that the recon-

nection gives exactly the right conditions. The reason for this is that the real magnetic rotating field is neither two-phase nor three-phase, but is just the same as if set up by direct current. This was described in Chapter II. Since this rotating field remains at the same value before and after the reconnection, it will clearly have the same effect on the rotor winding in generating counter-electromotive force. Hence there will be the same voltage between collector rings as existed with the original connection, and there need be no change in the controller or the external resistance used in starting and running the motor.

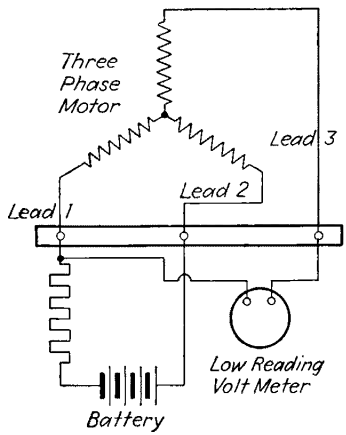


FIG. 186.

Matching Rotation of Induction Motors.—In the *Electric Journal*, May, 1935, Professor M. M. Flanders describes a useful test for determining the direction of rotation of several motors before alternating-current power is available, in case it is necessary to insure that they all run in the same direction of rotation when power is applied.

A storage battery, a resistor and a low-reading voltmeter were applied to the terminals of the first motor as connected in Fig. 186. The rotor was then turned through a small arc by hand and the movement of the meter needle was noted. It increased in value. The other motors were then connected to the battery and meter and the rotor turned in the same direction as in the case of the first motor. If the reading decreased, the leads were interchanged until the movement of the meter needle was the same in all cases. Corresponding leads were then marked 1-2-3 and when all were similarly connected to the power lines the rotation of the motors was the same.

This is a simple test and in critical cases would be most useful.

CHAPTER X

HOW THE FREQUENCY AFFECTS THE WINDINGS

The necessity for operating motors on a frequency differing from that for which they were originally designed may be the result of actually changing the frequency of the power supply and thereby affecting a number of motors in one installation, or it may result from applying used or repurchased motors on new circuits. At times such as those at the outbreak of the European War, when numbers of concerns were undertaking the manufacture of explosives and all sorts of munitions, the sudden demand for motors for the operation of machine tools and other purposes greatly overtaxed the available stocks and created a brisk demand for second-hand motors wherever they could be found. The installation of these machines on new circuits necessitated a change in frequency in many cases as well as changes in phase and voltage. Another instance of a wholesale change of frequency is the retiring of an existing isolated plant for the purchase of central-station power which may differ in frequency. This may result sometimes in changing the motors in a single plant, or it may involve a plant serving a town, in which case the motors in the entire district served must be arranged for the new frequency.

The commonest changes of this kind are from 25 cycles to 60 cycles and vice versa. There is also some changing from 60 cycles to 50 and infrequently 40-cycle motors are changed to 60 or the reverse.

Checking the Speed When Operating at Higher Frequency.—The most important and immediately noticeable change in the motor when the frequency is changed, is that the motor operates at a different speed. This change in speed is directly proportional to the change in frequency. It was explained in Chapter II that the so-called synchronous speed, or the number of revolutions per minute made by the magnetic field of the stator is equal to the alternations per minute of the supply circuit divided by the

number of poles, or it is equal to the expression $\frac{Cycles \times 120}{Number\ of\ poles}$. From this it follows at once that if the cycles are changed and the poles remain the same, the revolutions per minute will change exactly as the frequency.

As an example, a four-pole motor operated on 25 cycles will have a synchronous speed (practically the no-load speed) equal to $\frac{25 \times 120}{4} = 750$ revolutions per minute. The full-load speed is usually about 3 to 5 per cent less than the synchronous speed. If now this same motor is operated on 60 cycles, the speed will be $\frac{60 \times 120}{4} = 1800$ r.p.m. This immediately brings up two serious mechanical questions: First is the mechanical design of the rotor such that it will stand this increase in speed, 240 per cent of the original value? The peripheral speed of the rotor (*i.e.*, diameter in feet $\times$ 3.14 $\times$ revolutions per minute) should not be permitted to go beyond 7500 ft. per minute without consulting the manufacturer of the machine. Second, can the belting or gearing be suitably adjusted so that the speed of the driven machine or apparatus will remain practically unchanged? If these two questions cannot be satisfactorily taken care of, it will be necessary to change the number of poles in the motor winding also, so that the speed on the new frequency and with the new number of poles will be nearly the same as the speed on the old frequency and with the original number of poles.

For example, in the case just cited, a four-pole motor operated on a 25-cycle circuit runs at about 750 revolutions per minute. The nearest combination to give this speed on 60 cycles would be to wind the motor for ten poles, and the resulting revolutions per minute would be $\frac{60 \times 120}{10} = 720$. There are, therefore, two conditions in case of a change in frequency—the first, when the number of poles remains the same and the speed changes with the cycles, and the second, when the number of poles is changed so as to keep the original speed or as nearly so as possible.

Consider first the case where the frequency is changed and the number of poles remains the same. The resulting change in the speed in this case is assumed to be proper for the motor in question, and the gears or pulleys are changed so that the driven load will operate at the same speed.

Relation between Voltage and Frequency.—The next thing that is affected by the change in frequency is the operating voltage. That is to say, if the frequency is raised, the voltage should be raised also and vice versa, if the conditions in the magnetic and electric circuits are to be kept normal. Assuming that the rotating magnetic field is to be kept at the same value and the frequency raised, this field will rotate at a faster rate and cut more conductors in a given time, which will immediately result in the generation of more voltage, or counter-electromotive force as it is called in a motor. It will be remembered that in Chapter I attention was called to the fact that one of the easiest ways of thinking of an induction motor is as an alternating-current generator generating a counter-electromotive force almost exactly equal to the line voltage on which it is operated. In the present instance, then, if raising the frequency causes the motor to generate more of this back voltage, it will be necessary to oppose it by a higher applied voltage; or, speaking simply, if the frequency is to be raised the line voltage should be raised by the same amount to keep the same magnetic conditions as existed in the original motor.

Relation between Torque, R.P.M. and Horsepower.—Suppose that the frequency is raised and the voltage is not raised. If the same magnetic field existed and rotated faster, it has been shown that an increased back voltage would be generated. However, if the line voltage is not raised, the motor does not require any increased back voltage and hence it does the only other thing it can to keep the generated voltage equal to the line voltage, and that is automatically to decrease its own magnetic field to such a point that the new field rotating at the new speed will generate the same back voltage as the old field rotating at the old speed, and this electromotive force will be nearly the same as the applied line voltage, which has been assumed to be the same on both frequencies. The result of a decrease in the magnetic field would be a decrease in torque or turning effort, and this might result in a reduced horsepower output were it not for the fact that the speed increases and tries to make up for the decrease in torque

$$Horsepower = \frac{Torque\ at\ 1\ ft.\ radius \times r.p.m.}{5252}$$

From this it follows that if the frequency was raised and the voltage left the same, the magnetic field might decrease and the torque decrease without lowering the horsepower by the same amount, since the speed increases and partly makes up for it. On most of the loads that are driven by motors, the driving effort, or pull, or torque is practically the same at all speeds. This is not true of centrifugal pumps or fans or similar apparatus, but is generally true of a great deal of industrial machinery. Since this is the case, it may be seen from the horsepower formula just given that if the torque is constant the horsepower will vary directly as the speed; that is, a higher speed will call for more horsepower and a lower speed for less horsepower. Going back to the frequency, a higher frequency means a higher speed and hence, directly, a higher horsepower, and a lower frequency means a lower speed and a lower horsepower. All these things work out automatically if the voltage and frequency are varied on the motor at the same time and by the same amount. This is for the reason that torque is the product of the magnetic field acting on the currents in the windings. To keep the heating reasonably the same, the magnetic field and the currents in the coils should be kept as nearly the same as possible.

It was shown in the foregoing that if the field is kept constant and the speed increased, the generated voltage would increase, and hence the applied voltage should be increased also. This brings about a rule which may be most easily remembered in this form: *If the frequency on a motor is changed, the voltage should be changed in the same direction and by the same amount.* If this is done and the torque against which the motor is working is constant, the magnetic field in the iron will remain constant, the currents in the windings will remain constant, the speed and the horsepower will vary directly with the change in voltage and frequency, and the heating will vary somewhat due to the variation of the iron loss with the frequency and the variation of the ventilating effect with the speed.

A concrete instance of the foregoing would be to take a 50-hp. 440-volt 60-cycle motor and operate it on 25 cycles and $\frac{25 \times 440}{60} = 183$ volts, in which case it would develop $\frac{25 \times 50}{60} =$ 20.8 hp. If 183 volts was not available, a connection of the windings should be selected which would have been equivalent to

880 volts on 60 cycles and this would be suitable for $\frac{25 \times 880}{60} =$ 366.6 volts on 25 cycles, which in many cases would operate satisfactorily on a commercial 440-volt circuit. In the case which is most commonly met with, which is changing from 25 to 60 cycles, this condition can often be taken care of by impressing twice the voltage on the motor on 60 cycles that was used on 25 cycles, such as operating a 220-volt 25-cycle motor on a 440-volt 60-cycle circuit, at about double the horsepower.

Theoretically, to follow the rule already given, the voltage on 60 cycles should be 2.4 times the value on 25 cycles, since $^{60}/_{25} = 2.4$. This would result in 2.4 times the speed and 2.4 times the horsepower. Practically, it is easier to get twice the voltage than 2.4, so the voltage is doubled and the horsepower considered as double also. In case it is not possible to get double the voltage on 60 cycles, the same result can be secured in another way. Suppose the original motor is operating on a 220-volt 25-cycle circuit and is connected series star as in Fig. 162. Suppose, also, that the available 60-cycle voltage upon which it is to run is 220. To get the effect of doubling the voltage, the motor can have its pole-phase groups connected in two-parallel star, Fig. 187, for 60 cycles and it will then be affected in the same way as it would if the windings had not been reconnected but had been operated on 440 volts 60 cycles. On 60 cycles the motor would then run 2.4 times as fast and develop about twice the horsepower.[1]

In some cases it would happen that the same horsepower was required on the new frequency and at the increased speed as on the original frequency. Hence, it would be undesirable to reconnect the motor so as to raise the voltage with the frequency, since this would result in twice the required horsepower and would mean operating the motor at all times at half-load and consequently somewhat lower efficiency and power factor than if it were fully loaded.

Considering again the horsepower formula, it can be noted that if the horsepower is to remain constant, the torque must decrease as the speed increases and vice versa. Since the torque varies as the square of the applied voltage, it is evident that approxi-

[1] See G. B. Werner, *Electric Journal*, vol. 3, p. 400; R. E. Hellmund, vol. 7, p. 680.

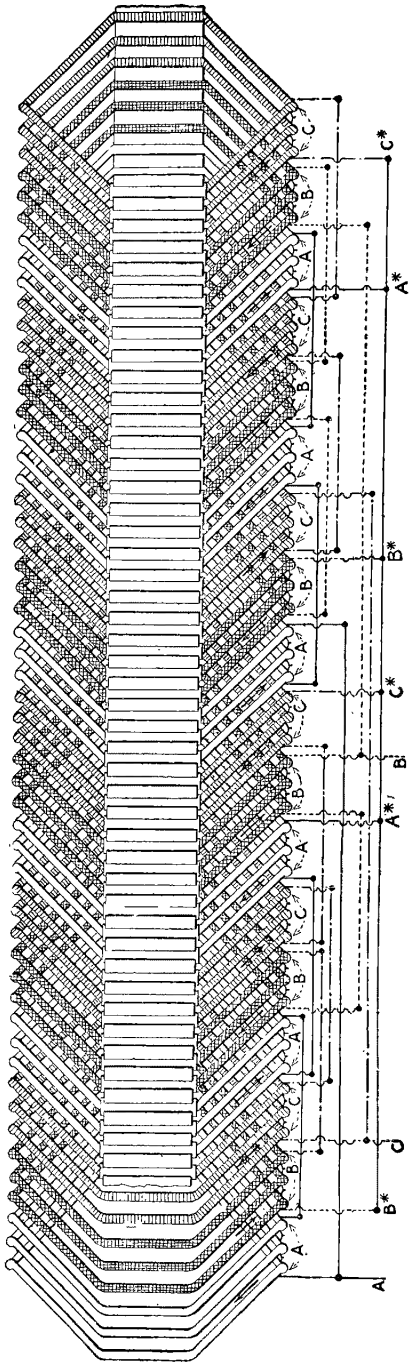


FIG. 187.—Three-phase six-pole, two-parallel star connection.

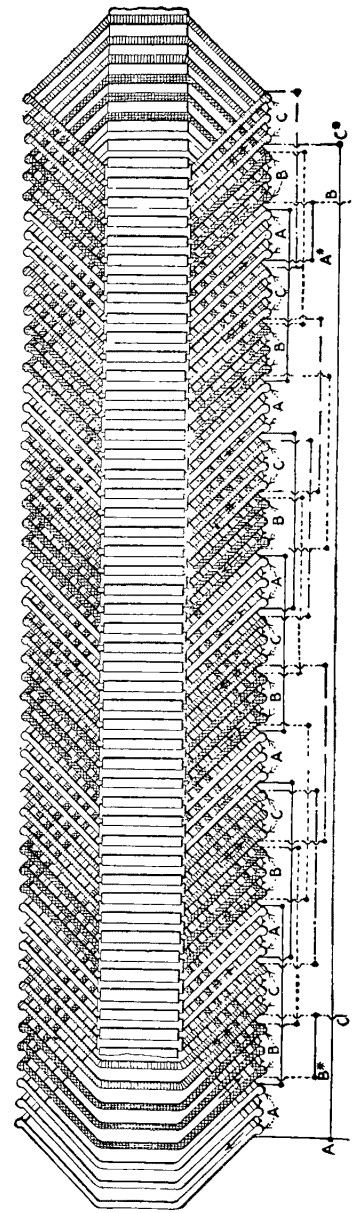


FIG. 188.—Three-phase, six-pole, series delta connection.

mately the same horsepower can be kept with a changing frequency by varying the voltage applied to the motor as the square root of the change in frequency instead of directly as the first power. An example of this would be operating a 440-volt 25-cycle motor on a 550-volt 40-cycle circuit. The square root of $^{40}/_{25} = 1.26$. Then if $1.26 \times 440 = 554$ volts be used on 40 cycles, the magnetic density in the iron will be about 80 per cent of its 25-cycle value and the torque will be $80^2/100 = 64$ per cent of the 25-cycle value. Since the speed will be $^{40}/_{25}$ of that on 25 cycles, the resulting horsepower will be $^{40}/_{25} \times {}^{64}/_{100} = 1.02$ times its 25-cycle value or practically the same.

A similar instance would be operating a 50-cycle motor on 60 cycles and at 110 per cent voltage to keep the same horsepower. Suppose in the latter instance it was not possible to juggle the generator or the transformers so as to get a 10 per cent increase in voltage. It would then be necessary to reconnect the motor so that there would be $^{100}/_{110} = 91$ per cent as many turns in series. One way of accomplishing this if the 50-cycle motor was originally connected series delta, as in Fig. 188, would be to reconnect it two-parallel star (Fig. 187) for 60 cycles and the same horsepower. This would have the effect of increasing the applied voltage $(200/1.73) - 100 =$ 15 per cent. However, since the frequency has increased 20 per cent (50 to 60 cycles) and the speed also has increased the same amount, if the voltage is increased only 15 per cent the magnetic density in the iron will be only $^{115}/_{120}$ of its 50-cycle value and the torque will be only $(^{115}/_{120})^2 \times 100 = 92$ per cent of its 50-cycle value. The resulting 60-cycle horsepower rating as compared with the 50-cycle will be $^{92}/_{100} \times {}^{120}/_{100} = 110$ per cent (since the torque is 0.92 and the speed 1.2 of its 60-cycle value). Instances could be multiplied of this, and some further examples will be given in a later chapter giving practical applications of the principles laid down here.

The fact that raising the frequency, and hence the speed also sometimes results in a horsepower rating greater than that actually required, leads at once to a word of caution regarding the converse proposition; namely, that in reducing the frequency on a motor and keeping the same number of poles, it should be figured that the horsepower will decrease exactly in proportion to the decrease in frequency and the consequent decrease in speed. The

physical conception of this is that if the frequency and voltage are varied together and the motor is working against the same torque, the magnetic density in the iron will remain the same and the current in the copper of the stator and rotor will remain substantially the same, but the horsepower will rise and fall with the voltage and frequency, since it is the product of the torque and the speed divided by a constant. If it is imagined that the voltage and the frequency are carried down to zero and the motor just comes to a standstill, it can be seen that the motor was developing full-load torque at standstill with no more than full-load current flowing in its windings.

The foregoing at once suggests a method that is sometimes used for starting a large squirrel-cage motor or a group of small motors where such motors constitute practically the only load on the generating unit from which they are operated. While the motors and the generator are at standstill, the motors are connected electrically to the generator by closing all line switches. The generator field is next excited to its normal value. The steam engine or the waterwheel is then started slowly from rest, and as the generator builds up in speed the motors come right up along with it and no more current is required in the motor windings than is represented by the torque against which they are starting. This gives the best physical picture of the voltage and frequency building up together from zero to normal value and yet the motors exerting a constant torque from standstill to normal full-load speed under these varying conditions.

The example just cited brings out the fact, also, which will be mentioned in Chapter XIII, that practically all changes in operating conditions can be considered equivalent to changes in voltage and so calculated and used. So it is with the change in frequency—if the torque is to be kept constant with the same number of poles and the horsepower is to vary with the speed, the voltage should be varied with the frequency or the winding connections changed to produce the equivalent. However, if the horsepower is to be kept constant at any and all speeds with the varying frequency, then the voltage should be varied as the square root of the change in cycles.

CHAPTER XI

THE NUMBER OF POLES AND THE R.P.M. AND THE POSSIBILITY OF VARYING THEM WITH THE SAME WINDING

The speed of an induction motor expressed in revolutions per minute = (cycles × 120) ÷ number of poles. The speed so determined is called synchronous speed and is very nearly the same as the no-load speed. When operating under full load the speed will be a few revolutions less than this—for ordinary motors, on an average of about 95 to 97 per cent of the synchronous speed. The synchronous speed is the speed at which the rotating magnetic field is traveling around in the stator, and the difference between this and the full-load speed of the rotor (3 to 5 per cent) is called the "slip" of the motor.

From the equation for revolutions per minute it can be seen at once that if the speed of the motor is to be changed, it is necessary to change either the cycles or the number of poles. Or, assuming that the cycles have been changed and that it is necessary to keep the same speed as before, it will be necessary to change the number of poles. So far as the cross connections themselves are concerned, and admitting windings where all the pole-phase groups do not have the same number of coils, as discussed in Chapter XII, it is evident that any winding might be connected for several different numbers of poles and for either two-phase or three-phase, by the simple expedient of changing the number of coils in each pole-phase group.

For example, a winding having 54 slots and 54 coils if arranged for three-phase six poles would have 3 coils per group and 18 pole-phase groups. If the same winding is rearranged for three-phase four poles there will be 12 pole-phase groups having alternately 4 and 5 coils per group. Or, if the same winding is arranged for two-phase four poles, there will be 8 pole-phase groups, 6 of which would have 7 coils and 2 of which would have 6 coils, or 54 total. There are practical limits beyond which this form of reconnection cannot properly be carried and which are

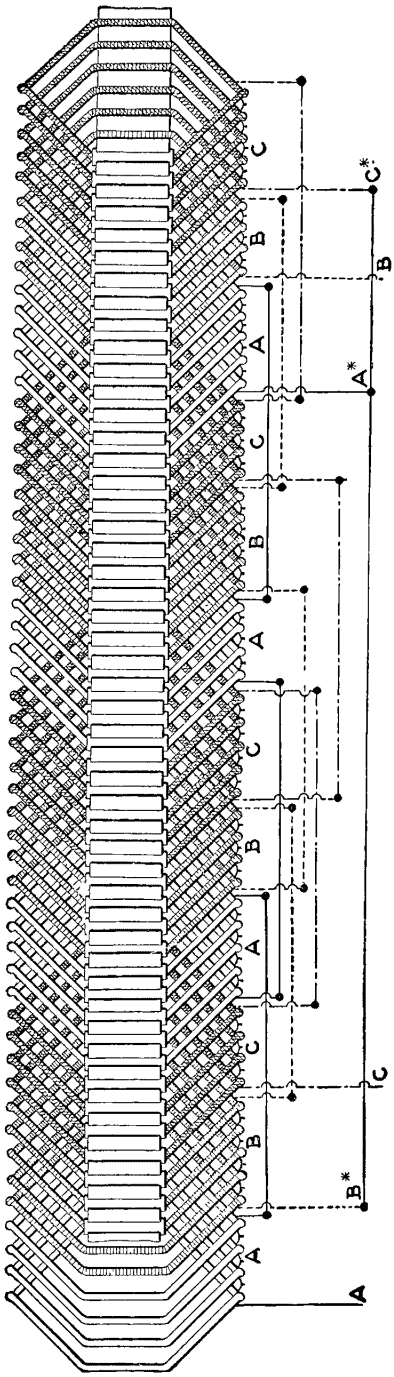


FIG. 189.—Three-phase, six-pole series star connection.

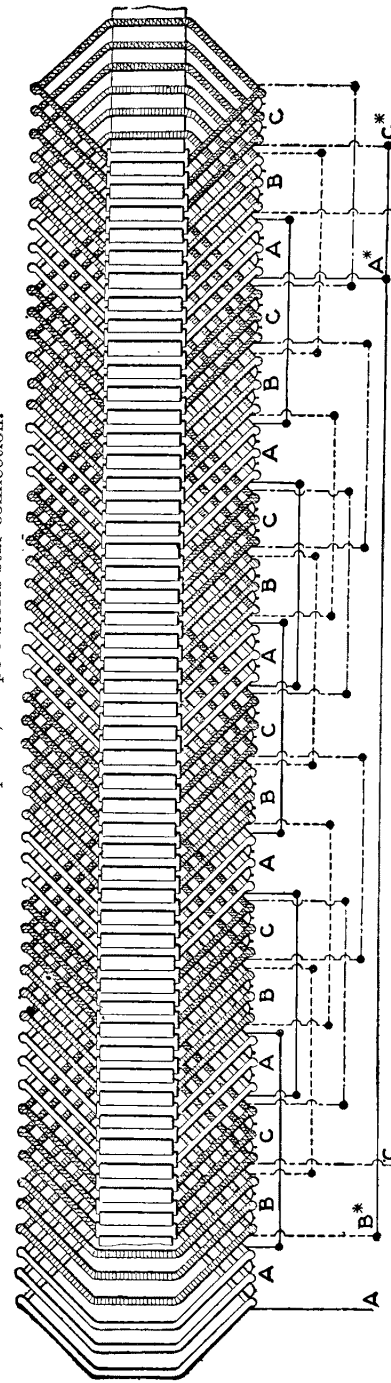


FIG. 190.—Same winding as Fig. 189 reconnected for three-phase, four-pole series star with uneven grouping.

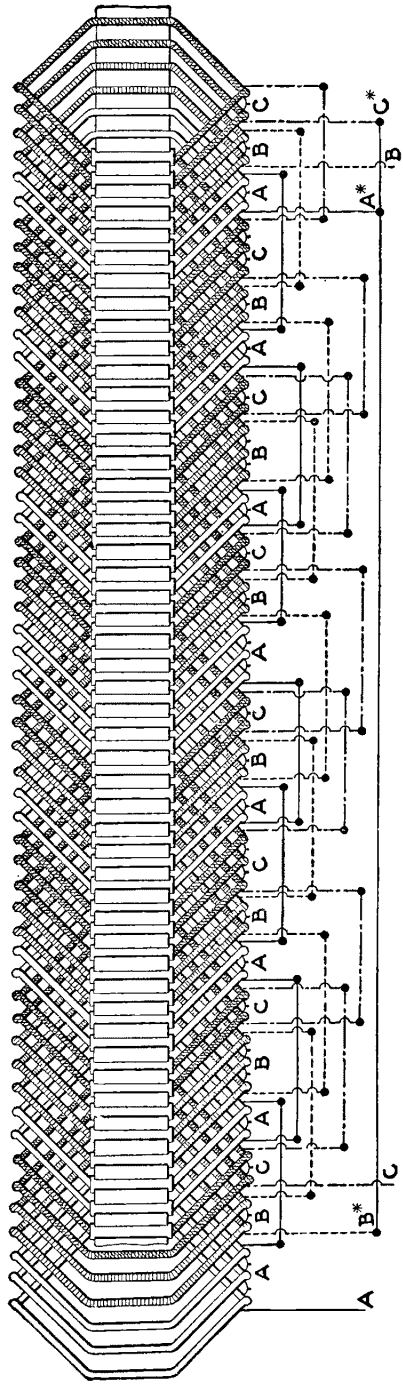


FIG. 191.—Same winding as Fig. 189 reconnected for eight poles.

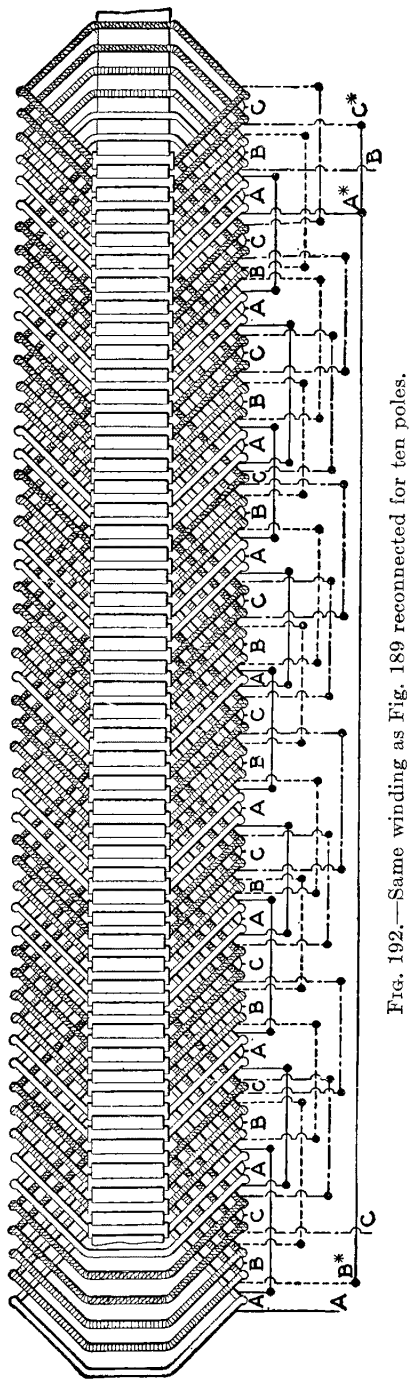


FIG. 192.—Same winding as Fig. 189 reconnected for ten poles.

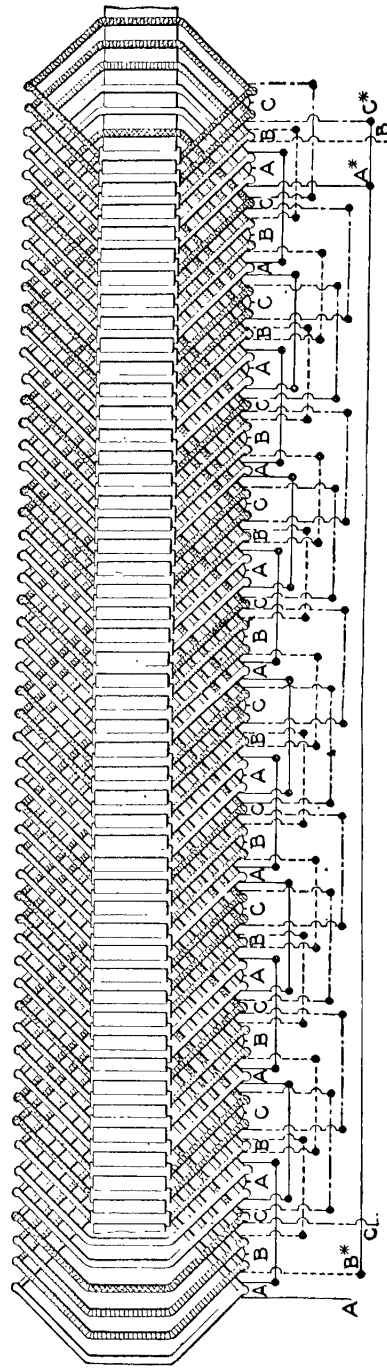


FIG. 193. Same winding as Fig. 189 reconnected for twelve poles.

discussed farther on in this chapter, but before proceeding to a discussion of them attention is called to some typical cases of reconnection of this nature.

Figure 189 shows a 54-slot winding having a coil pitch of 1 and 7 as arranged for six poles and connected series star. There are 3 coils in every group. Figure 190 shows the same winding as Fig. 189 except grouped and connected for 4 poles. It will be noted that there are now $3 \times 4 = 12$ pole-phase groups containing alternately 4 and 5 coils per group. Figure 191 shows the same winding as in Fig. 189 arranged and connected for eight poles; there are 18 pole-phase groups with 2 coils and 6 with 3, making total of 24 groups and 54 coils. Figure 192 is the same winding as Fig. 189 connected for 10 poles. There are 24 groups having 2 coils each and 6 groups with 1 coil, making a total of 30 pole-phase groups and 54 coils. Figure 193 shows the winding, Fig. 189, connected for 12 poles. There are 18 groups of 2 coils each and 18 groups of 1 coil each, making a total of 36 groups and 54 coils.

Of course, all these connections would not normally operate at the same voltage, nor would the horsepower developed be the same, and the speed would vary inversely as the number of poles. Assuming, for example, that the motor was 100-hp. 60-cycle three-phase 440-volts and run at 1160 r.p.m. on the 6-pole connection, the characteristics for the other connections are shown in Table VIII. Three-phase is assumed throughout.

TABLE VIII.—CHARACTERISTICS OF A THREE-PHASE MOTOR CONNECTED AS IN FIGS. 189 TO 193

Poles	Hp.	Voltage	R.P.M.	Connection
6	100	440	1,160	Fig. 189
4	110	484	1,750	Fig. 190
8	86	375	860	Fig. 191
10	68	300	690	Fig. 192
12	50	220	580	Fig. 193

The only commercial voltages in Table VIII are the first and last, 440 and 220. To operate the motor on the other connections would require special taps from the transformer, unless some other change could be made in the motor's winding at the same time that the number of poles was changed. For example, the

eight-pole connection requires 375 volts. If it so happened that the six-pole motor was connected in parallel star, then the eight-pole motor could be connected series delta, which would be the same thing as operating the motor on a voltage in the ratio of 1.73 to 2 or $\frac{375 \times 2}{173} = 434$, which is approximately the voltage required.

Table VIII of horsepowers and normal voltages is figured by taking account of the speed and of the chord factor in the following way:

One of the functions of the winding is to be acted upon by the rotating magnetic field and actually to generate a counter e.m.f. which is opposed to and almost equal to the applied line voltage. If, then, in reconnecting for a different number of poles, the assumption is made that the magnetic field in the teeth and air gap remains at a constant value irrespective of the connections, it is at once evident that the generated electromotive force, and consequently the applied line voltage, should vary directly as the speed of the rotating magnetic field, which is practically the same as the revolutions per minute of the motor at no load. For example, in the case cited in the foregoing, if the normal voltage on the six-pole connection is 440, everything else being equal, the normal voltage on the 12-pole connection should be 220, since the revolutions per minute of a 12-pole motor are just one-half those of a six-pole machine.

Practically, the only condition which enters to change the voltage from varying directly as the speed is the "chord factor," which is due to the throw or pitch of the coil. This is described under Fractional Pitch Windings in Chapter VI. It will be recalled that this is a factor which reduces the voltage generated in a coil because one side of a coil is not exactly under the center of a north pole when the other side is exactly under the center of a south pole. This numerical value of this factor is expressed as the sine of one-half the electrical angle which is spanned by the coil. It may appear in the example given in Figs. 189 to 193 that the chord factor should remain constant since the physical throw of the coils is unchanged. It should be carried in mind, however, that while the coil spread remains unchanged, the number of poles is changed, consequently the pole arc is changed; hence, the relation of the throw of the coil to the pole arc is

different in each case. The foregoing can be best shown by Table IX, remembering that the throw of the coils is slots 1 and 7 in all cases.

TABLE IX.—EFFECTS OF CHANGING THE NUMBER OF POLES IN AN INDUCTION-MOTOR WINDING

Number of poles	4	6	8	10	12
Throw of coil	1–7	1–7	1–7	1–7	1–7
Slots spanned by coil	6	6	6	6	6
Number of slots equivalent to 180 electrical degrees $= \frac{54}{\text{No. of poles}}$	13.5	9	6.75	5.4	4.5
Electrical degrees represented by six slots	80	120	160	200	240
Sine of half the electrical angle covered by the coil throw or pitch = chord factor	0.64	0.866	0.99	0.99	0.866

Table IX indicates that the normal six-pole voltage of 440 must be modified by two factors to find its value for other speeds. These factors and their results are combined in Table X.

On first comparison of Tables IX and X it seems peculiar that the 4-pole connection, having the lowest chord factor, which is 0.64, operates at 484 volts, which is the highest voltage, while the

TABLE X.—FACTORS, DUE TO CHANGE IN NUMBER OF POLES, MODIFYING INDUCTION-MOTOR VOLTAGE

Number of poles	4	6	8	10	12
Factor for changing voltage on account of changing speed	1.5	1	0.75	0.60	0.50
Factor for changing voltage on account of change in chord factor for new No. of poles ÷ 6-pole chord factor	0.74	1	1.14	1.14	1
Product of both factors	1.11	1	0.855	0.685	0.50
Resulting voltage = (440 × No. 4)	484	440	375	301	220

8- and 10-pole connections, having a high chord factor of 0.99, operate at 375 and 300 volts, respectively. It must be remembered that the speed at which the magnetic field is rotating comes into effect and changes the result of the chord factor. Throughout this book we have considered the induction motor as being an alternating-current generator, generating the counter-electromotive force, or back voltage. Hence, in this case, the

assumption has been made that the magnetic field in the air gap remains the same in density for all these connections, and when connected for four-pole this field will rotate twice as fast as when connected for eight-pole, and thus generate twice as much voltage. This is the reason that the two factors, one due to changing the speed of the field and the other due to changing the throw of the coil, are introduced, as shown in Table X. The product of these two factors governs the voltage which must be applied to the windings to give normal operation.

Table X determines the value of the proper voltage for the new connections as given in Table VIII. The horsepower is determined just as if it were an alternating-current generator by taking the product of the volts $\times$ amperes $\times$ 1.73 $\times$ power factor and dividing by 746. The cross section of the copper has not been changed, hence the amperes remain constant. The power factor is assumed the same, although it will be somewhat higher on high speeds and lower on low speeds. Therefore, the output in horsepower will vary as the voltage, assuming 100 hp. at 440 volts. The horsepower for the new connections is figured in this manner, as given in Table VIII. Some general observations might be made about the examples chosen in this chapter: First, the question of starting torque or maximum torque required, or the saturation of the core when connecting for higher speeds might require a voltage somewhat higher or lower than Table VIII; second, as pointed out in Chapter VI, on fractional-pitch windings it is not wise, in general, to chord up a coil so far that the chord factor is less than 0.707, which means that the coils span only halfway from the center of a north to the center of a south pole. The reason for this was shown in Chapter VI by plotting the shape of the magnetic field set up by windings having different coil pitches. For this reason the four-pole connection, as shown and discussed in this chapter, should be avoided in practice, but the 6-, 8-, 10- and 12-pole connections would be satisfactory if the proper operating voltage could be secured.

Check Points in Changing Number of Poles.—From the foregoing it may be seen that there are three factors to be taken care of in changing the number of poles. These are:

First, if the new speed is to be higher than the original speed, the peripheral speed should not be allowed to exceed 7500 to 8000 ft. This figure is the diameter of the rotor in feet $\times$ 3.14 $\times$ revolutions per minute.

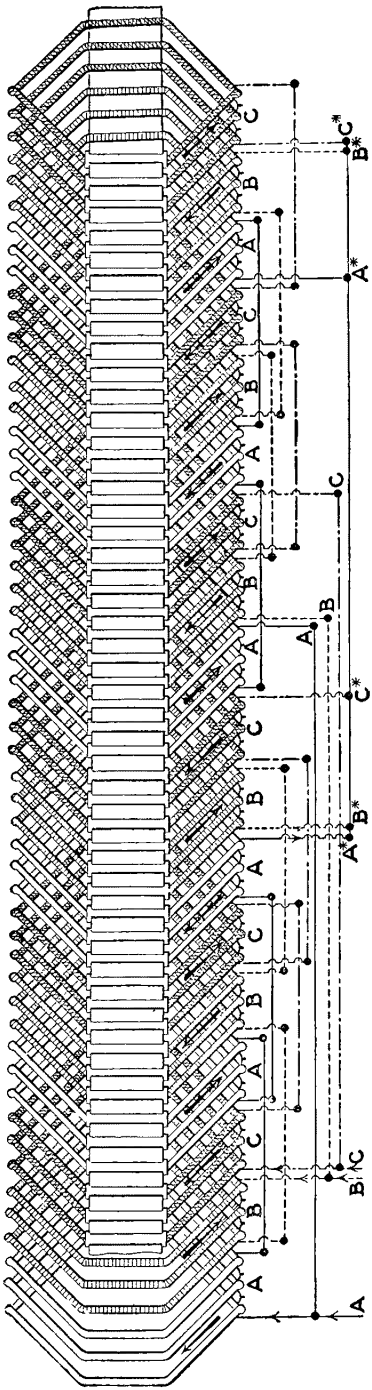


FIG. 194. Normal three-phase, six-pole, two-parallel star connection.

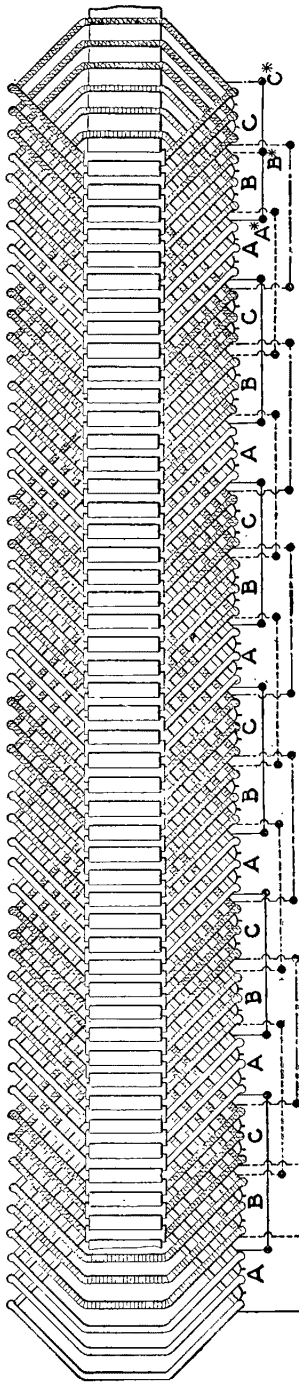


FIG. 195.—Same winding as Fig. 194 except connected for twelve poles by consequent-pole method. Note difference in position of leads between Figs. 194 and 195.

Second, the chord factor of the winding.

Third, the phase-insulation coils should be shifted so as to come at the beginning and ending of the new pole-phase groups, as discussed in Chapter VI.

Sometimes, when a winding is connected in parallel star it is possible to reconnect it in series star with consequent poles, as explained in Chapter XII, and have the motor operate at one-half its original speed. This reconnection is shown in Figs. 194 and 195. Conversely, if the motor was originally connected for series star, it might be reconnected for parallel star and operate at double speed if the motor would stand up mechanically. The counter e.m.f. generated by the consequent-pole connection is only 86.6 per cent as much as with the salient-pole connection, which means that if the motor was run on normal rated voltage on the consequent-pole connection it would operate as if it had an overvoltage of $\frac{100}{0.866} - 100 = 15$ per cent. Such a reconnection should not be attempted if the throw of the coils is exactly or nearly full pitch for the high speed. The reason for this was explained in Chapter VI.

The effect of chording the coils or making the throw less than full-pole pitch, as in Figs. 194 and 195, brings out the point that it is often possible in reconnecting a winding to raise the side of all the coils lying in the top of the slots, and to spring the coils one or two slots longer or shorter and thus help out materially on the operating conditions after the change is made. For example, in Fig. 195, if the coils are raised and wound in slots 1 and 6 instead of 1 and 7, the new chord factor would be sine one-half of $\frac{5}{4.5} \times 180$ deg. $= 200$ deg., or 0.98 instead of 0.866. The winding connected, as shown in Fig. 195, would then operate as if on 102 per cent of normal voltage instead of 115 per cent, which would have cut down the iron losses and improved the power factor.

In Chapter VI a graphical explanation was given of the effect of chord factor and reconnecting for a different number of poles. This was shown by plotting the shape of the magnetic field set up by a three-phase winding connected for different numbers of poles and whose coils had different pitches. It showed the magnetic conditions inside the motor which give rise to the practical results discussed in this chapter.

CHAPTER XII

LESS COMMON CONNECTIONS USED FOR UNSYMMETRICAL CONDITIONS OR IN AN EMERGENCY

Chapter III discussed the usual forms of connection for windings using "diamond" coils in open slots. It is the purpose of this chapter to present some of the less usual forms. These are often of more importance in reconnecting old machines than are the standard forms, because it is by their help and "judicious" use that a job is pulled through in a hurry or a temporary workable connection made that will carry on an essential part of a larger work until such a respite can be obtained as will allow a more permanent connection.

The word "judicious" is used for the reason that short-cut methods of this type are sometimes used where there is no need for them and where their use is a positive detriment, since the extra operating expense caused by them soon offsets any immediate apparent gain. Such a case, for example, would be represented by reconnecting a three-phase 440-volt series star winding for two-phase 440 volts with the same coils, making no other change. The machine would probably operate in many cases, but the increased power bill would pay the interest on a considerably larger sum than would be represented by the cost of a proper set of two-phase coils. If this point is understood and given proper consideration, it is desirable to know some of these semistandard or possible schemes, as they may be of service in an emergency.

Number of Slots Not a Multiple of Phases Times Poles.—Among these schemes one which is not usually found in textbooks, but which is perfectly legitimate and largely employed by all manufacturers, is the use of a core having a number of slots that is not an exact multiple of the number of phases times the number of poles—for example, a 90-slot core wound for three-phase, eight poles. This connection is represented by Fig. 196. The Roman numerals on each pole-phase group represent the number of coils in that group, and it will be seen that each

phase consists of 6 groups of 4 coils each and 2 groups of 3 coils each, or a total of 30 coils, and 90 coils in the complete winding. This irregularity introduces a slight displacement of the phase angle at certain places, but these places are so chosen around the machine that the net result is a perfectly balanced three-phase voltage at the terminals of the machine. E. M. Tingley originated an ingenious and simple method for arranging such

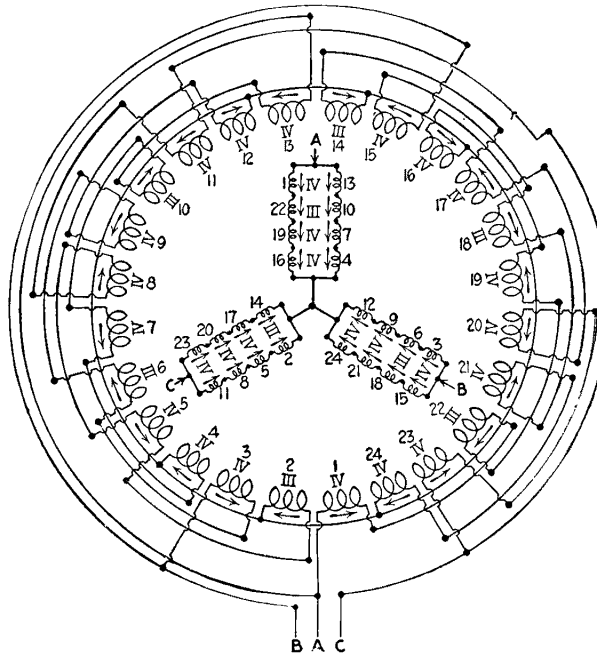


FIG. 196.—Three-phase, eight-pole, parallel-star diagram with uneven grouping for a ninety-slot stator.

windings with mathematical accuracy to give perfectly balanced voltage.[1]

It does not follow, however, that only the slot numbers recommended by Mr. Tingley can be made to give operating results. Other combinations are practically workable along the same general lines and can be laid out by inspection with reasonable regard to the best symmetry. But it is true that only the combinations pointed out by him can be made to give a theoretically perfect voltage balance at the motor terminals on all phases. This

[1] In the *Electrical Review*, vol. 46, pp. 116–118, Jan. 23, 1915.
See also Chapter XVIII, page 347.

explanation is made in reply to the question frequently asked as to whether it is essential that the number of primary slots shall be a multiple of the number of phases times the number of poles. It does not necessarily have to be such a multiple, and connections of the type shown in Fig. 196 give practically as good operating results as any other.

The manufacturers make use of this type of connection in order to use the same core for as many combinations of phase, voltage, poles, cycles and horsepower as possible, thereby greatly reducing the stock of punchings or stampings that must be carried and also the expense necessary for dies to produce these punchings.

Particular reference is made to such diagrams in this chapter to insure that no one who is contemplating a reconnection need be discouraged or give up the attempt if it is discovered that the number of pole-phase groups does not divide exactly into the number of slots. In general, if the total number of coils in the winding is right for the voltage to be used, it will be satisfactory to put as many coils in each group as can be obtained by the even division of pole-phase groups into total number of slots and then to distribute the odd coils equally among the phases and insert them mechanically in various groups to give the greatest symmetry. Of course, if there are two or more parallels in each phase, there must be the same number of coils in each parallel. For example, in the case of Fig. 196 there are three phases and eight poles; $3 \times 8 = 24$ and $90 \div 24 = 3\frac{3}{4}$; therefore there will be four coils in each group excepting in the case of six groups which will have three coils. Two of these six groups are in each of the three phases, and one of these groups is in each of the two parallel legs of each phase. If this be followed, it may not give the perfectly balanced condition of Fig. 196, but when done by a careful man, it will usually give a safe operating condition.

Consequent-pole Windings for Two Speeds.—A second expedient which may be employed to connect a given winding for twice the original number of poles is the use of what is known as a "consequent-pole" connection. This is illustrated by Figs. 197 and 198, which show the usual connections for the three-phase motor wound to give two sets of poles or two speeds in the ratio of two to one. This change is accomplished by a

single winding. In Fig. 197 the high-speed is parallel star and the low-speed series star. In Fig. 198 the high-speed is parallel star and the low-speed series delta. Either may be used at the discretion of the designer. Figure 197 usually gives better results where a constant torque is desired and gives twice the horsepower on the high-speed that it develops on the low-speed. Figure 198 gives somewhat better results where a constant horse-

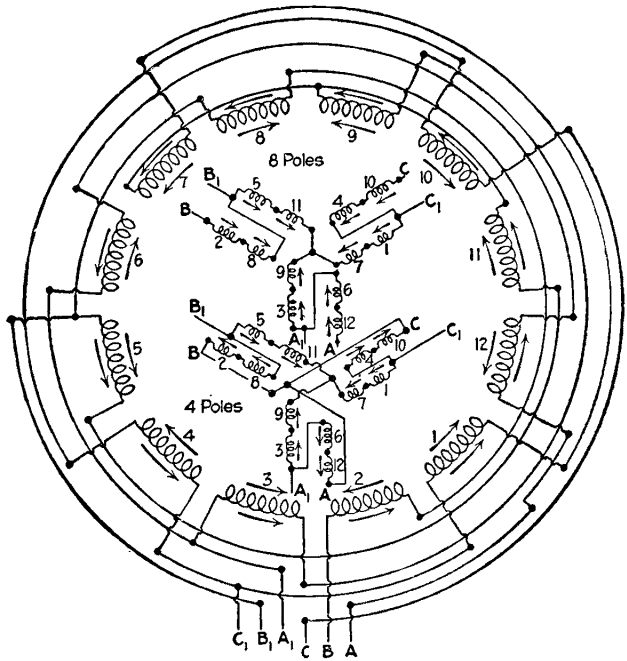


FIG. 197.—Two-speed, three-phase, four- and eight-pole parallel and series star diagram.

power is desired at both speeds, as is the case with most machine-tool applications.

Figure 199 is an explanatory diagram showing schematically how the two sets of poles are produced by such windings. Considered with Fig. 197, the inside set of arrows shows the parallel star connection where four salient poles are produced directly by the winding, two north and two south. The set of arrows outside the winding circle shows the winding connected in series star and the current direction such as to produce four north poles by the winding. Since it is not possible to have north poles alone, there immediately result four consequent south poles,

indicated by the dotted arrows, where the magnetic flux returns to the primary. This results in eight poles and half-speed. For the sake of simplicity the arrows shown are for one phase only. The three phases interact to produce the combined magnetic pole as in any normal three-phase winding. These diagrams are shown to indicate that it may be possible in some cases to reconnect motors for half-speed by making use of a diagram of

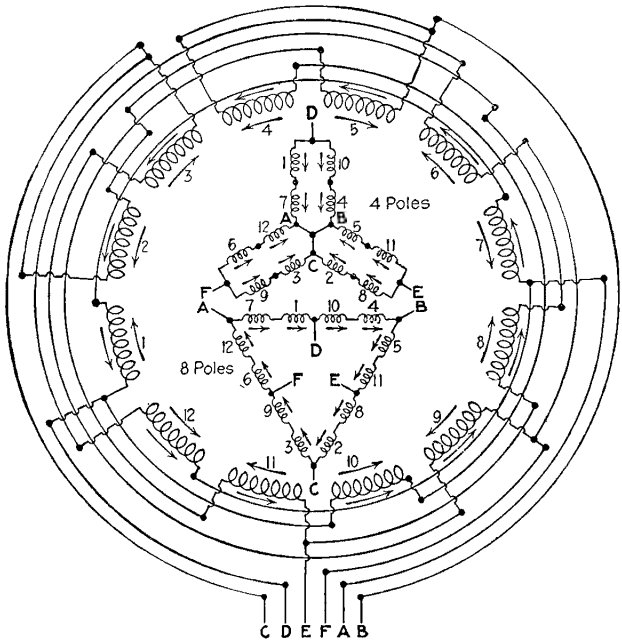


FIG. 198.—Two-speed, three-phase, four- and eight-pole parallel-star and series-delta diagram.

this nature. Such a connection, for example, makes it possible at times to reconnect a 25-cycle motor for 60 cycles and twice the number of poles, and so keep the r.p.m. of the motor nearly the same.

It will be noticed that the outside arrows on the pole-phase groups for checking the slow-speed, or eight-pole, connection in Figs. 197 and 198 all point in the same direction instead of alternately in opposite directions as the inside arrows do. This is because the eight-pole connection is "consequent-pole," or so connected that the current produces the same polarity in all the pole-phase groups, instead of alternate north and south as is

usually the case. It will be recalled that in Chapter III mention was made of the fact that in such a case the check with the alternate arrows did not hold. It will be seen from Figs. 197 and 198 that in checking windings of this type, or consequent-pole, by placing arrows on the pole-phase groups in the direction from the lead toward the star in all three phases, the arrows will all point in the same direction. This can be explained in another way by saying that in a winding of this type there are only half

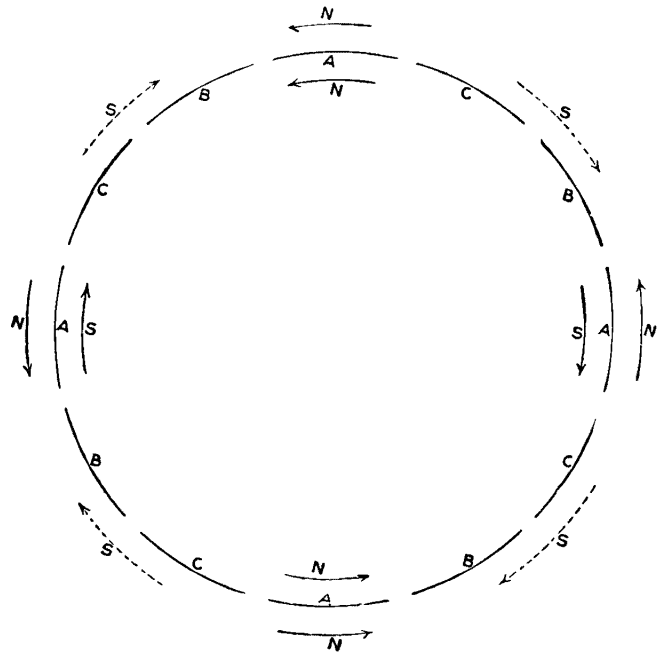


FIG. 199.—Schematic magnetic diagram explaining the eight-pole connection of Figs. 197 and 198.

as many pole-phase groups for the same total number of poles as there are in the usual form of winding. This is equivalent to saying that alternate pole-phase groups are omitted. Since in the check of the usual winding the arrows are alternately opposed, if alternate arrows are omitted the remainder will all be in the same direction, as is indicated in the check of the eight-pole connection of Figs. 197 and 198.

A diagram for a two-phase two-speed connection where the winding is in parallel on the high-speed and in series on the low-speed is shown in Fig. 200. This winding is of particular and

especial interest in that it overcomes one of the disadvantages of the corresponding three-phase connections shown in Figs. 197 and 198 by putting half of the winding in one phase for the low-speed connection and in the other phase for the high-speed connection. This is an advantage, because the so-called "winding factor," or "distribution factor," remains the same on both speeds as in a normal two-phase machine, while in the three-

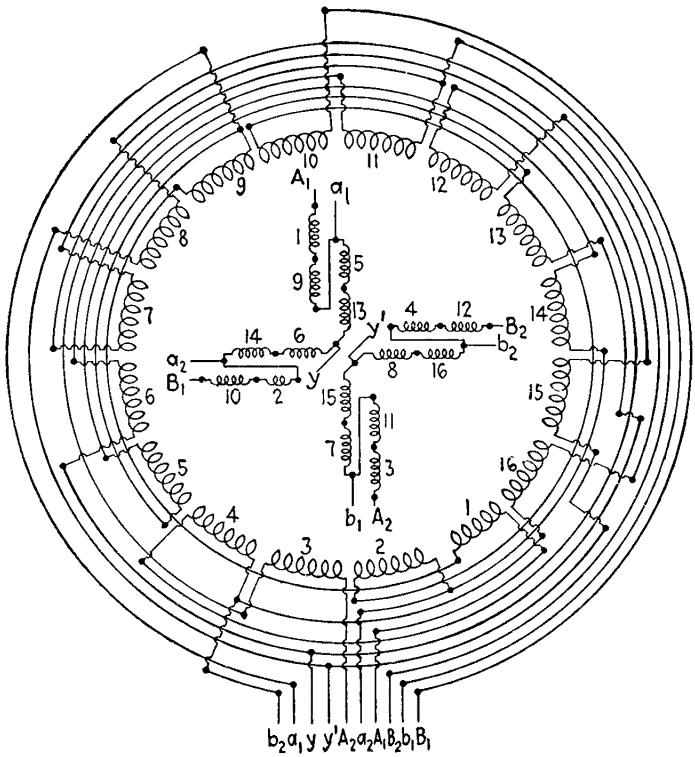


FIG. 200.—Two-speed, two-phase, four- and eight-pole, parallel and series diagram for same distribution factor on both connections.

phase connections shown in Figs. 197 and 198 the distribution factor is only 86.6 per cent as good on the low-speed connection as on the high. This is because there are only four winding groups per phase spread over the entire periphery, and yet eight poles are being produced.

Expressed in another way, the coils for one of the eight poles are spread over the usual span for a four-pole machine. Since the distribution factor is a measure of the induced voltage or

counter-electromotive force generated, and since the capacity of the motor may be measured by its current-carrying capacity multiplied by the induced voltage, it can be concluded at once that the loss of 14.3 per cent in the three-phase connection on the slow speed is avoided in the two-phase diagram, Fig. 200. In reality the gain is greater than this, for the reason that the

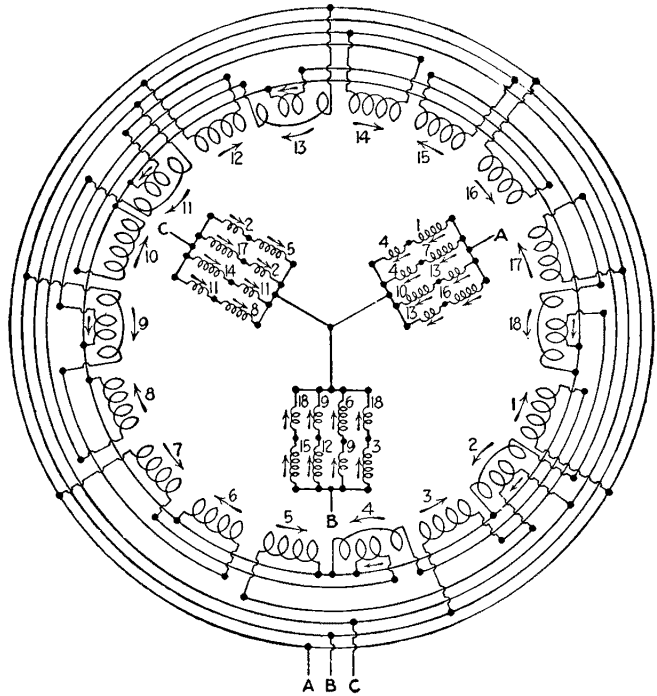


FIG. 201.—Three-phase, six-pole, series star diagram in four parallels. So-called "split group" diagram. Emergency makeshift.

two-phase distribution factor caused by consequent poles is only 70.7 per cent, as against 86.6 per cent in the three-phase.

Speaking simply, if a series-parallel two-phase connection were used, similar to the three-phase, Fig. 197, and without changing the coils from one phase to the other as does Fig. 200, the loss in horsepower on the slow speed would be approximately 30 per cent, which is certainly a matter of prime importance. It is mechanically possible to make such an arrangement on a two-phase winding, but there seems to be no practical way of accomplishing the same result on a three-phase winding.

As in the case of the three-phase two-speed diagrams, this connection shows the possibility of changing a standard motor to half-speed by the medium of such a connection.

When operating from a three-wire two-phase system or any system having the two phases interconnected in any way, all four of the leads that connect to y and y', Fig. 200, should be brought out instead of tying them together in pairs and bringing out y and y' as shown. This is in order that the phase windings may be kept clear of each other on both speed connections.

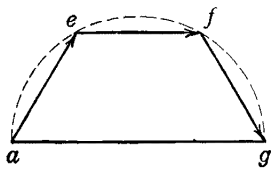


FIG. 202.

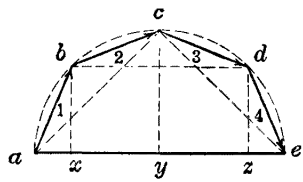


FIG. 203.

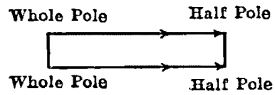


FIG. 204.

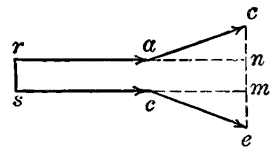


FIG. 205.

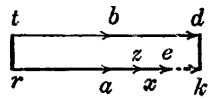


FIG. 206.

FIGS. 202, 203, 204, 205, 206.—Vector diagrams of group voltages in Fig. 201.

Splitting Groups.—Figure 201 illustrates a connection that is sometimes attempted, but usually with disastrous results. In all the foregoing diagrams the phase-pole group has been treated as a unit. That is to say, if there were four coils per pole per phase, these four were connected in series into a group and handled as a unit. Figure 201, on the other hand, breaks up some of the groups into halves. Suppose, for example, that a three-phase six-pole motor has 72 coils total and is connected in series for 440 volts and it is desired to reconnect it for 110 volts. It can be parallel for 220 volts, and there will be three pole-phase groups in each of the two parallel legs of the winding. It cannot be paralleled four times, since 6 is not divisible exactly by 4. Since there are six poles and three phases, there are 18 pole-phase

groups and $72 \div 18 = 4$ coils per group. It is therefore possible to split 6 of the 18 groups into halves of two coils each, and by putting a half-group in series with a whole group to get four parallels per phase having 1.5 pole-phase groups in each of the four parallel circuits. Such a connection is shown in Fig. 201. This is rather difficult to do properly unless there is an expert winder available, and it leaves the motor in an unsatisfactory operating condition when it has been done. This is explained by the vector diagrams in Figs. 202 to 206.

Let *ag* represent the voltage vector of one magnetic pole made by combining the three pole-phase vectors *ae*, *ef* and *fg*, Fig. 202. For clearness, one pole-phase vector *ae* is shown in Fig. 203 drawn to a larger scale and made up of the vectors of the four separate coils *ab*, *bc*, *cd* and *de*. The length of the line *ab*, for example, represents the voltage generated by the rotating field in a single coil of the winding, and four of them are considered together because there are four coils in series in any complete pole-phase group, for example, group 16 in Fig. 201. If two or more circuits, each made up of one whole pole plus one half-pole, are to be connected in parallel, the two resulting vectors should be the same length and have the same direction or phase. Such a condition is shown in Fig. 204. This is a true parallel, and there will be no circulating current around the closed loop formed by the two parallels in the winding, since two equal voltages in phase with each other are opposed.

An inspection of the four vectors of which *ae* is composed will show that it cannot readily be divided into two parts and paralleled without there being circulating current. Suppose, first, that the winding group is split in the middle at *c*, leaving $ab + bc$ for one half and $cd + de$ for the other. The two resulting vectors are *ac* and *ce*. When each of these vectors is added to another complete pole and the two connected in parallel, the result is indicated in Fig. 205, where $ra + ac$ is paralleled with $sc + ce$. Since *ac* and *ce* are not in phase, there is left a voltage equivalent to $em + nc$, which will set up current around the closed loop and produce increased heating. In order to avoid this to a certain extent, the two outside coils of the group, *ab* and *de*, are sometimes paralleled against the two inside coils, *bc* and *cd*. The two resulting vectors $ax + ze$ and *bd* are in parallel, but they are of different lengths. The results are shown in Fig. 206, where

a whole pole *tb* plus the half-pole *bd* is in parallel with *ra* + *ax* + *ze*. While these vectors are in phase, the difference in their numerical value leaves a component *ek*, which is unbalanced and which is free to cause circulating current in the closed loop of the parallel circuit.

TABLE XI.—COMPARISON OF A TWO-PHASE MOTOR CONNECTED "T" TO OPERATE ON THREE-PHASE WITH NORMAL WINDING

	Normal two-phase winding	Three-phase "T" connection	Normal three-phase winding
Full-load efficiency	88.0	86.9	88.5
Full-load power factor	89.0	84.8	90.0
Starting torque	1.75	1.20	1.94
Maximum torque	3.3	3.17	3.3
Deg. C. rise at full load:			
Stator copper	22.5	32.0	21.0
Stator iron	20.0	32.5	19.0
Rotor copper	22.0	30.0	22.0

In addition to the difficulty of making this connection properly and the fact that there is at all times some circulating current, there is also likely to be trouble in keeping the phases insulated from each other. All things considered, this is an expedient which had better be left untried except in cases of emergency. For all ordinary operating conditions much better results will be secured by replacing the old coils in the machine by new coils wound for the proper voltage.

Table XI shows comparative performances of a two-phase motor reconnected for operation on three-phase by a "T" connection and the performance of the same motor when supplied with new three-phase coils and connected in a normal three-phase manner.

Figure 176 shows a possible three-phase "T" connection which may be made from a two-phase winding by a method similar to the Scott-transformer connection. The effect of this connection upon the performance is shown in the table and was discussed in Chapter IX under Changes in Phases. It is a connection that should be used only as a temporary expedient until better arrangements can be made. It is possible to devise other makeshifts, but they are usually attended with so great a sacri-

fice in the heating and efficiency of the motor, that it is safer to leave them untried. It happens that a connection that looks feasible from the standpoint only of the number of coils in series, falls down on trial because these coils are not strictly in phase. Experiments of this nature are better left to the electrical manufacturing establishments.

CHAPTER XIII

RECONNECTING AN OLD WINDING FOR NEW CONDITIONS

General Fundamental Considerations.—An electric motor is a device for transforming energy in the form of an electric current into mechanical energy in the form of turning effort, or rotating force. This turning effort, or driving force, is called "torque" and is measured in the pounds pull that a motor would develop at the rim of a pulley one foot radius. This torque is produced by the force exerted by a current flowing through a conductor located in a magnetic field. From this it is evident that the capacity of a motor to produce torque is limited both by the capacity of the copper circuit to carry current and the iron circuit to carry magnetic lines of force.

The amount of current, or flux, that is being carried by a given cross section of copper or iron determines the heating of the motor. It may be assumed that in a normal motor operating under the conditions for which it was designed, there is a reasonable current flowing in the copper and a reasonable flux in the iron, which the designer believes will give the most satisfactory operating results. Therefore, if changes are to be made in the speed, phase, frequency and voltage at which the machine is to operate, the winding must be reconnected so as to have approximately the same number of magnetic lines per unit cross section of iron and the same current density in the copper that existed before the change was made in the motor. This statement is true over a wide range of conditions, and would be true universally if it were not for the fact that the high-speed machine will generally run cooler than the same machine operated at low speed with the same current density in the copper and number of magnetic lines per unit cross section of iron, because of the larger amount of air that the high-speed machine will force through its parts. For this reason it is generally true that the capacity of a motor may increase in the same proportion as the speed when the speed is being increased, but may decrease

somewhat faster when the speed is being reduced. As a concrete example of this, it may be stated that a 75-hp. motor operating at 450 r.p.m. may be made to develop 150 hp. at 900 r.p.m., assuming that the mechanical design will stand the stresses due to the increased speed; but, conversely, a motor originally designed for 150 hp. at 900 r.p.m., when cut down to 450 r.p.m., might not be able to develop more than 65 hp., on account of reduced ventilation.

There are certain fundamental mechanical relations that govern all motors whether alternating or direct current. The idea given in the foregoing of the reaction of the electric current upon a magnetic field concerns the production of a mechanical pull tending to rotate the movable member of the motor. This pull is usually expressed in pounds at one foot radius. This in turn is expressed in horsepower when multiplied by r.p.m. and by 2π and divided by 33,000, and may be expressed by the equation:

$$Hp. = \frac{Torque \times r.p.m. \times 2\pi}{33{,}000} = \frac{Torque \times r.p.m.}{5252}$$

from which

$$Torque = \frac{hp. \times 5252}{r.p.m.}$$

Since the current in the copper and the flux in the iron are to be held approximately constant whatever change may be made in the motor winding, it follows that the torque will be kept constant and the horsepower will vary with the speed. In other words, if the copper and iron are carrying the same current and flux at all times, twice the horsepower will be developed at twice the speed or approximately one-half the horsepower at one-half the speed.

It is essential, in getting a clear conception of the motor, either for purposes of making changes or for other reasons, that a plain distinction be made between torque and horsepower. It is the function of a motor to produce torque, or turning effort. It is incidental that when the same force is allowed to rotate at one speed or another, a different horsepower is produced. For this reason it is incorrect to say of a motor "It required 20 hp. to start the load," because, when starting, the motor was generally at a standstill; therefore there was no rotation and hence no

horsepower. The motor, however, was taking current and developing torque, and the correct expression would be the current taken at start was equivalent to the current taken by the motor when developing 20 hp. at full speed.

It is often possible to reconnect a motor and adapt it to new conditions leaving it entirely normal, and the performance in all essential respects remains the same as before reconnection. Such changes, for example, are represented by connecting the polar groups of a winding in series for 440 volts and in parallel for 220 volts. These are classified as strictly legitimate changes.

A second class of changes leaves the performance in some respects unchanged and alters it in others. These may be represented by operating a motor in star on 440 volts and in delta on 220 volts. In this change there is little change in efficiency or power factor; the starting and maximum torques on 220 volts, however, are only 75 per cent of their value on 440 volts. In such a case the advisability of the change depends entirely on the work that the motor is doing. If the torques at their altered values are sufficient to start and carry the driven load easily, there is no objection to operating the motor indefinitely as so reconnected, since the motor will not run any warmer than before and its efficiency and power factor may be better. Such changes may be classified as possible changes.

A third class of changes leaves a motor operative in the sense of producing torque enough to do the work required, but so alters its performance as to heating, or efficiency, or power factor, or insulation, that it is undesirable to leave the motor operating indefinitely in such a condition. Such changes might be exemplified by taking a three-phase motor and reconnecting the coils as they stand for two-phase. This is equivalent to operating the three-phase motor at 125 per cent of normal voltage, and in addition, the coils which should have extra insulation where the phases change, have only group insulation. The iron loss and heating may be increased to a dangerous degree and the power factor greatly decreased. Such changes should be used only in an emergency and the proper permanent changes made at as early a date as possible. These changes should be classified as makeshift or undesirable changes.

The main principles which operate to fix the limits of the different combinations, such as series, parallel, series star, parallel

star, series delta, parallel delta, etc., possible with a single winding, may be enumerated somewhat in the following manner:

1. The mechanical output of a motor is limited by the cross section of copper available to carry current and by the cross section of iron available to carry magnetic flux.

2. An induction motor is also at all times an alternating-current generator as well, and the voltage generated by its own rotating field cutting the conductors of its own stator coils must at all times very closely approximate the applied line voltage.

3. It is necessary that the pitch or throw of the coils bear some reasonable physical relation to the number of poles that the machine has. For example, in a four-pole motor the coils should throw somewhere near one-fourth of the circumference of the stator bore, in a six-pole motor somewhere near one-sixth the circumference, and so on. The practical limits to the throw are from one-half to one and one-half times this full-pitch value. That is to say, in a 72-slot six-pole motor the full or exact pitch for the coil throw would be $72/6 = 12$ slots, or the coil would be in slots 1 and 13. Using the limits one-half to one and one-half as given, the throw of the coil should be not less than 6 slots nor more than 18 slots for possible operation; that is, the coils should not spread less than slots 1 and 7 nor more than slots 1 and 19.

4. All changes in operating conditions whether of horsepower, voltage, phases, frequency or poles, may be reduced to terms of change in voltage and so considered.

5. An induction motor is similar to a transformer in that the number of turns in series in the winding must be varied in the same direction and by the same percentage as any change in the voltage applied. In addition to these principles the following practical considerations must be remembered:

a. The new voltage which is applied to a reconnected motor must not exceed the limiting value of the insulation which is on the coils. For example, 2200 volts should not be applied to a 550-volt winding even though it has been reconnected with four times as many turns in series.

b. In reconnecting for higher speeds the peripheral speed of the rotor must be kept down to a safe value so that the centrifugal force does not damage the rotor core or winding mechanically.

c. In a wound-rotor motor the rotor winding must be connected for the same number of poles as the stator winding.

d. In a squirrel-cage motor if radical changes are made in the number of poles, a change may also be required in the short-circuiting rings of the squirrel-cage rotor winding in order to keep the proper starting torque.

e. In a polar-group winding the individual coils at the beginning and end of the phase groups have usually heavier insulation than the inside coils of the groups. Where this is the case, when reconnecting for change in phase or poles the coils with the heavier insulation should be shifted to their proper new places in the winding.

These principles have been thoroughly covered in preceding chapters, but in recapitulation some additional comments may be made bearing on the practical application.

1. Cross Section of Copper and Iron.—From the existing connection of the winding in the machine which is to be reconnected, it is a simple matter to check the current flowing in the turns of the coils which are in series. This is done by checking the connection of the winding; that is, whether it is series or parallel, star or delta, etc. From this fact and the rated current of the machine can be derived directly the current in the coils themselves. For example, a three-phase machine has a normal rating of 50 amperes per terminal and is connected two-parallel delta, Fig. 207. The current in the individual coils themselves is $\frac{50}{2 \times 1.73} = 14.5$ amperes, as shown. Then the load which is put on the motor after reconnection should not be greater than that which would cause 14.5 amperes to flow in the coils themselves. Under the new connection the polar groups might be five-parallel star, as in Fig. 208, in which case the new current per lead would be 5×14.5 amperes $= 72.5$ amperes, but the current in the individual coils would still be 14.5 amperes as indicated.

If the new connection is for a greater number of poles and hence a slower speed, it would be well not to put quite so much current through as originally on account of the reduction in ventilation. Regarding the cross section of iron, this remains constant so far as the teeth are concerned, but in the core back of the slots this changes with the number of poles. This is illustrated by Fig

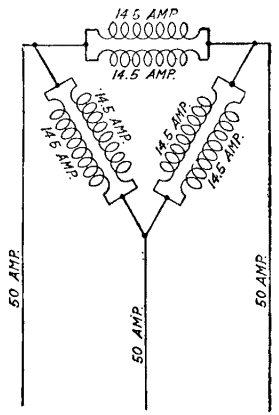


FIG. 207.

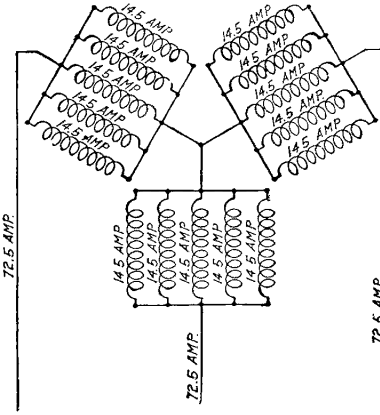


FIG. 208.

FIGS. 207 and 208.—Checking current-carrying capacity of a winding to be reconnected.

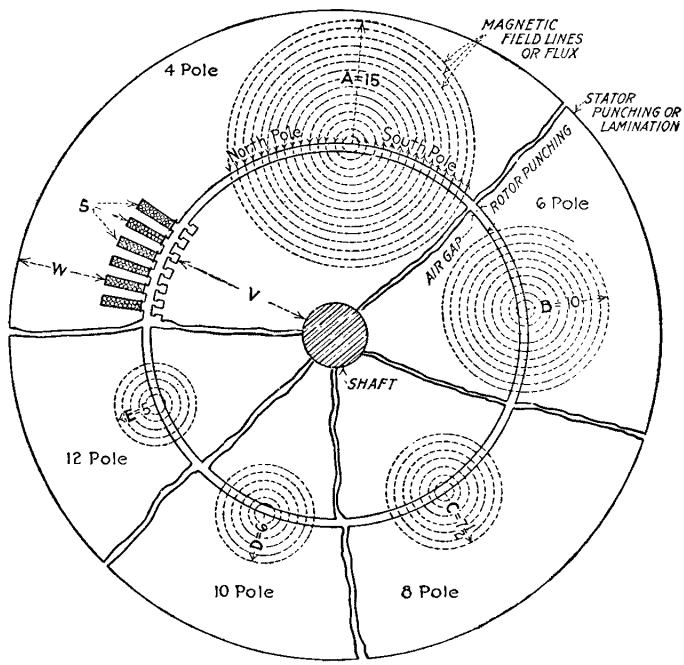


FIG. 209.—Cross section of stators wound for four, six, eight, ten and twelve poles showing radial depth of iron behind slots required for the magnetic field.

209, which shows a cross section of a motor indicating the magnetic conditions in the iron when the motor is connected for 4, 6, 8, 10 or 12 poles.

Considering the four-pole sector, the coils in the stator slots *S* set up a magnetic field represented by the 15 concentric circles causing a north pole where they leave the stator and a south pole where they reenter the stator, as indicated by the arrowheads. The proximity of these 15 circles at the air gap indicates the density of the magnetic field at this location. It will be noted that all 15 of these circles must pass through the core back of the slots or through a cross section represented by the dimension *W*.

If, now, consideration is given to the sector marked six-pole, it will be noticed that the magnetic density in the air gap as indicated by the proximity of the concentric circles is the same as in the case of four poles, but the iron back of the slots now has to carry only 10 circles and hence has only $^{10}/_{15}$ the magnetic density as in the case of four poles. There is still the same total flux in the machine, since $4 \times 15 = 60 = 6 \times 10$, and this explains why the air-gap density stays the same, but this total flux is now separated into six magnetic circuits instead of four and hence the iron in the core back of the slots is not worked nearly so hard.

Similarly, in the case of eight poles there are only $7\frac{1}{2}$ circles, since $8 \times 7\frac{1}{2} = 60$; in the case of 10 poles there are six circles and in the case of 12 poles there are five circles, since $10 \times 6 = 60$ and $12 \times 5 = 60$. In other words, there is the same total flux in the machine for all these connections and the same magnetic density in the air gap, but the core iron back of the slots works at a higher density the smaller the number of poles and at a lower density the larger the number of poles for which the winding is connected. The obvious precaution to be drawn from this consideration is that when reconnecting a winding for a smaller number of poles some check should be made to insure that the magnetic density in the core does not exceed a safe value. Reference will be made to this in Chapter XV on estimating a new winding for an old core.

2. Generator Action of the Winding.—This has been referred to several times and will not be elaborated here beyond calling attention to the fact that the rotating magnetic field will always assume such a value that as it cuts the stator coils it will generate

in them a voltage practically equivalent to the applied line voltage. Since both the number of turns in series in the coils and the magnetic density in the iron may be varied, there are evidently several combinations that would generate the line voltage, some having more turns and less field and some having less turns and more field. The practical difference between these combinations would be that the fewer the turns and the stronger the

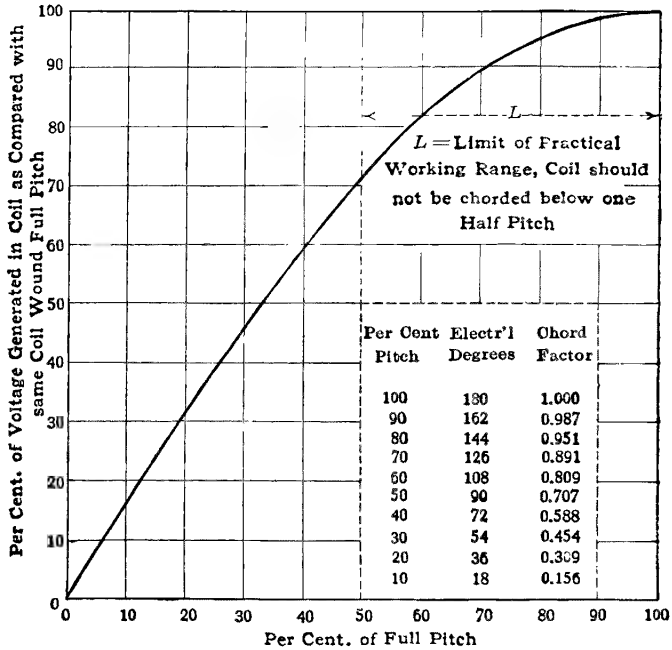

Per Cent. of Voltage Generated in Coil as Compared with same Coil Wound Full Pitch
Per Cent. of Full Pitch
L
L = Limit of Practical Working Range, Coil should not be chorded below one Half Pitch
Per Cent Pitch | Electr'l Degrees | Chord Factor
100 180 1.000
90 162 0.987
80 144 0.951
70 126 0.891
60 108 0.809
50 90 0.707
40 72 0.588
30 54 0.454
20 36 0.309
10 18 0.156

FIG. 210.—Curve showing the variation of the "chord factor" with the throw of the coil.

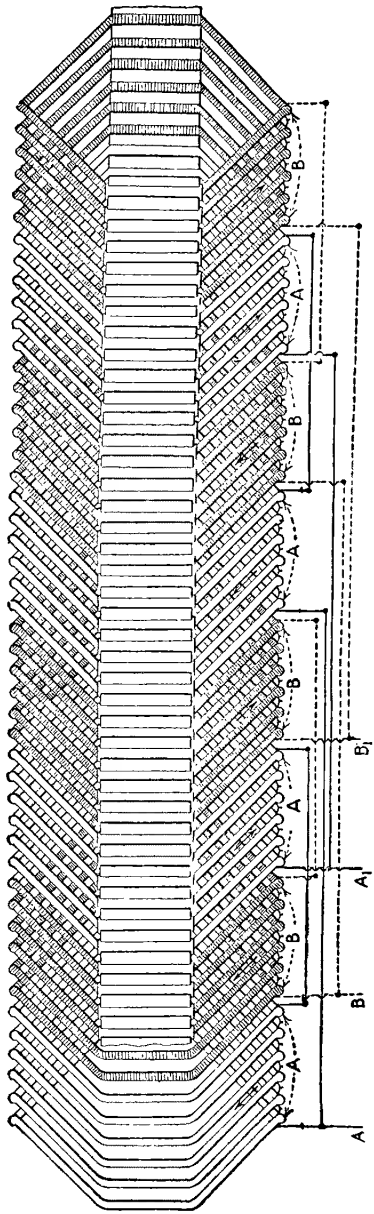
field the greater would be the maximum torque, this being limited by the saturation of the iron in the core. A little thought brings out the fact that this is equivalent to raising and lowering the voltage on a fixed winding. The higher the voltage the greater will be the magnetic field and the greater the torque. This consideration of the generated voltage or counter e.m.f. or back e.m.f. is one of the simplest checks on the number of turns required in a winding.

3. Changing the Throw.—The effect of changing the throw has been thoroughly covered, and only the effect on the applied voltage will be shown here as a curve, Fig. 210. In this figure full

pitch is called 100 per cent. For example, in a 72-slot six-pole motor the winding would be 100 per cent pitch if the coils lay in slots 1 and 13, it would be 66.66 per cent pitch if the coils lay in slots 1 and 9 or 50 per cent if they lay in slots 1 and 7. The curve indicates how the voltage applied to a coil or a winding should be reduced as the coil is chorded up if the same magnetic conditions are to be kept, or the reciprocal of the curve values indicates how the density of the magnetic field will increase if the voltage is held constant while the throw of the coils is decreased.

4. All Changes Can Be Handled as Voltage Changes.—The statement is here made that any change in the operating characteristics of a motor may be reduced to terms of a voltage change and that if the corresponding voltage be applied the operation under the new conditions will approximate the normal operating conditions under the original conditions. Since there are five main operating characteristics—namely, volts, phase, poles, cycles and horsepower—a brief résumé is in order stating how each one of these may be considered as a voltage change. In other words if, for example, the horsepower or phase of a motor is to be arbitrarily changed, what will be the new operating voltage to secure this result? Taking these characteristics in order, a voltage change is self-evident since everything is to be reduced to voltage. In the case of a phase change, two to three or vice versa, the voltage on a three-phase connection should be five-fourths that on the corresponding two-phase connection. For example, if a two-phase motor is connected for three-phase and everything else left the same, the three-phase connection should be operated at five-fourths the rated voltage of the two-phase, or a two-phase 440-volt motor when reconnected for three-phase becomes a 550-volt motor, etc. In Fig. 211 is shown a 48-coil winding grouped for two-parallels two-phase four-poles; if this winding will operate on 220 volts two-phase, it will also operate on 550 volts three-phase when grouped four-pole series star, as in Fig. 212.

In the case of a change in the number of poles, if the voltage be changed in the same direction and by the same amount as the change in speed, the torque will remain essentially constant and the horsepower will vary with the speed, being greater at higher speed and less at lower speed in exact proportion. How-


A
B
A
B
A
B

ever, if for reasons explained in connection with Fig. 209, there is not enough iron back of the slots to permit of keeping the same total flux and dividing it into fewer circuits with greater flux per circuit, the voltage may be kept constant and the horsepower will remain practically constant. The latter condition would mean that there is less total magnetic flux and less torque at higher speeds and greater total flux and greater torque at lower speeds, as must necessarily be expected since the horsepower is constant and horsepower = torque $\times$ r.p.m. $\div$ 5252.

A varying frequency can be readily reduced to a corresponding voltage change by remembering that a change in frequency without any other change would result in a change in speed and, since the basic idea of this method is that the motor is also an alternating-current generator, generating the applied voltage, it is evident that with an increased speed the generated voltage will be increased and with decreased speed the generated voltage will be decreased. Hence, it follows directly that when the frequency or cycles of the supply circuit are changed the voltage should be changed by the same amount and in the same direction. For example, if a 60-cycle motor is run on 50 cycles it should have applied only five-sixths the voltage if the same magnetic condition is to be kept, and consequently the horsepower will be only five-sixths of the 60-cycle value. Viewed mechanically, the torque remains the same on 50 cycles, but the speed is only five-sixths as great, hence there is only five-sixths the horsepower, which checks the electrical result.

There remains only a change in horsepower to be converted into a voltage change, and this is apparent from the fact that in any motor the horsepower is proportional to the product of the voltage and amperes. Since the cross section of the copper conductor remains the same and hence the amperes remain the same, the only thing that can vary is the voltage, and it follows directly that to get more horsepower out of a motor requires the application of a higher voltage and less horsepower will permit the use of a lower voltage.

From these considerations it appears that the effect of a change in any of the characteristics of the motor can be balanced by the proper change in the voltage. This statement at once arouses the comment that while it might be found that 273 volts or 346 volts or something of the kind was proper to give normal opera-

TABLE VII.—COMPARISON OF MOTOR VOLTAGES WITH VARIOUS CONNECTIONS AND PHASES

If a motor connected originally, as shown in any horizontal column, had a normal voltage of 100, its voltage when reconnected, as indicated in any vertical column, is shown at the intersection of these two columns.

Form of connection	Three-phase series star	Three-phase 2 parallel star	Three-phase 3 parallel star	Three-phase 4 parallel star	Three-phase 5 parallel star	Three-phase 6 parallel star	Three-phase series delta	Three-phase 2 parallel delta	Three-phase 3 parallel delta	Three-phase 4 parallel delta	Three-phase 5 parallel delta	Three-phase 6 parallel delta	Two-phase series	Two-phase 2 parallels	Two-phase 3 parallels	Two-phase 4 parallels	Two-phase 5 parallels	Two-phase 6 parallels
Three-phase series star	100	50	33	25	20	17	58	29	19	15	12	10	81	41	27	20	16	14
Three-phase 2 parallel star	200	100	67	50	40	33	116	58	39	29	23	19	162	81	54	40	32	27
Three-phase 3 parallel star	300	150	100	75	60	50	173	87	58	43	35	29	243	122	81	60	48	41
Three-phase 4 parallel star	400	200	133	100	80	67	232	116	77	58	46	39	324	163	108	80	64	54
Three-phase 5 parallel star	500	250	167	125	100	83	289	144	96	72	58	48	405	203	135	100	80	68
Three-phase 6 parallel star	600	300	200	150	120	100	346	173	115	87	69	58	486	243	162	120	96	81
Three-phase series delta	173	86	58	43	35	29	100	50	33	25	20	17	140	70	47	35	28	23
Three-phase 2 parallel delta	346	173	115	87	69	58	200	100	67	50	40	33	280	140	94	70	56	47
Three-phase 3 parallel delta	519	259	173	130	104	87	300	150	100	75	60	50	420	210	141	105	84	70
Three-phase 4 parallel delta	692	346	231	173	138	115	400	200	133	100	80	67	560	280	188	140	1 1	93
Three-phase 5 parallel delta	865	433	288	216	173	144	500	250	167	125	100	83	700	350	233	175	140	117
Three-phase 6 parallel delta	1,038	519	346	260	208	173	600	300	200	150	120	100	840	420	280	210	168	140
Two-phase series	125	63	42	31	25	21	72	37	24	18	15	12	100	50	33	25	20	17
Two-phase 2 parallels	250	125	84	63	50	42	144	73	49	37	29	24	200	100	67	50	40	33
Two-phase 3 parallels	375	188	125	94	75	63	216	111	73	55	44	37	300	150	100	75	60	50
Two-phase 4 parallels	500	250	167	125	100	84	288	148	97	73	58	49	400	200	133	100	80	67
Two-phase 5 parallels	625	313	208	156	125	105	360	165	122	91	73	61	500	250	167	125	100	84
Two-phase 6 parallels	750	375	250	188	150	125	433	217	144	108	87	72	600	300	200	150	120	100

tion on a motor under changed conditions of phase or poles or what not, still such information would be of little use since there are no commercial circuits having such voltage values. The answer to this is that the number of turns in the winding or the connection of the groups may be changed so as to increase the total number of turns in series by the amount that the voltage should be decreased; and vice versa, it may be possible to decrease the total number of turns per phase in series by the amount that the voltage should be increased.

Consideration of a simple case under each of the five characteristics of horsepower, poles, cycles, phase and voltage will bring out the manner of applying the "voltage method" to any and all changes in the motor-operating conditions.

1. Change in Voltage.—A motor is connected series star for three-phase 440 volts, as in Fig. 213. How should it be connected for 220 volts? [For convenience Table VII is reproduced (p. 224).] Looking at the table and following the horizontal line "Three-phase series star," there appears under vertical heading "Three-phase series star," also, the figures "100." That is to say, the motor as it stands on 440 volts is considered 100 per cent. The new voltage is to be 220, which is 50 per cent of 440. Hence, the same horizontal line in the table, namely, "Three-phase series star," is followed along until the desired figure of 50 is found, which is under the vertical heading "Three-phase 2-parallel star." This is the correct answer: that is, if a motor is connected three-phase series star for operation on 440 volts, it must be connected three-phase 2-parallel star, as in Fig. 214, to operate correctly on 220 volts.

2. Change in Phase.—Refer again to the table and assume that a three-phase 440-volt motor is to be reconnected for two-phase 440 volts. Inspection shows that the winding as it stands on 440 volts is four-pole three-phase series delta, as in Fig. 215. Select the horizontal column in the table marked "Three-phase series delta" and follow it across, looking for a vertical column showing the value "100," since the desired two-phase voltage is the same as the present three-phase voltage, or 100 per cent. Inspection shows that there is no "100" under any two-phase connection. This indicates at once that a three-phase series-delta connected motor which is normally operated on 440 volts cannot be changed and operated on two-phase

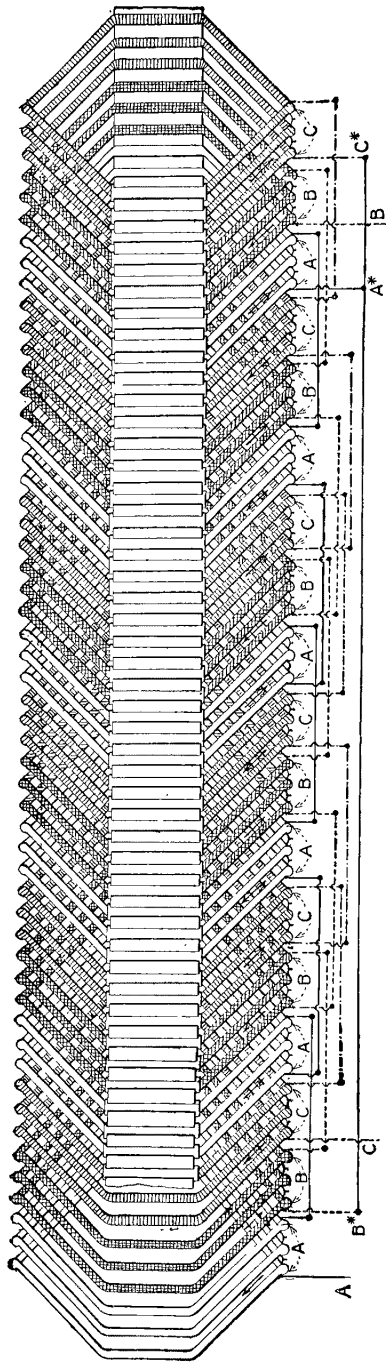


FIG. 213.—Three-phase, six-pole, series star connection.

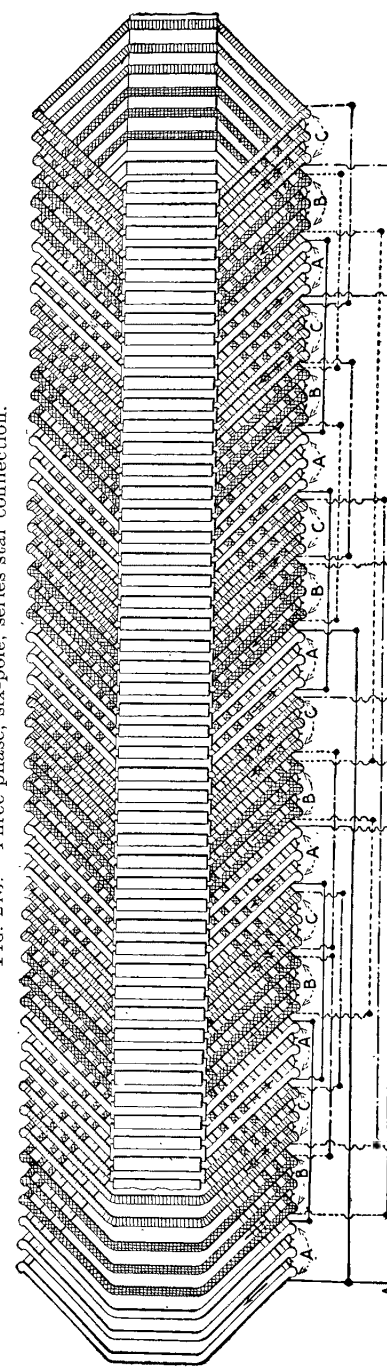


FIG. 214.—Three-phase, six-pole, two-parallel star connection.

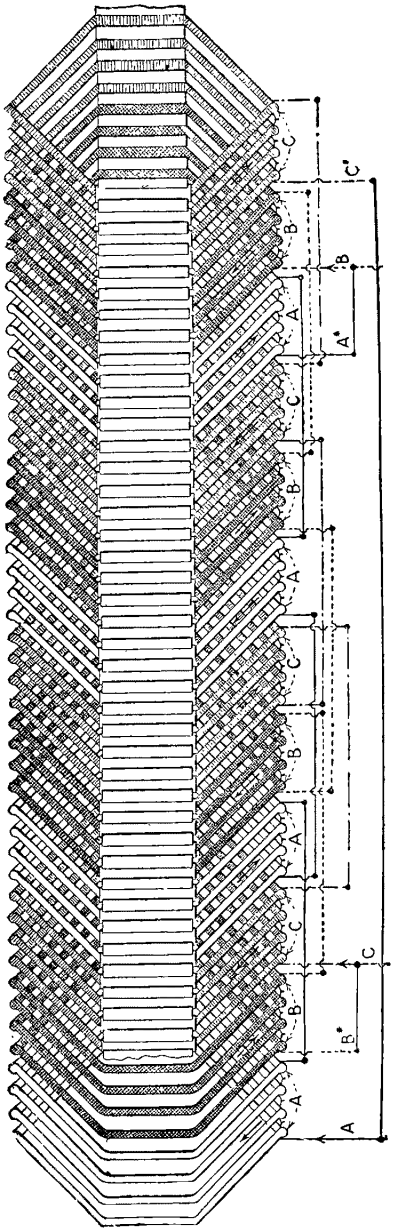


FIG. 215.—Three-phase, four-pole, series delta connection.

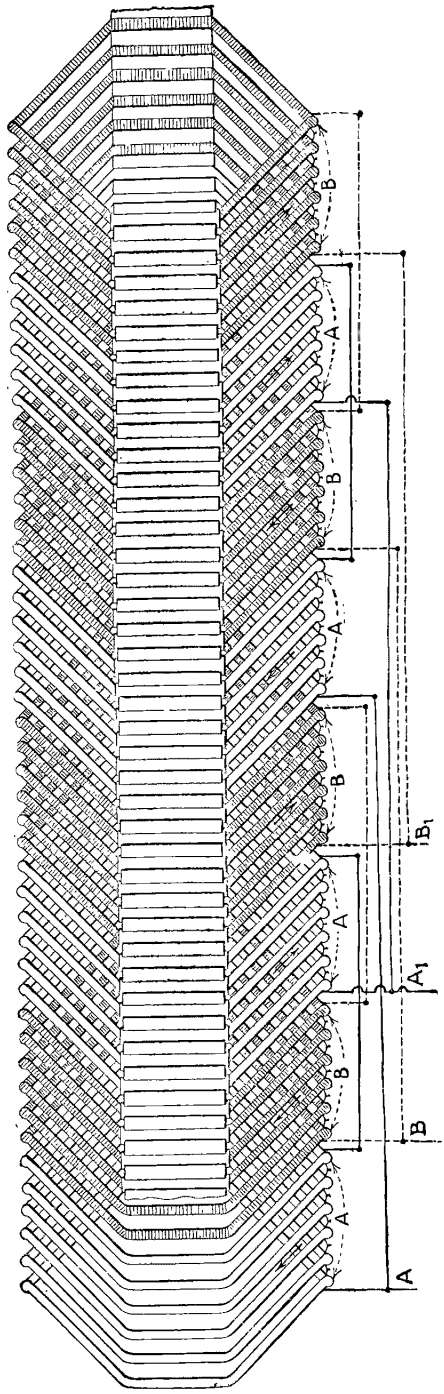


FIG 216.—Two-phase, four-pole, parallel connection

440 volts, without rewinding. The nearest value to "100" under a two-phase column is "70," shown under "Two-phase 2-parallels." This means that if a three-phase 440-volt motor which is connected series delta, be reconnected for two-parallel two-phase, as in Fig. 216, it should be operated on 70 per cent of 440, or 308 volts.

3. Change in Frequency.—It is desired to operate a three-phase 440-volt 60-cycle motor on 50 cycles at the same voltage. What change should be made in the connections? Inspection indicates that as the motor stands it is connected for three-phase five-parallel star on 60 cycles. A change in frequency should be offset by a change in voltage in the same direction and by the same amount; hence, if a motor is operated on 100 per cent voltage on 60 cycles, it should be connected for five-sixths of 100, or 83⅓ per cent, voltage on 50 cycles. However, the voltage is to remain the same on 50 cycles as on 60 cycles, so this result must be obtained in another way. If the voltage cannot be decreased the number of turns in series can be increased. Another way of saying this is that we can reconnect the winding so that ordinarily it would be good for a higher voltage and then if it is operated on the same voltage the effect will be the same as if a lower voltage had been applied to the original connection. In the case in hand the motor should, when connected on 50 cycles, be operated on 83⅓ per cent of the 60-cycle voltage. Only 100 per cent is available, so the winding will have to be reconnected with $100/83\frac{1}{3} = 120$ per cent of the original number of turns in series. This would ordinarily mean the winding was good for 120 per cent of the original voltage. Hence, in looking up the change in the connection table the figure "120" is located instead of 83⅓.

Referring to the table and following the horizontal line "Three-phase 5-parallel star" across, search is made for the figure "120," the nearest thing to it is "125," found under the vertical heading "4-parallel star." The number of poles in the motor would have to be divisible by both 4 and 5, in order to make this change possible; or, in other words, it would have had to be either 20 poles or 40 poles. As it may have been 10 poles, for example, the nearest connection that could be made would be for 144 under "Three-phase 2-parallel delta." This would mean the

correct operating voltage on 50 cycles would be $\frac{144 \times 440}{120} = 528$ volts; or, if operated on 440 volts, it would be working under $440/528 = 83\frac{1}{3}$ per cent normal voltage, which would usually not be permissible on account of lowered torque and increased heating.

4. Change in Number of Poles or Speed.—A 60-cycle three-phase motor is operating on 550 volts at 850 r.p.m.; it is desired to operate at 690 r.p.m. on the same voltage. What change in

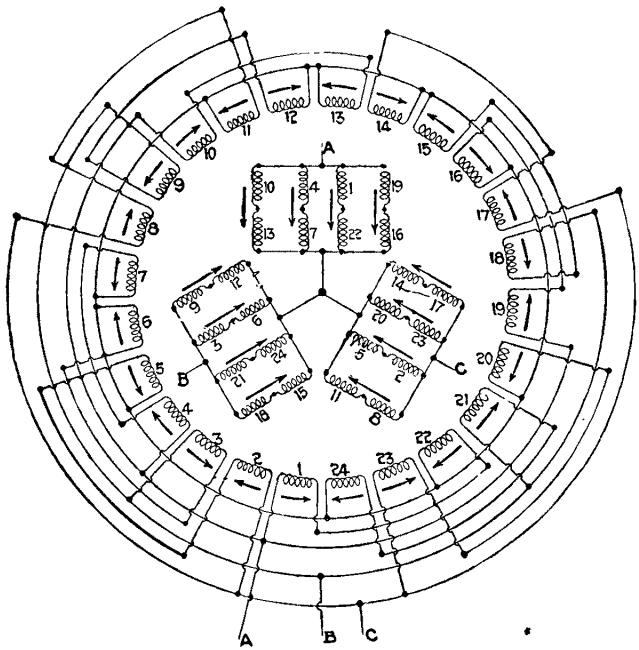


FIG. 217.—Normal three-phase, eight-pole, four-parallel star connection.

connections should be made, if any, in addition to changing the number of poles? Inspection shows the motor is connected four-parallel star for eight poles, as in Fig. 217. To get 690 r.p.m. would require to connect for 10 poles, since this would give a no-load speed of about 720 r.p.m. and a full-load speed of about 690 r.p.m. Since the motor is a generator also, it will generate only $\frac{690 \times 550}{850} = 446$ volts when connected for 10 poles and a slower speed. However, it is desired to continue at 550 volts, so that the connections will have to be changed to get the effect of

550/446 = 123 per cent of the old voltage. In the table opposite the horizontal line "Three-phase 4-parallel star," the nearest figure to "123" is "116," which is found in the vertical column headed "Three-phase 2-parallel delta." Hence, the conclusion is drawn that, if an eight-pole motor, Fig. 217 is connected three-phase four-parallel star and operated on 550 volts and it is reconnected for 10-poles two-parallel delta, Fig. 218, it may be still operated on 550 volts, although, strictly speaking, its normal

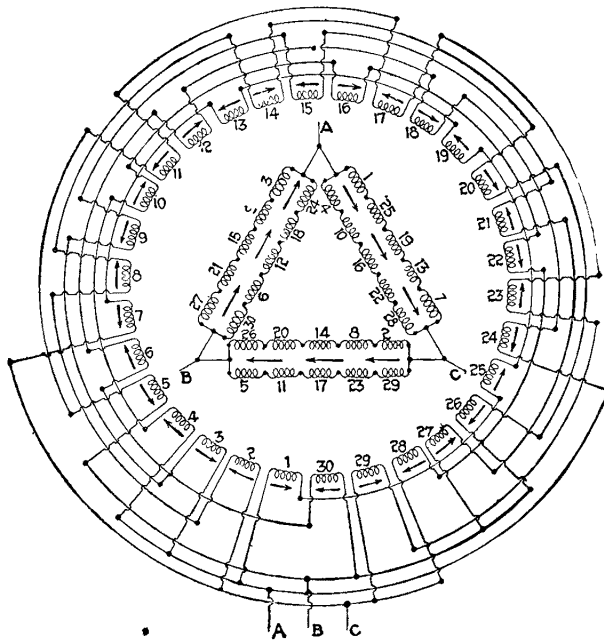


FIG. 218.—Same winding as Fig. 217 reconnected for three-phase, ten-pole parallel delta.

voltage would be $\frac{116 \times 550}{123} = 520$ volts. In this example no consideration was given to the fact that the throw of the coil in electrical degrees was changed in changing from 8 poles to 10 poles. This can be taken account of in the following way:

Suppose the motor as it stood had 120 stator slots and the coils lay in slots 1 and 13. Full pitch would be 1 and 16, since 120/8 = 15. Since the coils throw 12 slots and full pitch is 15 slots, the per cent pitch = 12/15 = 80 per cent, and from Fig. 210 the chord factor for 80 per cent pitch = 0.95. When recon-

nected for 10 poles, the throw of the coils is still 1 and 13, but this is now 100 per cent pitch since 120/10 = 12 and 1 and 13 does span 12 slots. Therefore, when connected for 10 poles the coils are more effective in the ratio of 1.00/0.95 since the chord factor for 100 per cent pitch = 1.00 from Fig. 210. Therefore, when the change in chord factor is also taken account of, the new normal operating voltage is 520, as obtained in the foregoing, multiplied by 1.00/0.95 = 548 volts, or almost exactly right for operation on 550 volts.

5. Change in Horsepower.—A 10-hp. 220-volt motor is operating above the allowable safe temperature, on its normal voltage, and it is found by experiment that when the voltage is raised to 250 its temperature is reduced to within safe limits. Can any change be made in the connections which will allow the motor to be operated still on 220 volts and duplicate the conditions when operating on 250 volts? An inspection of the winding shows the motor to be connected three-phase series delta, as in Fig. 215. The experiment which was made showed that the voltage should be increased to 250/220 = 114 per cent of its original value. It has been pointed out that reducing the number of turns in series in a winding has the same effect as increasing the voltage on the same number of turns. In this case if the voltage was raised to 114 per cent the same effect could be obtained by reducing the turns to 1.00/1.14 = 87.5 per cent. Consequently, in referring to the voltage change table, in this case, search is made for "87.5" and not "114."

Selecting, therefore, the horizontal line "Three-phase series delta" in the table and looking across, the nearest figure to "87.5" is "86," which occurs under the vertical heading "Three-phase 2-parallel star." Consequently, the conclusion is at once drawn that if a 220-volt motor has its connections changed from series-delta, Fig. 215, to parallel-star, Fig. 214, it will act in every way as though 220/0.86 = 256 volts had been applied to the series-delta connection. This is equivalent to increasing the horsepower of the motor, since on the original connection the motor was overloaded when carrying its rated load, but when the connections of the winding were changed the machine could drive its rated full load without distress. The reason for this is that, although the density of the magnetic flux was increased the cross-section of the copper in the winding was increased, con-

sequently the copper losses were reduced. The latter being considerably greater than the former resulted in a reduced temperature. The capacity in horsepower has actually been increased to 256/220 = 116 per cent of its original value.

From these five examples, which could be multiplied many times and from all sorts of combinations that could be made by changing the characteristics in pairs, it can be readily seen that any contemplated change can be reduced to an equivalent change in the applied voltage and the proper connection, if it is a feasible and rational change, selected from the table of phase and voltage given herewith.

CHAPTER XIV

LOCATING FAULTS IN INDUCTION MOTOR WINDINGS

NOISE AND VIBRATION

After the coils have been placed in a motor and the cross connections completed according to the desired diagram, a check is necessary to insure that the connections are properly made before load is put on. The simplest way of making this check is to start up the motor and run it light on a circuit of the proper phase, frequency and voltage. Observation of the behavior of the motor under these conditions indicates to the trained observer whether there are any serious discrepancies in the winding or connections. This observation should cover five points; namely, speed, noise, mechanical vibration, general heating of the whole winding and local heating of one or more separate coils.

The speed, if correct, should be of nearly synchronous value when the motor is running without load; that is, equal to cycles times 120 divided by the number of poles.

The motor should give a low humming noise similar to that made by transformers, but there should be no irregular or "growling" noise. There may also be a considerable volume of air noise or whistle caused by the ventilating air passing through the air ducts in the rotor and stator. The magnetic noise may be distinguished from the air noise by the expedient of opening the switch for a second or two while the motor is running full speed without load. Opening the switch breaks the current and removes the magnetic field; consequently the magnetic noise ceases, but leaves the rotor running at practically the same speed, owing to its inertia or stored energy; hence the windage, or air noise, is practically unaffected. In this way, by opening and closing the switch two or three times, it becomes readily apparent what part of the total sound made by the motor is magnetic and what part is windage. It also indicates whether either or both of these sounds are abnormal. If the speed is correct and the motor makes no more than a reasonable singing or humming

noise, the hand should be placed on the frame to note the mechanical vibration.

If there is noticeable mechanical vibration, it may be due to purely mechanical causes or to magnetic causes or possibly to both. By opening and closing the switch, as described in the foregoing, the mechanical vibration due to the magnetic field can be easily separated from that due to strictly mechanical causes, because when the switch is open there is no magnetic field present. Suppose, for example, that when the motor is running at full speed there is a marked vibration or trembling that can be felt when the hand is laid on the frame of the motor. Suppose, then, that when the switch is opened for a second or two the vibration disappears and the motor rotates smoothly at nearly the same rate of speed. This, then, is evidence that the vibration was caused by the action of the magnetic field on the stator and rotor. However, if the motor vibrates whether the switch is open or closed, it is evidence that the action is purely mechanical and is affected little or not at all by the presence of the magnetic field.

When the trouble is traceable to the magnetic field, it may indicate improper connection of the winding or it may indicate that the mechanical clearance between stator and rotor is not symmetrical or that there is some similar combination of mechanical and magnetic features that is responsible for the vibration noticeable. The commonest mechanical causes for vibration are rotor out of balance, either standing or running; bent shaft; too great clearance between shaft and bearings; unbalanced or eccentric coupling or pulley or a combination of two or more of these faults. These mechanical conditions are easily determined and can be corrected. The commonest causes of mechanical vibration due to a combination of mechanical and magnetic conditions are rotor out of round, stator out of round, too great clearance in the bearings, or rarely, uneven or eccentric air gap or clearance between stator and rotor. The latter point seldom gives trouble and a polyphase motor will practically always run without giving any trouble until the bearing wear allows the rotor to strike on the stator. Single-phase motors are more sensitive to eccentricities in the air gap or clearance between stator and rotor and sometimes show a considerable variation in torque in motors otherwise duplicate due to such irregularities.

There are a number of elements that may cause the rotor or stator to be out of round. In the first place there is a slight variation due to the punch-and-die work, which may amount to 0.005 inch between individual punchings. In the second place some allowance around the outside of the punching must be made in the fixture or frame in which they are built up so that they will assemble readily, and this allows the punchings to stagger more or less. In the third place, when the punchings are actually assembled in the frame, the frame may spring out of shape slightly after machining, owing to the release of casting strains when removing the material in the cut. Of course, none of these variations is in itself large, but when they all accumulate in the same direction, perceptible eccentricity may result, amounting to a good many thousandths of an inch. This is not serious, since it is present to some extent in all motors, but under extreme or extraordinary conditions it may cause mechanical vibration.

Mechanical vibration caused by the windings may be due to either the rotor or the stator. For example, in a squirrel-cage rotor there may be bad contacts between certain bars and the short-circuiting rings, resulting in more resistance in some parts of the winding than in others. This in turn affects the distribution of current in the different bars and hence affects the magnetic field and varies the mechanical pull from point to point. Or if the winding on the rotor of a wound-rotor type motor is grounded in a number of places, it will also cause unequal distribution of the current in the windings, which in turn causes severe vibration during the starting period. However, this generally disappears to a large extent after the motor comes up to full speed. From this it may be seen that where mechanical vibration is absent the conclusion may be drawn that the windings are symmetrical and are functioning properly, but where vibration is present it may be caused by a number of things, some of them obscure, and must not immediately be attributed to improper winding connections until a further examination is made.

The next point to be observed is the general temperature of the entire winding as determined by passing the hand around the ends of the windings. It is best practice in making this examination to shut down the motor after it has run three to five minutes. If the examination is made while the motor is running, care should

be taken to avoid injury by coming in contact with moving parts and also to avoid injury from electric shock, if the circuit is 550 volts or over. If the winding as a whole is cool, inspection should be made for individual coils that are much hotter than the rest of the winding, as these may indicate short circuits or improper connections in that particular coil.

If a motor is operating freely and easily at the proper speed without undue noise or mechanical vibration and if there is no general or local heating of the winding, the next step is to measure the current in each phase. This may be done as indicated in Figs. 219, 220 and 221. If possible an ammeter should be con-

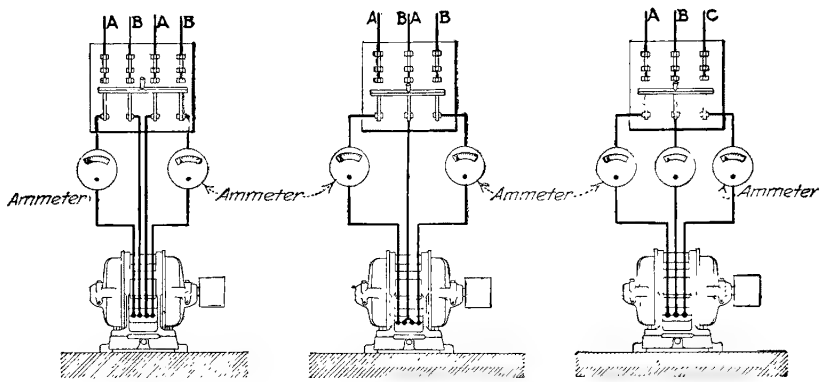


FIG. 219.—Two-phase, four-wire circuit.

FIG. 220.—Two phase, three-wire circuit.

FIG. 221.—Three-phase circuit.

Measuring the current in each phase.

nected in each phase so that the readings of all phases may be taken simultaneously. For a two-phase motor two ammeters are required, as in Figs. 219 and 220, and for a three-phase motor three ammeters are required, as in Fig. 221. The no-load, or magnetizing current as it is called, will usually be somewhere between 15 and 35 per cent of the full-load current with an average value of perhaps 25 per cent. If the no-load current in all phases is equal and approximately 25 per cent of the full load, it is safe to assume that the winding connections are properly made. If a wattmeter is available, a further check might be made on the total watts taken by the motor running light, but that does not add greatly to the ammeter check. The connections for connecting two wattmeters in a two-phase four-wire circuit are given in Fig. 222 and for a three-phase circuit in Fig.

223. The connections for a three-wire two-phase would also be the same as those in Fig. 223; where only one wattmeter is available, it may be connected into a three-phase circuit with a single-pole switch, as in Fig. 224, so that the two readings may be taken by simply throwing the switch. In a two-phase circuit the total watts will always be the sum of the two readings, but in a three-phase circuit this is true only when the power factor is greater than 0.50. Where two wattmeters are used to measure the no-load watts of a three-phase motor, the difference of the two readings gives the correct value of the watts, since the power fac-

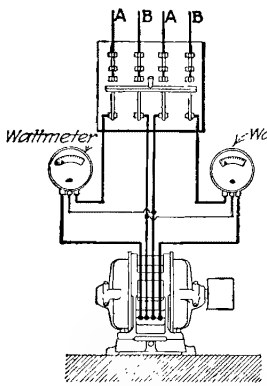


FIG. 222.—Two-phase, four-wire circuit with two wattmeters.

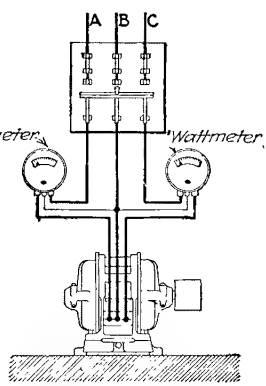


FIG. 223.—Three-phase circuit with two wattmeters.

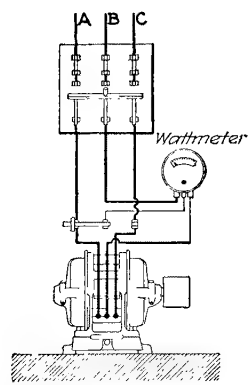


FIG. 224.—Three-phase circuit with one wattmeter.

Measuring the total watts.

tor of an induction motor at no load is always less than 0.50. The watts taken at no load and full speed and voltage cover the iron loss, bearing friction, windage and a small amount of copper loss. The total no-load watts will in general be in the order of 5 per cent to 8 per cent of the rating of the motor in watts varying with the capacity and speed of the motor. The motor rating in watts would be the horsepower from the nameplate multiplied by 746.

If the foregoing checks indicate that the motor is not acting normally, they should also give some evidence that there is a fault in the coils of the winding or in the manner in which these coils are connected, and further search is made to analyze the nature of this fault so that it may be located and corrected.

The winding of an induction motor is made up of a number of similar coils connected into groups. These groups in turn are

connected in such a manner that, when an alternating current of the proper characteristics flows through them, a magnetic field having alternate north and south poles is set up and caused to rotate in the motor. The coil itself is usually made up of two or more turns of wire or strap so that there are at least ten chances for defects in the winding after the coils are all in place and connected. Some of these faults are simple and readily rectified, while others are more obscure and difficult to handle.

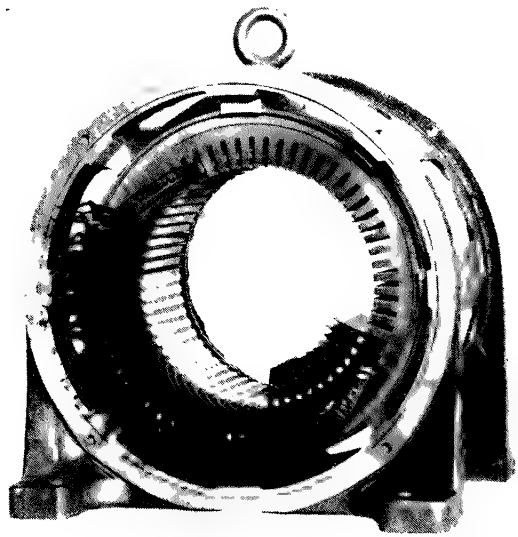


FIG. 225.—Placing the coils in the core of an induction motor.

The Ten Most Common Defects.—These ten most common defects in the order of their likelihood are:

1. The winding grounded on the core.
2. One or more turns in one or more coils short-circuited.
3. One or more complete coils short-circuited at the coil ends or at the "stubs."
4. A complete coil reversed or connected so that the current flows through it in the wrong direction.
5. A complete group of coils or pole-phase group is reversed; that is, connected so that the current flows through the group in the wrong direction, making a north pole where a south should be or vice versa.
6. Owing to lack of care in counting, two or more pole-phase groups may include the wrong number of coils.

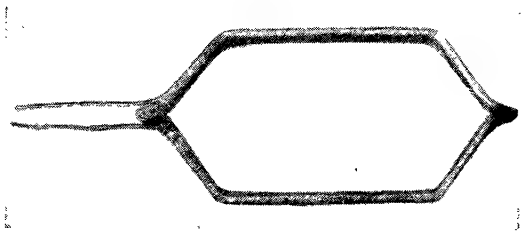


FIG. 226.—Typical "diamond" coil.

FIG. 227.—Coils in place unconnected.

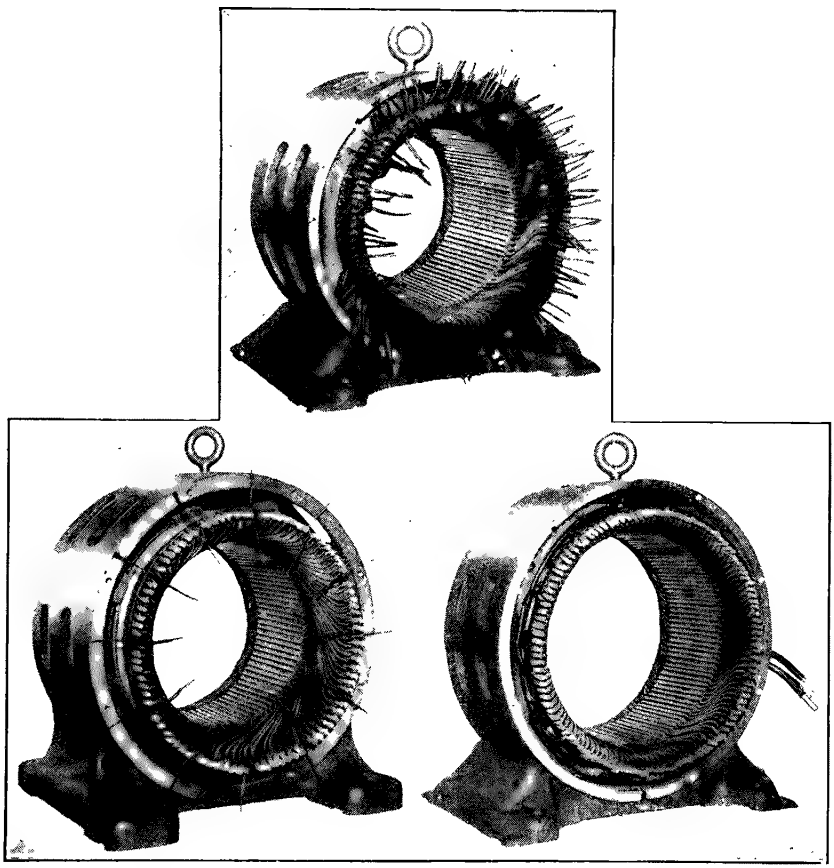


FIG. 228.—Coils connected into pole-phase groups.

FIG. 229.—The completed connection of winding.

The three stages of connecting a winding.

7. A complete phase in a three-phase star or delta winding is reversed.

8. The winding connections may be properly made in themselves, but not right for the voltage upon which the motor is to be operated. That is, the motor may be connected properly for 110 or 440 volts, but the motor is to operate on 220 volts.

9. The winding connections are properly made, but they are for the wrong number of poles, and hence the motor runs at a different speed from that which was intended.

10. An open circuit somewhere in the winding, or one or more coils are omitted and left out of the winding, known as "dead" coils.

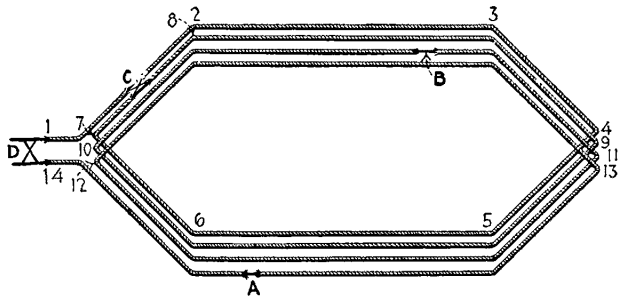


FIG. 230.—Individual coil with insulation removed to show "shorts" and "grounds."

The manner in which these various faults occur can be best understood by referring to what takes place, first, in winding and insulating the coils, and, second, in placing them in the core and connecting them.

Figure 226 shows a coil of the usual form wound up from several turns of wire and insulated ready to be used in the slot; Fig. 225, the operation of winding these coils in place in the core; and Fig. 227, the coils all in place ready for connecting. The coils connected into pole-phase groups, with the coil ends at the beginning of each group bent into the bore and the coil ends at the end of each group bent out toward the frame are shown in Fig. 228. The cross connections are made in Fig. 229, thus completing the winding connections. Figure 230 shows the coil in Fig. 226 as it would appear if the insulation were stripped off and individual turns of wire separated.

Grounds.—The first fault listed—grounding of the winding on the core—occurs when in some manner the insulation becomes

stripped from the coil and also the cotton covering from the wire so that at some point, as at *A*, Fig. 231, the bare-copper conductor touches the laminated-iron core and by so doing "grounds" the winding. This means that a live current-carrying part is touching the metal structure of the motor, and when this condition exists anyone who touches the frame of the motor actually touches a live conductor. This may not be detected if the entire winding and the supply circuit otherwise is free from grounds, but it often happens that other grounds are present somewhere in the system so that in standing on the ground and touching

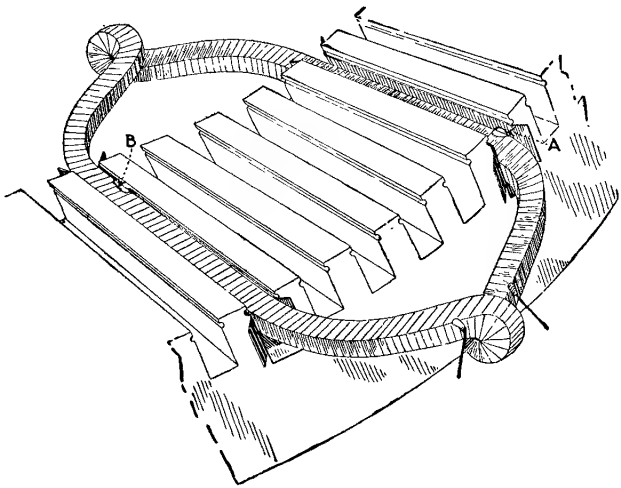


FIG. 231.—Coil grounded on core.

the frame of the machine the chances are very good of getting a shock at a voltage that may equal that of the supply circuit.

Referring to Figs. 230 and 231, should two grounds occur simultaneously, as, for example, at *A* and *B*, a short circuit would be formed in the loop, Fig. 230, from *B* through 11, 12 and 13 to *A*; and if the normal voltage remained on the motor, this short-circuited turn would immediately become hot enough to destroy the insulation on the complete coil. This is the second fault listed and may occur without grounding by the touching of the bare conductor of adjacent turns as at *C*, where the complete short circuit follows the path of *C* 2, 3, 4, 5, 6, 7 and *C*.

Short Circuits.—The third fault—short-circuiting a complete coil—can also be seen from Fig. 230 and exists when the insulation of the ends of the coil 1 and 14 become damaged and allow

these two wires to touch, as at *D*. A current then flows in the entire coil, in addition to and aside from the line current, equal to the voltage of the coil divided by its impedance. In other words, what happens is equivalent to removing that particular coil from the main winding where it is generating its share of the useful counter-electromotive force and using up this same generated or induced counter-voltage, simply, to force current through the coil itself. This coil would heat up practically as fast as would any induction motor winding if the rotor was held from rotating and full-line voltage applied to the stator winding.

Reversed Coil.—The fourth fault occurs when the two leads of a coil are interchanged, as at *X*, Fig. 232. This has the effect of causing the one coil, or in this case coil *Y*, to "buck" all the other coils in the same pole-phase group. Expressing this in another way, the cross-connected coil is trying to produce a magnetic north pole when all the other coils in its group are producing a south pole. The effect of this is magnetic dissymmetry and manifests itself, as do most irregularities in winding, in noise and heating.

Reversed Group.—The fifth fault, and one that can occur readily in connecting, is when an entire pole-phase group is reversed, as at *Z*, Fig. 232. This can be understood from Fig. 228. The beginnings of all pole-phase groups are bent in toward the center of the bore, and the endings are all bent out. Should one of the ends bent out be used as a beginning and the other end as an ending, the entire group would be reversed with consequent magnetic distortion and trouble due to noise and heating.

Wrong Grouping.—The sixth fault is one due wholly to wrong counting in grouping the coils. In a three-phase four-pole motor with 48 coils there should be in each group $48 \div (3 \times 4) = 4$ coils, and the presence of three coils or five coils in any group constitutes the sixth fault as they are here listed. This is also shown in Fig. 232, where all the groups have four coils except A^1 and B^1, which have five and three coils respectively.

Reversed Phase.—The seventh fault is present only in the case of three-phase motors and consists in reversing the ends of one-third of the winding so that one leg of the star or one side of the delta is connected in such a way that the voltages generated in the three phases are only 60 electrical degrees apart, whereas the currents supplied from any normal three-phase generator

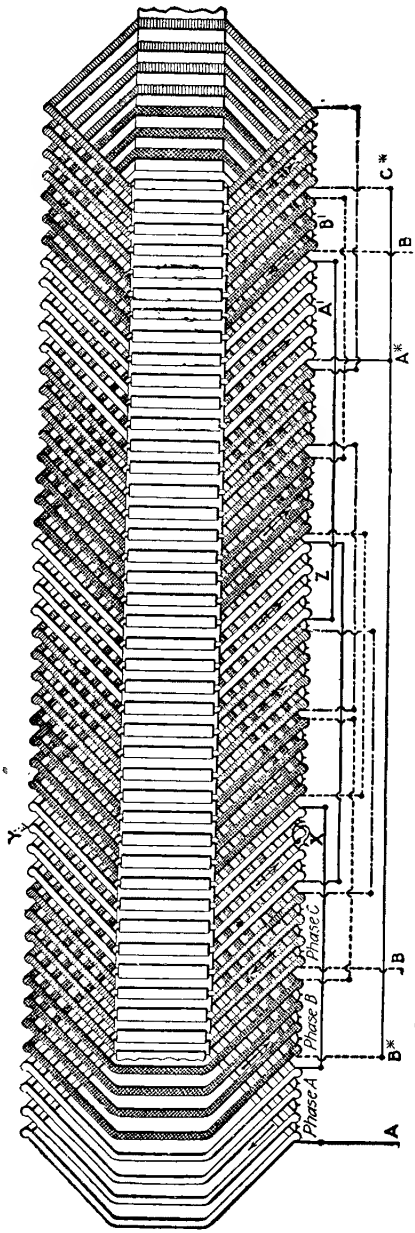


FIG. 232.—Reversed coil, reversed group and wrong number of coils in a group.

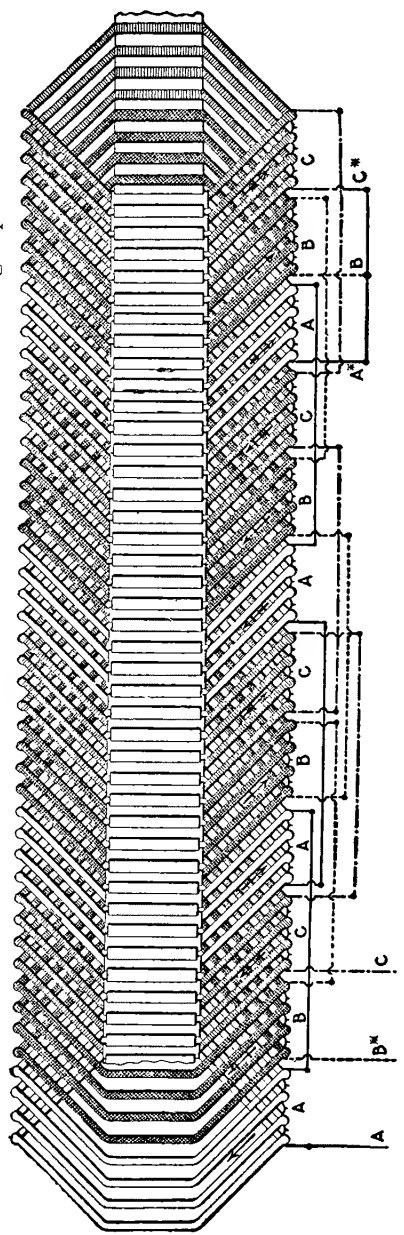


FIG. 233.—Reversed phase.
Faults in completed connections.

are 120 electrical degrees apart, and hence these three voltages and currents cannot combine to produce power as they properly should. This can be understood by referring to Figs. 234 and 235. Figure 234 shows the three voltages generated in a three-phase winding as represented by three arrows or vectors arranged 120 deg. apart. If, however, one phase of the winding was reversed and the lead connected to the star point and vice versa, the back, or counter-electromotive, force generated in that winding would be reversed and would no longer be 120 deg. from the voltages in the other two phases, but would be 60 deg. from them, as in Fig. 235. This would mean that the magnetic field in the stator, instead of being a balanced succession of north and south

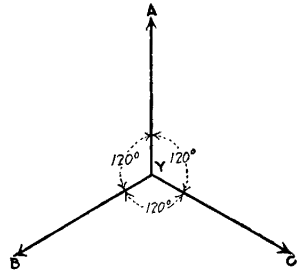


FIG. 234.—Normal winding relation.

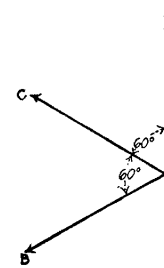


FIG. 235.—Wrong connection in one phase as in Fig. 233.

Effect of reversing a phase.

poles rotating and pulling the rotor around, would become unbalanced and would no longer rotate properly. According to another method of looking at the matter, there would be one field rotating clockwise and another different kind of a field rotating counterclockwise, and the natural result would be that these two fields would interfere, and instead of rotating, the motor would remain at a standstill, emitting an unusual amount of noise and reaching a dangerous temperature in a very short time.

A four-pole three-phase winding with the *B* phase reversed is shown in Fig. 233. It will be observed that instead of the arrows on the pole-phase groups pointing in alternate opposite directions, as they should for a correct connection, they point in opposite directions in groups of three. Further consideration will be given this feature later in this chapter.

Connected for Wrong Voltage.—In the eighth fault the winding connections are all made properly to form the magnetic poles

in their proper sequence, but there are only half as many turns in series or perhaps twice as many as there should be. When this is discovered, the winding may be such as to permit connecting in series instead of parallel or vice versa, but under the worst conditions it may be necessary to remove the entire set of coils and replace with a new set having the proper number of turns for the required voltage.

Wrong Number of Poles.—The ninth fault is sometimes overlooked unless the speed is taken with a tachometer or speed counter, in which case it is readily detected. Its correction is not always either evident or simple, but can often be accomplished without change in coils by following some one of the various methods described in Chapters VI and XI.

Open Circuits.—The tenth fault, "open circuits," may be due to failure to solder a joint properly or to a joint being broken mechanically after having been once made. "Dead" coils are usually purely inadvertent and are sometimes present without being discovered at all. Such an occurrence could hardly happen unless there were a large number of small coils crowded together.

After the enumeration of the commonest errors made by the winder, as outlined above, the next step is to consider them in turn with particular reference to how each may be detected and corrected.

Correction of Faults. *First Fault.* Grounds.—If the ground is fairly low resistance—*i.e.*, if the bare copper of the winding touches the core—the defect may be detected by using an incandescent lamp arranged as shown in Fig. 236. One of the lamp leads is touched to a bare spot on the winding, for example, a terminal connector or a "stub" where two adjacent coils are connected, and the other is touched to the bare metal of the motor frame at some point not protected by paint. If there is a ground present, the lamp lights up. Another common method is by "ringing out" with a magneto similar to that used in telephone work. In this method the terminals of the magneto are applied, one to the winding and the other to the frame similar to the procedure in Fig. 236, and the handle is turned. If the bell rings, there is probably a ground in the winding. A third method employs a "testing box," which is really a transformer for obtaining voltage much higher than the normal voltage of the motor under test. These boxes give 2000 or 3000 or more

volts and readily detect grounds on windings of 550 volts and below. The test box is so arranged that when the terminals are applied as in Fig. 236, the presence of a ground instantly opens a circuit-breaker on the side of the box.

Having established the fact that the winding is grounded by some one of the foregoing methods, the next problem is to locate in which coil or what part of the winding it has occurred. This can sometimes be done by inspection, but sometimes requires other means. The most usual of these is to put enough voltage on the ground with the lamp device of Fig. 236, or the test box, so that the resulting current heats up the contact that is causing

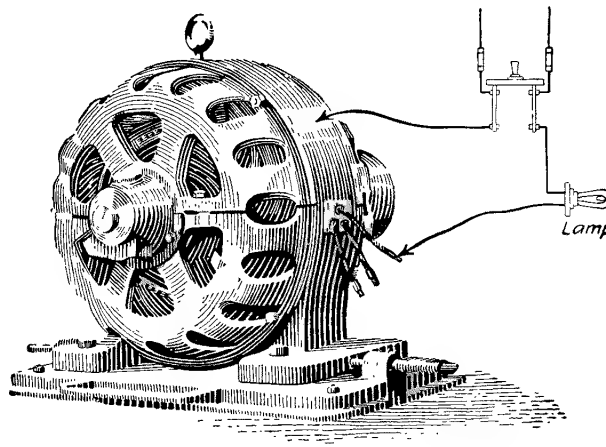


FIG. 236.—Lamp test for grounds.

the ground and it becomes evident through smoke or slight arcing. This will generally require two or more lamps connected in parallel. When the ground is definitely located, it is corrected by repairing the insulation at this point by retaping the coil, or replacing the defective slot cell or whatever may be causing the trouble. Sometimes the ground cannot be "smoked out" in this manner, and it then becomes necessary to open up the winding at two or three places and test out the different pieces to find in which one the ground is present. If it is still not evident, the defective section of the winding is further broken into smaller pieces and the search pursued until the trouble is finally run down to the individual coil which is defective. It is seldom necessary to go so far, as the ground furnishes evidence of its location as soon as the voltage is put across it.

Second and Third Faults.—Short circuits of a few turns in a coil, or a single coil completely short-circuited, become hot in a short time if the motor is run light on normal voltage. Their presence can be detected by feeling around the winding with the hand immediately after starting the machine and noting if some individual coils are much warmer than others. A device for detecting such short circuits before the rotor is put in the stator and without applying any voltage to the winding itself is shown

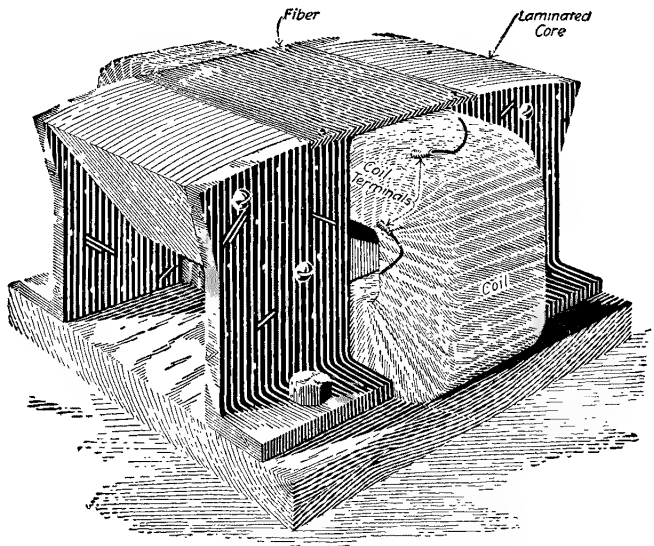


FIG. 237.—Transformer device for locating short-circuited coils in a completed winding.

in Fig. 237. This device is somewhat similar to a large horseshoe magnet excepting that the iron part is built up of laminations, or it may be considered as a core-type transformer having a primary coil only with one side of the iron core missing. The coil is excited with alternating current of suitable voltage, and then the complete device is passed slowly around the bore of the machine being tested as shown in Fig. 238. In passing around, if the testing device passes over any short-circuited turn or coil, such short circuit immediately acts as a short-circuited secondary coil on a transformer of which the exciting coil on the testing device is the primary and whose magnetic circuit is made partly by the testing device and partly by the core of the machine under test. As in any short-circuited transformer, an increased cur-

rent flows in both the primary and secondary coil and can be detected by an ammeter in series with the device or by the heating that immediately takes place in the defective coil, or by the attraction that the short-circuited coil has for a strip of sheet iron. By passing the device slowly around the core and observing its behavior from point to point, short circuits can readily be

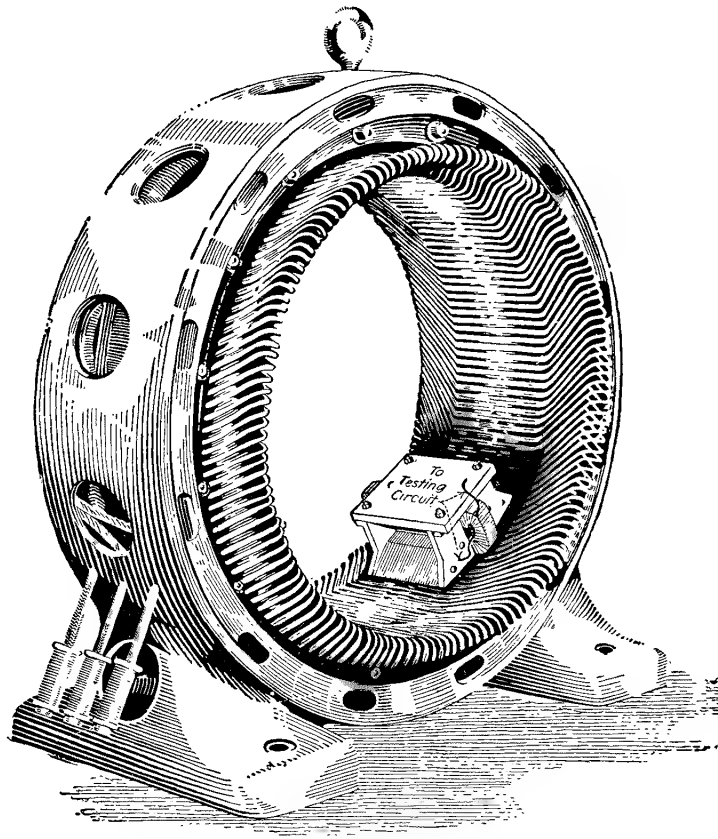


FIG. 238.—Method of using the device shown in Fig. 237.

detected. This refers particularly to short circuits in individual turns or in one complete coil. A short circuit of a complete pole-phase group is more readily located by a compass test, and a short circuit of an entire phase can be located by a "balance test."

The *compass test* referred to in the preceding paragraph consists in passing a compass slowly around the bore of the stator

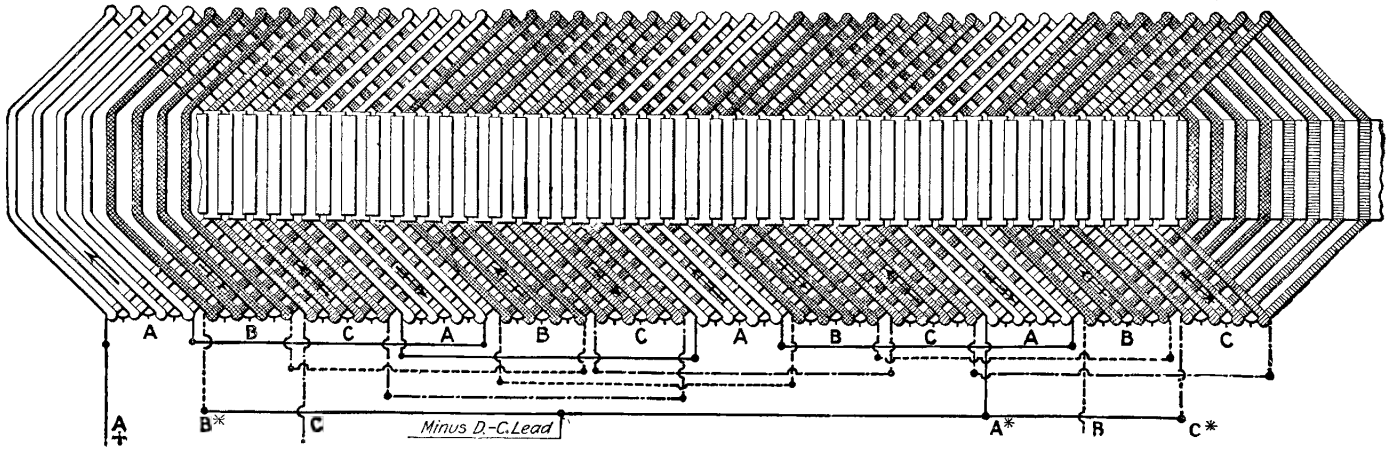


FIG. 239.—Manner of connecting d.c. in making compass test for reversed pole-phase group.

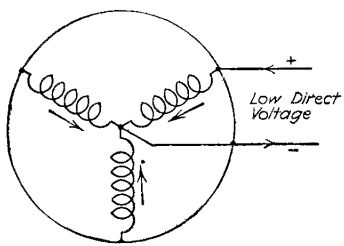


FIG. 239*a*.—Schematic diagram for Fig. 239.

from which the rotor has been removed and which has the winding excited by direct current of the value of about one-third the full-load alternating current. The effect of this direct current is to set up north and south poles alternately in the phase which is excited, and as the compass is passed slowly around the bore its needle reverses with the polarity, and by marking the polarity plus and minus with chalk marks in the bore, the chalk marks immediately indicate the correctness or faults in the winding. If it is a two-phase machine, the direct current is put on each phase separately and the check is made. For a three-phase star winding cause the direct current to flow from each lead to the star by making three observations, and mark the polarity only on the groups from the lead to the star in each phase separately. This can be readily understood by referring to Figs. 239 and 239*a*. For the first observation put the direct-current plus lead on *A* and the minus on the star connection, then pass the compass around the bore and mark the polarity of the groups from *A* to the star point with an arrow, the arrow pointing in the same direction as the compass needle. For the second observation put the direct-current plus lead on *B* and the minus lead on the star connection and passing the compass around marking the polarity of the groups from *B* to the star point. For the third observation put the direct-current plus lead on *C* and the minus on the star, and by means of the compass determine and mark the polarity of the groups from *C* to the star point. If the three observations have been made correctly, there will be a chalk arrow on each pole-phase group of the winding, and if the winding is correctly connected, these chalk arrows will alternate north and south, as shown in the Fig. 239. In case of a short circuit of a complete pole-phase group the compass needle will not be deflected. If a three-phase delta winding is being checked, open the delta connection at one lead, as in Figs. 240 and 240*a*, connect the direct-current source in so that the current flows through the three phases in series, and if the pole-phase groups be checked for polarity, the arrows will reverse as just described for the star winding.

The *balance test* referred to consists in checking each phase of the winding separately with low-voltage alternating current, say 20 per cent of normal full voltage, and measuring the amperes to check the impedance roughly and see if it is the same in all

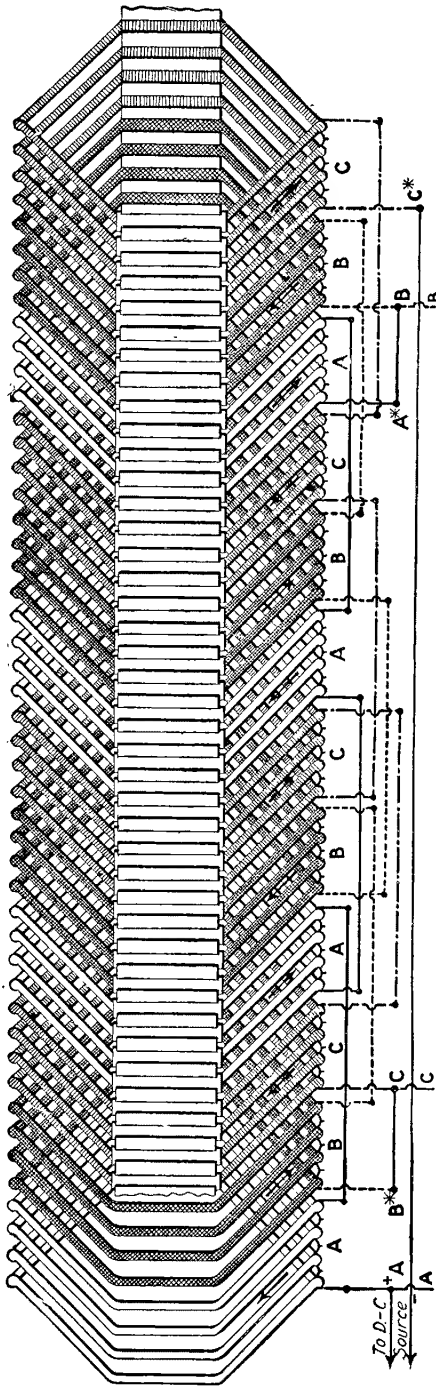


FIG. 240.—Checking a delta winding with d.c. and compass to locate a reversed group.

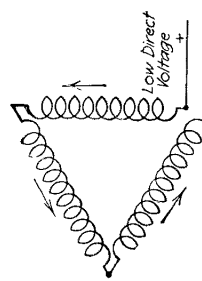


FIG. 240a.—Schematic diagram for Fig. 240.

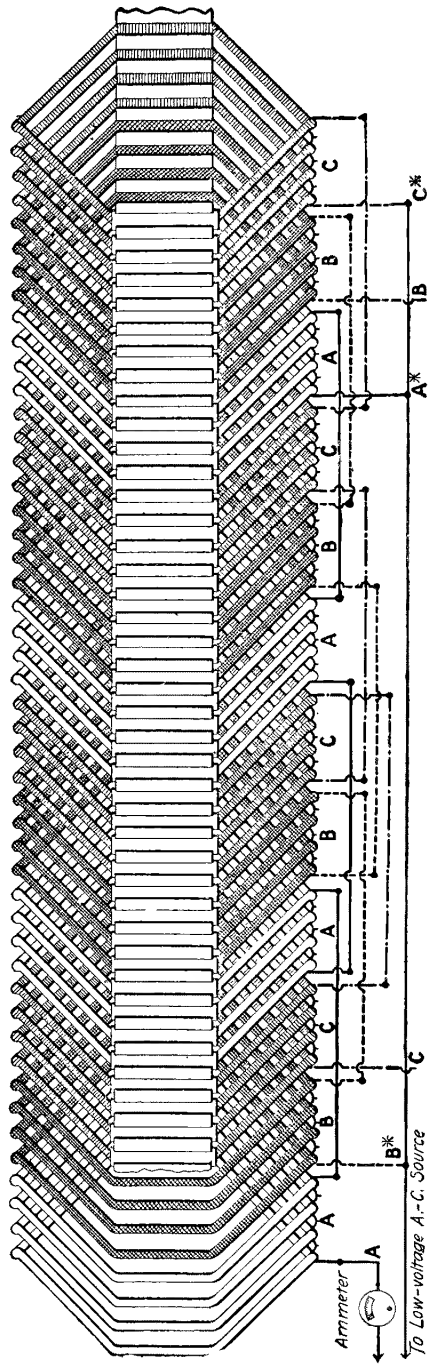


FIG. 241.—Connections for making a "balance test."

phases. The connections for a star-connected winding are made, as in Fig. 241, so that the current can be measured in each phase, with an ammeter. The low-voltage alternating-current source is, in all cases, connected across one terminal, *A*, *B*, or *C*, and the star as in the figure. The ammeter should read the same in all three leads. For a delta-connected winding it is necessary to open the delta connections at some point, as at *A*, then test across each phase separately. This test is made on the stator only and with the rotor removed.

Fourth and Fifth Faults.—Reversal of one or more coils in a group or group of coils. It happens that individual coils or sometimes entire groups are connected in backward. If the error is confined to one coil it does not usually show up on a "balance test" and would not be found on a resistance test, since the resistance would be the same no matter which way the coil was connected. Such reversed coils or groups can be located by means of the compass test described under Short Circuits. If an individual coil is reversed, it will show a tendency to reverse the compass needle when the needle is directly over that coil. If an entire pole-phase group is reversed, the compass needle will indicate the same direction of field on three successive groups, as at *Z*, Fig. 242. Also if a coil is left out of circuit, or "dead," as listed under the tenth fault, the compass needle will indicate an irregularity at the instant of passing over that particular coil. By checking the three phases of a three-phase winding separately, with a compass, as described under the second and third faults, it is possible to check for the reversal of an entire phase.

Sixth Fault.—This is the case where one coil too many or too few is connected in a pole-phase group, as at *A'* and *B'*, Fig. 242. The best check on this is a visual inspection and count of the "stubs" at the end of each group, and when the trouble is located it is corrected by disconnecting, regrouping and reconnecting.

Seventh Fault.—The reversal of an entire phase in a three-phase winding usually manifests itself in a very pronounced manner when the motor is run light. If the rotor turns over at all it is probably at a speed very much less than normal and emits a loud, growling noise and immediately becomes hot. This fault may also be detected by the compass test, as described under faults two and three. The arrows on the windings will point in groups of three in opposite directions, as in Fig. 243. The

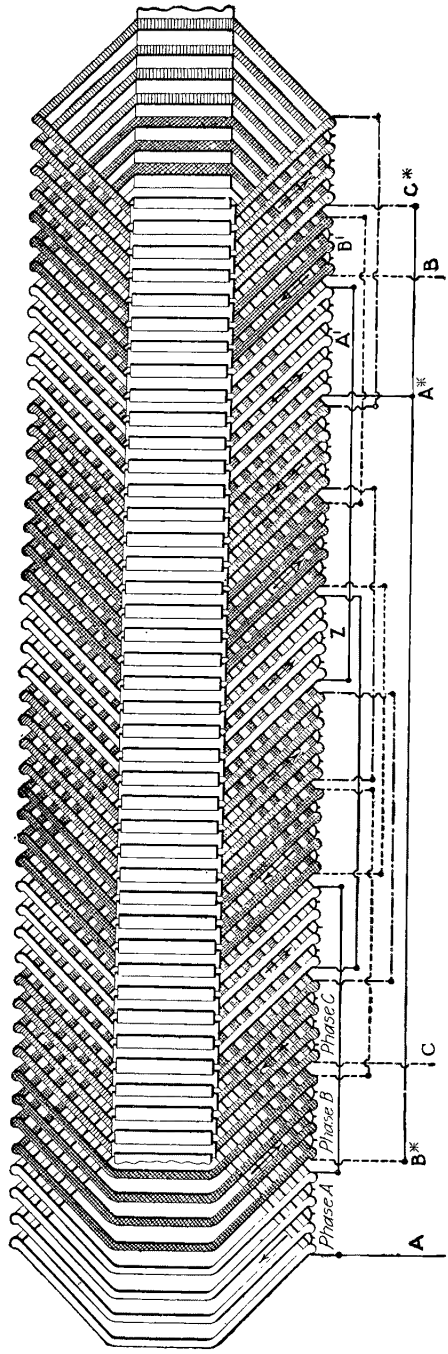


FIG. 242.—Reversed group and wrong count in a three-phase winding.

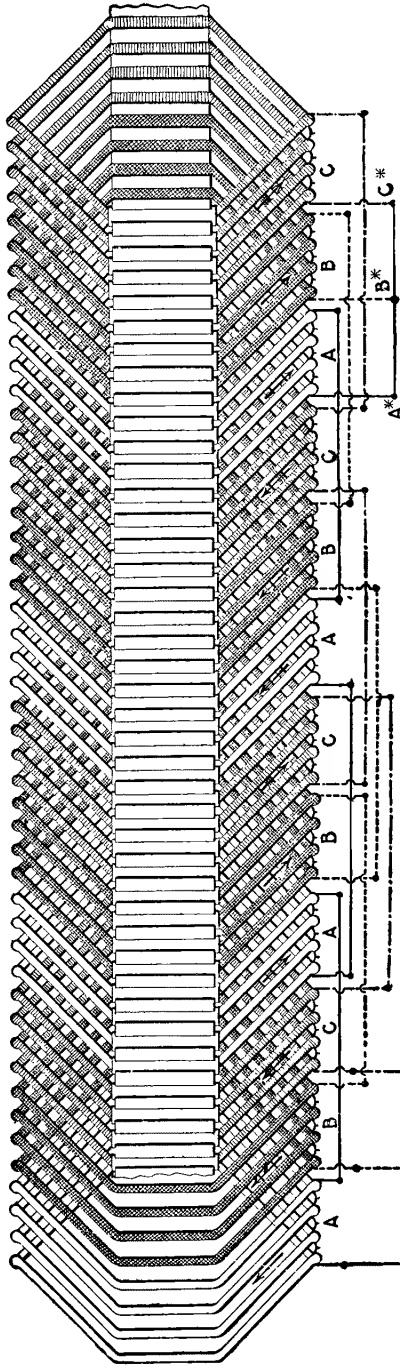


FIG. 243.—Reversed phase in a three-phase winding.

remedy when the defect is found is to open the star point and use the star point on the defective phase, which is the *B* phase in Fig. 243, for a lead and bringing the end that was a lead to the star, thus giving the connection, Fig. 239. In a two-phase winding there is no trouble with reversed phase for the reason that if the direction of rotation of the motor is wrong, the leads may be easily reversed outside of the motor and the correct rotation secured.

Eighth Fault.—Connection for wrong voltage. If a motor is connected for a lower voltage than the circuit upon which it is operating, the no-load current becomes excessive and may even approach full-load value. There is a pronounced magnetic hum with a vibration indicating that the field is very strong, which is the case. On the other hand, if the motor is connected for a higher voltage than that upon which it is being tried, the no-load current is very small and the motor apparently "pulls out" on much less than its rated full load. If these faults are a matter of half-voltage or double voltage, for example, they can usually be detected without much trouble; but if the variation is less, this becomes a more difficult matter and in the absence of any other official data it sometimes becomes necessary to take a brake test to determine what the trouble is. After the difficulty and its extent have been determined, a reconnection of the groups can usually be made which will give the proper operating conditions. For example, if it is found that the winding is connected series star as in Fig. 239, and the motor is connected for 440 volts, when it is to be operated on a 220-volt circuit the winding should be changed to parallel star, as in Fig. 244, and the operation will be normal.

Ninth Fault.—The easiest way to detect a connection for the wrong number of poles is to run the motor light and take the speed with a tachometer or speed counter. When it is found that the winding is connected for the wrong number of poles, the possibility of reconnecting can be determined by methods suggested in Chap. XIII.

Tenth Fault.—Open circuits are manifest from the fact that the motor will not start, but acts as if it were operating single-phase. It is easy to determine, in a star-connected winding, in which phase the open circuit exists by connecting all phase leads to the starting transformer and opening them one at a time to

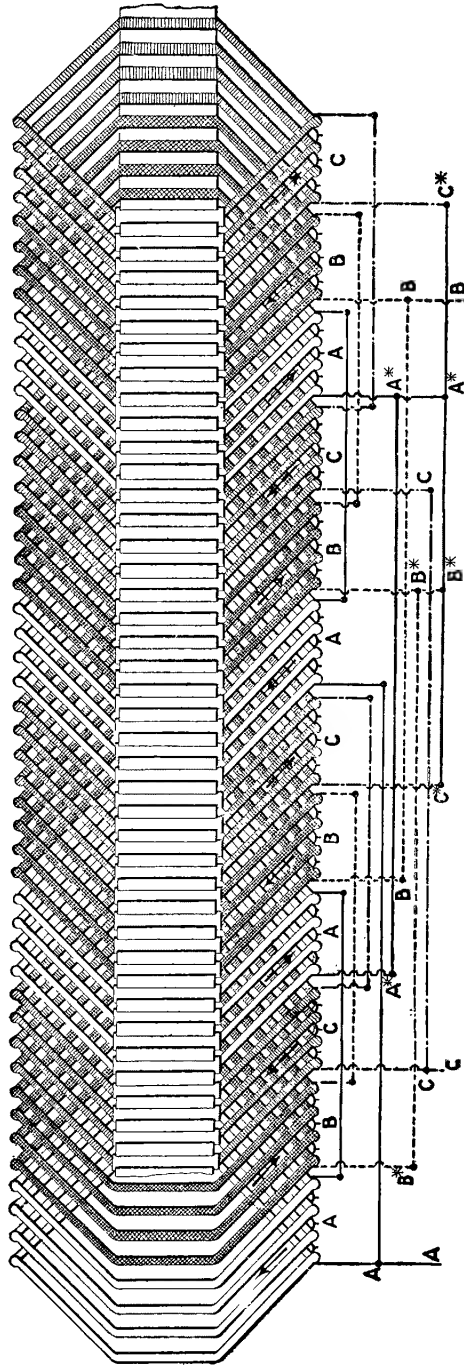


FIG. 244.—Normal three-phase, four-pole parallel star connection.

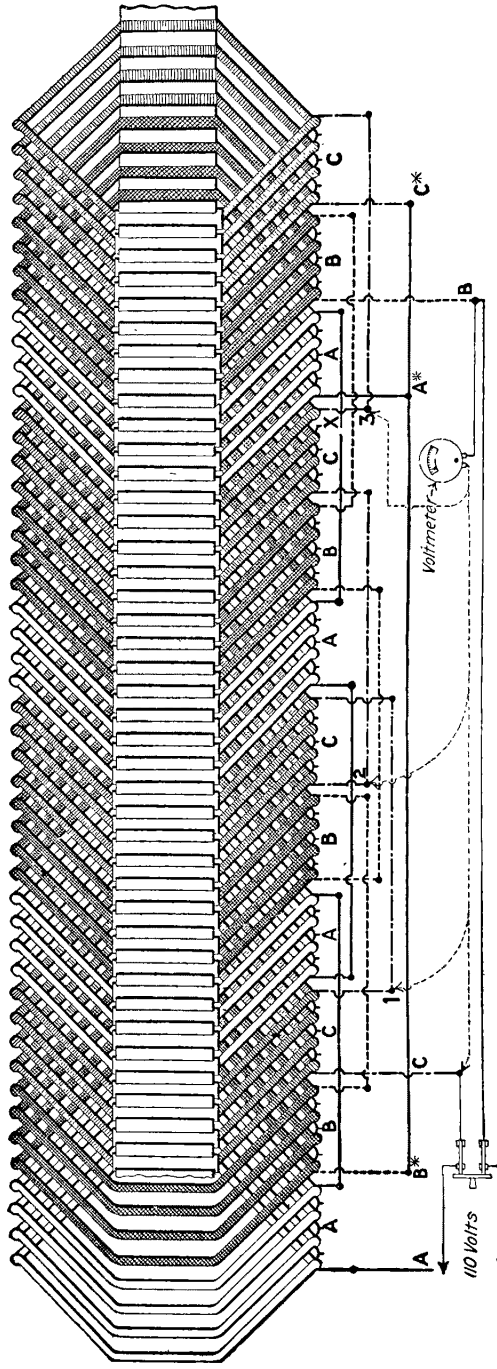


FIG. 245.—Checking for open circuits.

see in which lead no current is flowing. In Fig. 245 assume that the open is in phase *C* at *X*. Then, if lead *A* is open, no current will flow through the motor, since the path is from *B* to *C* and is open at *X*. If the *B* lead is disconnected with *A*, and *C* connected, no current can flow, since the *C* phase is still in circuit. If *C* is disconnected with *A* and *B* lead connected in circuit, then *C*, the defective phase, will be cut out of circuit and current will flow in the *A* and *B* windings of the motor and it will act as if operating single-phase, which will be indicated by the motor emitting a humming sound. When the defective phase is located, it is not always apparent just where the break is. A visual inspection may fail to show the break on account of tape over the defect or for some other reason. If this point cannot be located by inspection, a simple method of finding it electrically is indicated by referring to Fig. 245. A test voltage somewhat lower than normal or whatever is convenient is then applied to *B* and *C*, and a suitable voltmeter is used to measure the voltage between *B* and various points along the *C* phase, as, for example, 1, 2, and 3, which are chosen at random along the "stubs," or coil-to-coil connections, or on the group cross connections, as in the figure. With the condition as shown in Fig. 245, assume that 110 volts has been applied to the *B* and *C* terminals of the winding, as shown. If one lead from the voltmeter be attached to *B* and the other lead touched successively to *C* and 1, 2 and 3, the voltmeter will read 110 volts between *B* and *C*, *B* and 1, *B* and 2, and zero volts between *B* and 3, since the *C* phase is open at *X*. The conclusion is immediately and properly reached that the break is between 2 and 3 and with the inspection narrowed down to this small section of the winding the break is usually apparent. However, should the break not be discovered by inspection, points can be selected with finer steps between 2 and 3 and voltage readings taken until the defect is narrowed to the exact coil or piece of cross connection where it exists.

In the case of a delta connection one of the simplest ways to detect an open circuit would be to open the connection at one terminal of the delta, such as *A* in Fig. 240, and connect a test circuit across the open. If the winding is open no current will flow. The phase with the open in may be located by testing across each phase separately. If a lamp is used to make the test, the defective phase will be indicated by failure of the lamp to

light. After the faulty phase has been located, the location of the defect can be determined as for the star connection, Fig. 245. There are all manners of parallel star and other groupings in which it is difficult to locate an open circuit, since an open in one parallel group does not open the circuit through the phase, but in only one of the parallel groups. For example, in Fig. 246 an open phase C at X will not open the phase between terminals B and C, but only through C''. Therefore, to detect the open group it will be necessary to break the winding up into its parallel

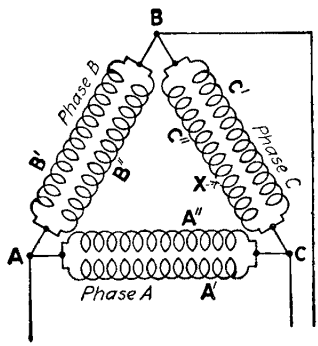


FIG. 246.

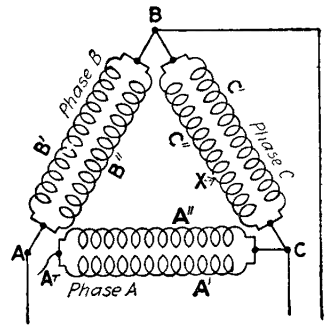


FIG. 247.

FIG. 246 and 247.—Locating open circuits in parallel delta connection.

groups and test each group separately. The defective phase could be detected by the balance test as previously described. First, open the delta connection, for example, at A, Fig. 247, then apply the low-voltage alternating current between points A and B and measure the current with an ammeter, test between A and B, B and C and between C and A_T. The phase with the open circuit, which in this case is C, will show a lower reading than the other two phases, after which all that is necessary is to break the phase up into its parallel groups and test the defective group for opens, as explained in Fig. 245.

Usual Order of Locating Defects.—These are the defects that commonly occur and the usual method of locating them. In checking for these defects the order usually observed is as follows: After the winder has completed the connection of the entire winding, his work is checked, preferably by a second winder, against the winding diagram specified for that particular job. The coils per group are counted and a visual inspection made for short circuits, open circuits and reversed coils, groups or phases.

A balance test is made on the stator alone with low voltage to see if, roughly, the same current flows in the various phases. A high-voltage test is then made on the insulation to insure that the coils are not grounded on the iron core, or that there is no short circuit between the conductors of the different phases. If everything is satisfactory up to this point, the rotor is then assembled in the stator and the machine prepared for a running test. The resistance of the winding is measured on all phases, and if alike, the machine is passed for running test without load. Sufficient voltage is applied to start the rotor, and if it comes up to speed quickly without apparent distress or irregularity of any kind, the speed is checked, to verify whether the winding has the proper number of poles. The temperature of the winding is then tested with the hand, passing completely around the machine and using care that the rotating member and its parts do not strike the observer. If neither general heating nor hot spots are observed, the voltage is raised to normal and the no-load current in all phases and the total watts are read. If these values check with the previous tests on similar machines or with calculations, the windings are considered to be correctly connected. If the motor does not readily come up to speed or the phases do not balance or there are signs of unequal heating in the winding or other distress, the rotor is removed and the connections again checked. If the error is still not apparent and a source of direct current is available, the compass test may be applied. Having exhausted this resource without avail, the problem is one that can be solved only by some expedient at the command of an experienced designing engineer, but such appeals are very seldom required, as the trouble usually appears from the simple tests described.

CHAPTER XV

HOW TO FIGURE A NEW WINDING FOR AN OLD CORE

It is felt by the author that this book is not quite complete without giving some idea of how a winding may be figured for a given core without reference to any winding that might previously have been on the core, but simply with a view to getting a given horsepower out of it at a given voltage, speed, phase and frequency. Obviously in a chapter with the limited space assigned to this one, there cannot be attempted a complete treatise on the design of induction motors with detailed methods of calculation which will make him who reads a finished designer. There are many excellent books on this subject and a few which are so written as to be useful to the practical man in his work, If the foregoing chapters have aroused sufficient interest in the general matter of design, some of these longer works can be consulted for an exhaustive treatment of the entire subject The author feels, however, after personal knowledge of many cases of windings roughly estimated by practical winders which performed satisfactorily, that an approximate idea of what is required in a winding to do a certain job can be had without involving so great a mass of calculation that errors creep in through the volume of figures alone, or without an advanced theoretical training in all the phenomena of alternating currents which are involved in the operation of induction motors. It should be understood that with the short-cut methods and the abbreviated consideration herewith presented, it is not intended or expected that anyone will produce finished and elegant designs; but it is believed that in an emergency, when time is of the essence of the consideration and some chances can cheerfully be taken, the method presented will give an approximation to the correct winding which will be satisfactorily operative in a high percentage of cases. If the writer is checked by his peers, the designing engineers, he should like to have it understood that he is not attempting to tell all the experience he has accumulated or to elaborate a new system of design calculation,

but he is attempting to tell our friends, whose concern it is to make motors run and keep them running, what they may do to help themselves when all these designing engineers are a thousand miles away and the job has to be running next week. Therefore in this discussion while reference is made to all the points considered by the designing engineer, only those points are covered in detail which it is felt are the most vital and these are handled in as elementary a manner as possible.

Effect of the Winding on the Performance.—The performance of an induction motor is made up of a number of different things. It must be able to start its load without drawing from the supply circuit an abnormal amount of current. It must be able to carry its load, as long as it runs, with a reasonable temperature rise and at a reasonable power factor.

It must have a good efficiency, that is to say it must not draw from the supply circuit an amount of energy greatly in excess of that represented by the work being done. It must have as much mechanical clearance as possible between the stationary and rotating members so as to increase the life of the bearings. It must have a momentary overload capacity of from one and one half to two times normal full-load torque without "pulling out" or stalling. And it must have all these things without an appreciable amount of noise due to magnetism or windage. Some of these characteristics may be favored at the expense of others as, for example, it is possible to get a high power factor at the expense of having a very small clearance between stator and rotor, or it is possible to have a high efficiency at a cost of low starting torque and high starting current. For this reason in selling motors the selling talk is often confined to those points which are high in that particular design and the corresponding points of disadvantage are dwelt upon lightly; but to get a true comparison of the relative merits of two competitive ratings or designs all these points must be considered and given their due weight in view of the service in which it is intended to use the motor.

It is understood that all these characteristics are affected in various ways by the different features of the design, that is to say by the axial length of the iron core as compared to the rotor diameter, or by the number of slots, or the kind and thickness of the laminated steel used and matters of this kind; but the thing

which has the greatest effect and which can most easily be modified is the number of turns in the stator or primary winding. In figuring this detail, which is of prime importance, it is therefore wise to have at all times a mental picture of what happens to each characteristic when the cross section of the conductors or the number of turns in the primary winding is changed. In order to summarize this quickly the various characteristics are listed in order and considered separately. The main considerations in the operation of any induction motor are: starting torque, starting current, air gap or clearance, power factor, efficiency, heating, maximum torque, or pull out, noise, and mechanical vibration.

If there were two motors which were exact duplicates in materials and all mechanical dimensions, except that one motor had more turns in the winding than the other, when comparing the characteristics just named, the motor having the most turns would have a lower starting torque and a lower starting current. It would probably have a higher power factor. It might have a higher or a lower efficiency for the reason that the copper loss would be higher and the iron loss lower and whichever one preponderated would determine whether the efficiency was higher or lower, in other words, whether the copper loss increased faster than the iron loss decreased and vice versa. Similarly the heating would be more or less, depending on the sum of the losses. In general this motor would be a little more quiet and have less tendency toward mechanical vibration.

On the other hand, considering the motor with the fewer number of turns, it will have relatively a higher starting torque and a higher starting current. It will probably have a lower power factor. It will have a higher or lower efficiency depending on the proportion of iron to copper loss, as explained in the preceding paragraph; similarly, the heating will vary with the amount of total losses. This motor would have a tendency to be noisier and have more mechanical vibration.

It will be noted that these changes are the same as would occur if the voltage were raised or lowered on any motor. Increasing the number of turns in a winding has the same effect as lowering the voltage and decreasing the number of turns has the same effect as raising the voltage on the winding. This can be seen from Fig. 248 where three windings are shown across 100 volts in

parallel. Winding *A* has eight turns in series and there are 12½ volts effective on each turn. Winding *B* has 10 turns and there are 10 volts effective on each turn; similarly winding *C* has 12 turns and the effective voltage on each turn is 8⅓ volts. Since the performance of the motor as regards torque and other characteristics is proportional to the voltage per turn in the winding, the *A* or 8 turn winding will operate as if on overvoltage and the *C* or 12 turn winding will operate as if on undervoltage. Expressing this another way, if we consider the *B* winding as the normal winding for 100 volts, the *A* winding on 100 volts would operate and give the same result as the *B* winding if there were 125 volts applied to the *B* winding, and similarly the *C* winding

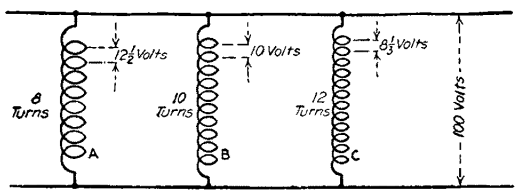


FIG. 248.—The voltage per turn or "transformer volts" on a winding.

on 100 volts would operate and give the same result as would the *B* winding if the *B* winding had 83⅓ volts applied to it. From this it may be seen that perhaps the most essential thing to determine in figuring a winding is the proper number of turns in series in the stator winding which will be put across the line voltage. Another vital consideration is the cross section of the copper wire or conductor used in the winding, necessary to carry the amperes required to develop the desired horsepower. In order to get an idea of all the points which have to be considered in making the complete design of an induction motor a brief enumeration is here given of the different items considered by the designing engineer with a brief statement of how and why each is taken in to account.

1. Diameter and length of laminated iron core necessary to get the horsepower desired at the given speed and voltage.

2. Magnetic flux or field required to generate the line voltage.

3. Number of turns of wire in series in the stator winding which, when cut by the rotating field, will generate the line voltage.

4. Cross section of stator conductor to carry the current required to develop desired horsepower at the power factor and efficiency that the design will probably give.

5. Number and size of stator slots, width and depth, to accommodate winding (3) and (4) when insulated for the required voltage.

6. Magnetic densities in the stator teeth, core, rotor teeth, core and air gap due to magnetic field (2).

7. Magnetizing or no-load current required to set up the field mentioned in (2) with the number of turns in (3) with lengths of path required by (1) and (5).

8. Iron loss due to densities (6).

9. Iron loss due to primary slot openings.

10. Number and size of slots in rotor.

11. Is rotor winding squirrel cage or phase wound?

12. Figure rotor volts and amps. if phase wound.

13. Figure "slip" or rotor copper loss.

14. Figure stator copper loss.

15. Estimate bearing friction and windage.

16. Figure leakage reactance for stator and rotor slots and coil ends, also zigzag, and belt, or differential leakage.

17. From (7) and (16) figure power factor.

18. From (13) and (16) figure starting and maximum torque.

19. From output and (8), (9), (13), (14) and (15) figure efficiency.

Since the consideration for the moment assumes an old core which already exists, many of these things are already determined and some can be assumed. The facts that require checking in determining a new winding for new conditions of speed or horsepower or voltage or phase or frequency, and which may be considered as fundamental are:

1. Is the core large enough to wind for the horsepower and speed that are desired?

2. Is there cross section of iron enough below the slots to carry the magnetic field that is needed in the air gap to do the work desired?

3. How many turns are required in the stator winding?

4. What should be the cross section or size of the wire or conductor used in the stator winding?

5. What should be the cross section of the bars in the rotor and what should be the cross section of the resistance rings at the ends of the rotor bars, assuming a squirrel-cage rotor winding?

6. Will the rotor diameter permit operating at the proposed r.p.m.?

These are comparatively few questions that can be readily answered and broad general limits laid down against which the individual case can be checked. This will assume some points but in general if the winding falls within these limits the motor will be sufficiently operative to fill the immediate requirement.

Proceeding at once to the determination of these quantities, (1) is answered by checking the so-called "output coefficient," that is to say, the horsepower of which a given core is capable at a given r.p.m. This may be expressed by the formula:

$$hp. = K \times D^2 \times L \times r.p.m.$$

where K is the so-called "output coefficient," which varies somewhat with the size and speed of the motor and the operating voltage, D = diameter of the stator bore in inches, L = axial length of the laminated iron core in inches measured parallel to the shaft and *r.p.m.* = revolutions per minute. Suitable values for this output coefficient may be found in several textbooks.[1]

TABLE XII.—OUTPUT COEFFICIENT VALUES

Pole pitch in inches	Values of output coefficient, K, when output is expressed in horsepower, linear dimensions in inches, and speed in r.p.m.					
	4 pole	8 pole	12 pole	16 pole	20 pole	24 pole
5	.....	0.000025	0.0000265	0.0000263	0.0000254	0.0000246
7	0.0000222	0.0000329	0.0000331	0.0000331	0.0000331	0.0000331
10	0.0000336	0.000039	0.0000394	0.0000394	0.0000394	0.0000394
12	0.0000392	0.0000436	0.0000438	0.0000440	0.0000443	0.0000443
16	0.0000434	0.0000482	0.0000484	0.0000486		
20	0.0000454	0.0000505				

The following example is given to illustrate the use of this table. A stator core having a bore of 17 inches and an axial length of 6 inches was brought into a repair shop and a request

[1] Perhaps the most convenient reference is to the "Standard Handbook for Electrical Engineers," 6th ed., McGraw-Hill Book Company, Inc. The table given in Sec. 7, paragraph 246, of the fourth edition is reproduced herewith.

made to put in a winding for 50 hp. at about 730 r.p.m. on 25 cycles. To determine whether it was physically possible the following calculation was made.

$$\textit{Pole pitch in inches} = \frac{\textit{Diameter} \times 3.14}{\textit{Number of poles}} = \frac{17 \times 3.14}{4} = 13\ 4.$$

The nearest figure to this in the table above is 12 inches and opposite 12 inches under 4 poles is the figure 0.0000392. Then

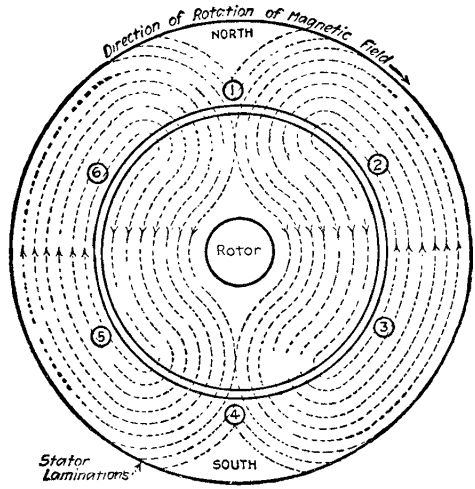


FIG. 249.—Cross section of two-pole motor showing distribution of magnetic field.

the horsepower that this core will develop at 730 r.p.m. is given by the equation:

$$hp. = 0.0000392 \times 17^2 \times 6 \times 730 = 49.6.$$

Hence the conclusion is reached that this core would wind satisfactorily for 50 hp. at 730 r.p.m. since the output coefficient for 13.4 inches would be a little greater than for 12 inches in the table which was used in the trial calculation.

The second question as to whether there is sufficient cross section of iron in the core between the bottom of the slots and the outside periphery can be determined by figuring the actual amount of magnetic flux per pole that must be set up to do the required work. This can be readily understood by a reference to Figs. 249 and 250, which illustrate the manner in which the magnetic flux is divided into as many groups or circuits as

the motor has poles. In passing from the stator to the rotor through the teeth, then behind the rotor slots and back to the stator and again behind the stator slots to the starting point, it will be noted that there must be enough iron behind the slots to carry the flux or the motor will overheat. Referring again to Fig. 250 it is evident that the more poles the motor has, the less iron is required in the core behind the slots of both stator and rotor. Therefore, the correct way to determine this point is to

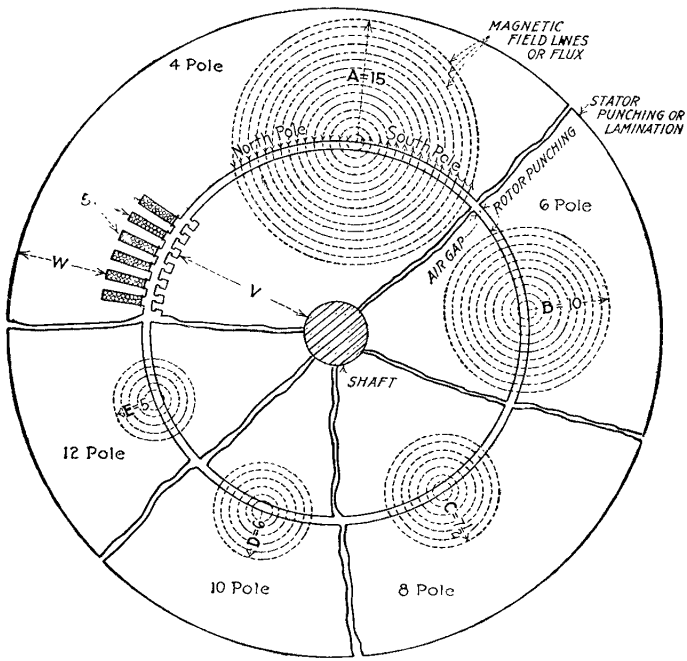


FIG. 250.—Core section, showing effect on magnetic field by changing number of poles.

figure the amount of magnetic flux per pole and the cross section of the core behind the slots and see that there are not more than 80,000 to 100,000 magnetic lines per square inch; if this condition is satisfied, and if other conditions are proper, the core should be satisfactory for the assumed conditions of the winding. Here we are confronted with a peculiar problem which often faces the designer; he must know part of his answer before he can solve the problem and find the rest of it. In other words the amount of magnetic flux in the core will depend on the number of turns of wire in the coils and the problem which he is trying to solve is

how many turns should be put in the coils. So it is apparent that he must either guess the number of turns required and find out if the amount of magnetic flux is reasonable or else he must figure how much flux can be carried in the core he is using and from that figure check back and see how many turns are required in the winding to give this magnetic result. When the number of turns is settled and the cross section of the copper is figured for the desired horsepower and voltage, there is at once a question whether the slots will accommodate that many conductors of that cross section after taking room enough to allow for the insulation required on the coil at that particular voltage. If the result is unfavorable and the copper so figured will not go in the slot at all, it means that the motor is not good for that much horsepower and the desired rating will have to be reduced. The number of turns cannot readily be reduced as that would mean more magnetic flux and the core back of the slots is already figured for 80,000 to 100,000 lines per square inch which is all it will stand. The reason why the number of conductors and the magnetic flux are tied in together in this way is because the conductors which are in series, when cut by the rotating magnetic field must generate or produce practically line voltage. This fact has been referred to many times in previous chapters.

The formula for the field flux per pole or per magnetic circuit is

$$\textit{Flux per pole} = \frac{45{,}000{,}000 \times \textit{volts per phase}}{\textit{Cycles} \times \textit{conductors per phase} \times K_1 \times K_2}$$

where volts per phase = line volts in the case of a two-phase winding or a delta-connected three-phase winding, or $\frac{\text{line volts}}{1.73}$ in the case of a star-connected three-phase winding

cycles = the frequency of the supply circuit as expressed in cycles, that is, 60 or 25 or whatever the circuit may be.

conductors per phase = number of wires per slot which are in series × number of slots ÷ number of phases.

K_1 = so-called "distribution factor"; is 0.905 for two-phase and 0.955 for three-phase.

K_2 = the so-called "chord factor"; depends on the pitch or throw of the coil. Its technical value is the sine of one-half of the electrical angle spanned by the coil.

A practical method of getting this factor which is close enough for general purposes is to use the expression

$$\textit{Chord factor} = K_2 = \sqrt{\frac{(\textit{Number of slots per pole})^2 - 2(\textit{number of slots dropped})^2}{(\textit{Number of slots per pole})^2}}$$

or taking a concrete example: suppose there is a 72-slot motor wound for six poles and having a coil throw of 1 and 8, what is the chord factor or K_2, which is under discussion? Since there are 72 slots and six poles there are 12 slots per pole and full pitch would be slots 1 and 13. Winding 1 and 8 drops 5 slots and thus our formula above becomes the square root of twelve squared minus two times five squared divided by twelve squared, or mathematically

$$\sqrt{\frac{12^2 - 2(5)^2}{12^2}} = \sqrt{\frac{144 - 50}{144}} = \sqrt{\frac{94}{144}} = 0.80$$

A comment has been made by readers that the formula for flux per pole apparently takes no account of the number of poles and hence gives the appearance that the flux per pole would be the same regardless of the number of poles. This is obviously not the case since other terms could be substituted for cycles from the formula:

$$\textit{Cycles} = \frac{\textit{Poles} \times \textit{r.p.m.}}{120}$$

so that the expression for flux per pole could be rewritten in the form:

$$\textit{Flux per pole} = \frac{45{,}000{,}000 \times \textit{volts per phase} \times 120}{\textit{Number of poles} \times \textit{r.p.m.} \times \textit{conductors per phase} \times \textit{chord factor} \times \textit{distribution factor}}$$

In this form it can readily be seen that the number of poles appears but it is more convenient to use the expression containing cycles.

To illustrate how this flux formula is applied, assume a core having dimensions as shown in Fig. 251 which it is desired to wind for 50 hp., 25 cycles, three-phase, four poles, 440 volts and 730 r.p.m. full-load speed. The outside diameter O of the stator laminations = $25\frac{1}{2}$ inches, the inside bore D of the stator

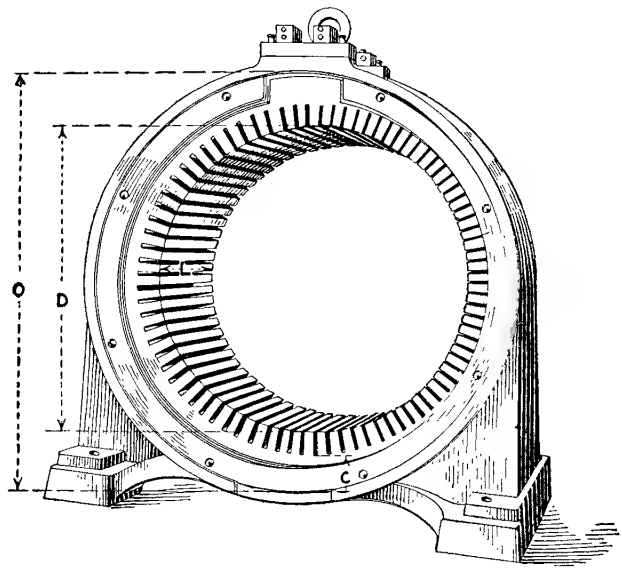


FIG. 251.—Stator core in frame.

laminations = 17 inches. The axial length of the core $L = 6\frac{3}{4}$ inches but it contains two ventilating ducts each $\frac{3}{8}$ inch wide so that the net iron core length = 6 inches. The primary slots are 1.7 inches deep, so that the dimension C or the radial depth of the laminations below the slots = $(25\frac{1}{2} - 17) \div 2 - 1.7 = 2.55$ inches and the actual cross section of the core below slots through which all of the flux per pole must pass is equal to $C \times L$ or in this case $2.55 \times 6 = 15.3$ square inches. A reference to Fig. 250

NOTE.—Do not figure the new winding from the core density alone, but check the density in the teeth also, as cautioned on page 275, since the density in the teeth is frequently the limiting factor.

indicates that when the flux per pole passes from the rotor into the stator, it divides and half goes one way and half the other

way. Hence in the present case the total available cross section of iron to carry the flux per pole is not 15.3 square inches, but twice that or 30.6 square inches. As stated above 80,000 lines per square inch is a permissible density, so that a total flux per pole of 30.6 × 80,000 can be used or 2,448,000 lines. The only other factor missing from the flux per pole formula which is necessary to give at once the total number of conductors per phase is the chord factor. This depends upon the slots in which the two sides of any coil are placed. In the core which is under consideration there are 48 slots and since a four-pole winding is under calculation, the full pitch for the winding would be slots 1 and 13. Full pitch is too long mechanically and some space endwise can be saved and some copper as well by chording it a few slots, so for illustration it is assumed that the coils lie in slots 1 and 10. Using the approximate formula given for chord factor above, this factor becomes

$$\sqrt{\frac{12^2 - 2(3)^2}{12^2}} = \sqrt{\frac{144 - 18}{144}} = \sqrt{0.0875} = 0.93$$

Expressing the flux per pole formula in terms of conductors per phase, this expression follows:

$$\textit{Conductors per phase} = \frac{45{,}000{,}000 \times \textit{volts per phase}}{\textit{Cycles} \times \textit{flux per pole} \times \textit{chord factor} \times \textit{distribution factor}}$$

Remembering that the distribution factor for three-phase equals 0.955 and substituting the values calculated above, and assuming a delta-connected winding,

$$\textit{Conductors per phase} = \frac{45{,}000{,}000 \times 440}{25 \times 2{,}448{,}000 \times 0.93 \times .955} = 362$$

Since there are three phases, there will be required a total number of conductors 3 × 362 = 1086 and since there are 48 slots there will be 1086 ÷ 48 = 22.6 conductors per slot. What a designer would do in this case would be to either wind 22 conductors per slot and throw the coil 1 and 11 instead of 1 and 10 or else wind it 24 per slot and throw the coil 1 and 9, either of which would be a good winding without much difference between the two.

The reason for this is that there are two coils per slot and hence with 22 wires per slot there would be 11 wires in each coil. As the wires are arranged in two or three layers, 11 would not be exactly divisible by either two or three, hence, in the case of a two-layer coil there would be one layer of five wires and one layer of six wires side by side, or in the case of a three-layer coil there would be two layers of four wires each and one layer of three wires. Either of these arrangements would be wasteful of space and hence it would be preferable to have 12 wires per coil which is evenly divisible by either two or three. If the coil is wound in slots 1 and 11 the chord factor is 0.97 and if it is wound in 1 and 9 the chord factor is 0.866. Hence, the real, effective number of wires in one case is $22 \times 0.97 = 21.3$ and in the other case is $24 \times 0.866 = 20.78$, which would give very close to the same result so far as torques are concerned.

In this calculation it was noted that the figure 440 was used for the voltage. This assumed a series delta connection. If, for example it had been desired to connect the winding in two-parallel delta for the same voltage, there would have been required twice as many conductors per phase and each conductor would have had one-half the cross section, since there would be two paths in parallel for the current instead of one in series. Similarly, if the winding was to have been connected in series star instead of series delta the voltage used in the equation would have been $440 \div 1.73 = 254$ instead of 440. Hence, in the result, the conductors required per phase would have been $362/1.73 = 209$ instead of 362. It is well to remember this fact: that with a star connection only about one-half as many turns are required in series as with a delta connection. It sometimes makes an easier coil to wind and a coil which is mechanically stiffer and stronger, if less turns of a larger size wire can be used. This is one of the principal reasons why a star connection is used much more frequently than a delta connection.

Having found the number of conductors per slot from the above equation there would seem to be nothing more to do but figure the required cross section of the conductor to carry the full-load current, and the space required for insulation and see if the coil so figured and insulated would go into the slot. There is a check calculation that should be made first to see how hard the iron

is working in the stator teeth. The calculation that was made concerned itself only with the density of the magnetic flux in the stator core behind the slots and was checked first to make sure the required field had room to get through the core. However, before accepting this figure the teeth should be checked also to see how hard they are working. This is a simple check from the figures already employed. The diameter of the stator bore of the core under calculation is 17 inches. The depth of the slots is 1.7 inches; therefore the diameter to the middle of the slot = 18.7 inches and the slot pitch at this point or the dimension P from Fig. 251*a*.

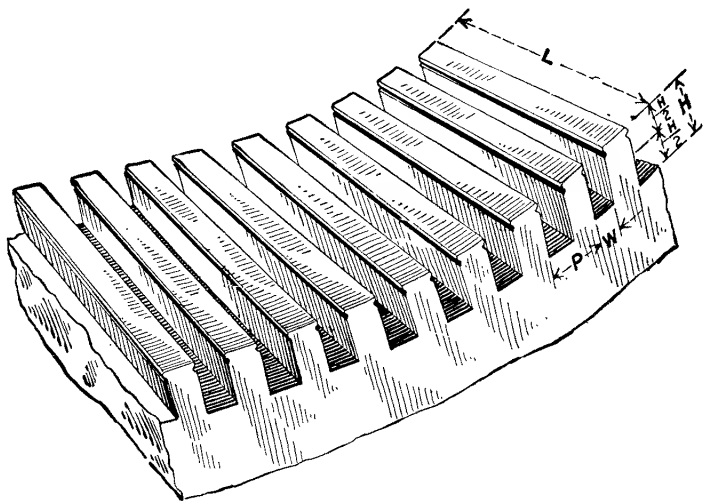


FIG. 251*a*.—Section of stator core.

$$C = \frac{18. \times 3.14}{48} = 1.22 \text{ inches.}$$

The slot width $W = 0.65$ inch. Hence the tooth width $P - W = 1.22 - 0.65 = 0.57$ inch and, since the net core length $L = 6$ inches, the cross section of one tooth at its mid-section $= 6 \times 0.57 = 3.42$ square inches. There are 48 teeth total and four poles, hence there are 12 teeth per pole through which the magnetic flux of one pole may pass. Therefore, the total iron cross section of 12 teeth $= 12 \times 3.42 = 41.04$ square

inches. It was calculated above that there were 2,448,000 magnetic lines per pole and it would seem that all that was necessary to check the tooth density would be to divide this figure by 41.04. This is not the case as in the core for the reason that all the teeth do not carry the flux equally but those in the center of a pole at a given section carry a maximum and those half way between poles carry nothing so that in order to take care of the maximum, the result above is divided by 0.636. Hence in the problem in hand the maximum density in the teeth is $\frac{2{,}448{,}000}{41.04 \times 0.636} = 94{,}000$ lines per square inch. As a matter of fact it is actually about 96,000 lines since the 2,448,000 was figured with 362 conductors per phase and a throw of one and ten, whereas there are now $24 \times 48 \div 3 = 384$ conductors per phase, but the throw is only one and nine and substituting back in the original flux equation,

$$\textit{Flux per pole} = \frac{45{,}000{,}000 \times 440}{25 \times 384 \times 0.955 \times 0.866} = 2{,}480{,}000 \text{ lines.}$$

This value, namely 96,000 for density in the teeth, is perfectly permissible. It should not be allowed to exceed, say, 130,000 for 25-cycle machines, or about 110,000 for 60-cycle machines.

Calculating the Cross Section of the Stator Conductor.—Having determined the number of conductors required in the slot, that is 24, the next step is to figure the necessary size of the conductor or cross section and see if the coils will go in the slot after being properly insulated. In order to figure this it is necessary to know what the full-load current of the motor will be. The formula for finding the full-load current of a two-phase motor is,

$$\textit{Full-load current per lead} = \frac{\textit{hp.} \times 746}{2 \times \textit{volts per phase} \times \textit{efficiency} \times \textit{power factor}}$$

Where the efficiency and the power factor are the full-load values and are expressed in hundredths, that is with a decimal point in front of each. For example 90 per cent is written 0.90 and 85 per cent power factor is written 0.85 etc. For a three-phase motor the formula changes to,

Full-load current per lead =

$$\frac{hp. \times 746}{1.73 \times volts\ per\ phase \times efficiency \times power\ factor}$$

which it will be noted is similar to the two-phase formula except 1.73 is used in the denominator instead of 2. One thing must be specially noted about the three-phase and that is that the full-load current so found is the current in the outside motor lead or the current drawn from the line. If the motor is star connected inside this same current flows in the motor winding itself and hence in the conductors in the slots, unless the winding is in two or more parallels, in which case, of course, the lead or line current splits up into as many parts as there are parallel paths. On the other hand if the windings inside the motor are delta connected as they are in the case we are considering, the current in the windings will be less than the current coming in the lead as figured above and it is necessary to divide by 1.73 a second time to find out what the current is, which must actually be provided for in the coils themselves.

Preparing to apply the above formula, at once the problem arises, What is the full-load efficiency and the full-load power factor of the motor for which this winding is being figured? Of course there is a wide variation in these figures between small and large motors, and between high and low speeds, and between 25 and 60 cycles and these variations are shown as well as may be in the "Standard Handbook for Electrical Engineers," referred to previously. For the purpose here, which, as has been stated, is somewhat rough and ready, an approximation must be assumed. The handiest approximation the author has ever used and one that has given good results is to assume that a three-phase, 550-volt motor, draws from the line in each lead just about one ampere per horsepower. This is very closely true in most lines of commercial motors over a wide range of sizes and speeds. Then if the motor in question is not a three-phase motor or if it is not 550 volts, the current can readily be changed to other voltages. For example assume a 40-hp. motor. Then at 550 volts three phases it follows that its full-load current per lead is 40 amperes, at 440 volts its full-load current would be $\frac{550}{440} \times 40 = 50$ amperes and

at 220 volts it would be $\frac{550}{220} \times 40 = 100$ amperes and at 110 volts it would be $\frac{550}{110} \times 40 = 200$ amperes and so on. Similarly to convert to two phase multiply these values by 1.73/2.00 = 0.86 because the current of any two-phase motor is always that much less than the corresponding three-phase motor.

Referring again to the formula above for the full-load current of a three-phase motor, to give 1 ampere per horsepower at 550 volts would mean that the product of the efficiency and power factor would be 0.785. This might be assumed to be 89 per cent efficiency and 88 per cent power factor or any other combination whose product gave 0.785. At all events this is an average value and sufficiently nearly correct for the present purpose.

Since the present calculation assumes a 50-hp. three-phase 440-volt rating, it may be assumed that the full-load current per lead is $\frac{50 \times 550}{440} = 62.5$ amperes. Since the winding is to be delta connected the current in the coils themselves will be 62.5/1.73 = 36.1 amperes. There is no fixed rule that can be followed for the cross section of copper required in the coil per ampere. It may be as low as 400 circular mils in some cases and may have to be as high as 1000 circular mils in others. Slow-speed motors and higher voltages (where there is more insulation to pass the heat through) require larger copper than do higher speeds and lower voltages. In the present case and in most average cases a figure of 750 circular mils can be used. For the present case then the circular mils required would be $36.1 \times 750 = 27{,}075$ circular mils. Looking in a Brown and Sharpe wire table, the nearest size to this is No. 6 round wire which shows 26,250 circular mils. This is near enough and it is selected. The problem now is, will 24 No. 6 wires go in a slot 0.65 inch wide by 1.70 inches deep and allow for the retaining wedge at the top and the proper insulation for 440 volts? To answer this it is necessary to know something about insulation requirements. As there are commonly only two voltage classes met with, it can be stated that voltages up to and including 550 volts will require a space in the width of the slot of about 0.1 inch and in the depth of the slot of about 0.15 inch and volt-

ages above 550 up to and including 2200 will require about 0.16 inch in width and 0.26 inch in depth. These figures in depth do not include any retaining wedges or so-called "top sticks," but must be allowed in addition to the wires between the bottom of the wedge, and the bottom of the slot. In the case just being figured the wires will evidently go in better 3×8 than any other way. The diameter of No. 6 round wire over double cotton covering is 0.178 inch. Three wires in width would be $3 \times 0.178 = 0.534$ inch. Adding 0.1 inch for insulation gives $0.534 + 0.1 = 0.634$, which goes very well in the width of the slot, which is 0.65 inch. In depth eight wires would require 8×0.178 inches $= 1.424$. The allowance for insulation is 0.150 inch and the usual coil-retaining wedge requires 0.125 inch so that the total required depth will be $1.424 + 0.150 + 0.125 = 1.699$ inches which just exactly fills the available depth. It should be understood that the 24 wires are not 3×8 in one coil but 3×4 in each coil and two coils in the slot according to the usual practice. If the wires had not fitted in the slot as shown it would have been necessary to choose a wire small enough to go in the space and then by trial after the winding was complete find out how many horsepower the winding would carry without overheating. If it were not possible to get 50 hp., it would probably develop 45 hp. without trouble if the output coefficient checked to 50 as shown in the beginning of this chapter.

With regard to the rotor winding if it is of the wound rotor type the number of wires per slot can be made any number that is convenient, provided the total weight of copper in the rotor winding is made approximately 80 per cent to 85 per cent of that in the complete stator winding.

Voltage between Collector Rings.—In the case of a wound rotor motor it is often useful to know the voltage at standstill between the rotor collector rings in order to determine how much resistance should be used in the starting or speed regulating controller. This may be determined very closely from the formula:

$$\textit{Volts between collector rings} = \frac{E_1 \times W_2 \times K_2}{K_1 \times W_1 \times K_3}$$

where E_1 = line voltage applied to the stator.
W_1 = number of conductors in series per phase in the stator.
W_2 = number of conductors in series per phase in the rotor.
K_1 = 1 if stator winding is two-phase or three-phase delta.
K_1 = 1.73 if stator winding is three-phase star.
K_2 = 1 if rotor is connected delta.
K_2 = 1.73 if rotor is connected star.
K_3 = chord factor of the stator coils as explained in Chapter VI.

The number of conductors in series per phase in either stator or rotor is equal to the total number of slots multiplied by the number of wires in each slot, divided by the number of phases and divided by the number of parallels in which the winding diagram shows the winding to be connected.

For example, what is the voltage between collector rings on a wound rotor motor with the following data? The line voltage is 220. There are 72 slots in the stator and 10 wires per slot. The stator winding is three-phase, two-parallel star, six-pole and the coil throw is slots 1 and 9. There are 54 slots in the rotor, two conductors per slot and the rotor winding is connected series star.

Setting down the data for use in the formula given above, $E_1 = 220$, $W_1 = \frac{72 \times 10}{3 \times 2} = 120$, $W_2 = \frac{54 \times 2}{3} = 36$, $K_1 = 1.73$, $K_2 = 1.73$, K_3 = primary chord factor = sine of 60 deg. = 0.866, because 72/6 = 12 slots = 180 deg. and one slot = 15 deg. Hence, a throw of 1 and 9 spans 8 slots or $8 \times 15 = 120$ deg. and the chord factor = the sine of one-half the angle spanned by the coil = $\frac{1}{2} \times 120$ deg. = 60 deg. = 0.866. Therefore

Volts between collector rings =

$$\frac{E_1 \times W_2 \times K_2}{K_1 \times W_1 \times K_3} = \frac{220 \times 36 \times 1.73}{1.73 \times 120 \times 0.866} = 76 \text{ volts}$$

If they are phase wound, the coils must, of course, be connected for the same number of poles as the stator. If there should be an old winding on the rotor for a different number of poles, it may be possible to reconnect it for the number desired,

but as rotor windings are nearly always of the "wave" type or something of the same order it is usually impossible to reconnect for any other number of poles.

If the rotor winding is squirrel cage, the number of bars and their cross section is probably fixed. The cross section of the end rings if of rolled or drawn copper should be so chosen that the weight of bars plus rings is about 75 per cent to 80 per cent of the total weight of the stator coils. If the rings are cast copper or cast brass it should be remembered that a larger cross section will be required since the conductivity of the best cast copper is only 80 per cent to 85 per cent of the conductivity of rolled copper and cast brass is as low as 18 per cent to 25 per cent of the conductivity of drawn or rolled copper. This would mean that a ring of cast brass would have to be four to five times the cross section of the corresponding rolled copper ring to carry the same current. It should also be remembered that a two-pole motor would have proportionately the heaviest ring on the squirrel cage, a four-pole the next heaviest, then a six-pole, and so on; when 10 poles or 12 poles are reached, the ring would probably be no larger in cross section than would be required for mechanical strength and construction.

Such in its briefest form is the simplest calculation that can be made which it is safe to make in the hope of getting the desired result. It will be noted that no attention has been paid to calculating the leakage reactance, or to the no-load current, the starting and maximum torques, the circle diagram or any of the refinements which the designing engineer commonly employs; and yet if care is used in employing the checks that are made the experimenter should be rewarded with reasonable results.

To sum up, the available core is first checked by the output coefficient to see if it will develop the horsepower at the desired speed. Next a check is made to see how much magnetic field can be handled in the core and teeth. Then the proper number of conductors is chosen to generate the line voltage when acted upon by the permissible magnetic field. These conductors are then made of the proper size to carry the working current and insulated for the working voltage and fitted in the slots. This is all that is attempted and it is assumed that if these conditions are met, all the other conditions of operation will fall reasonably

in line or can be adjusted after trial without too much change to meet the desired requirements.

Naturally, such broad assumptions may not result in a design of finished nicety, but they may sometimes give quick results where results must be had quickly or not at all.

On account of the interest in this chapter in previous editions there has been added to the present edition Chapter XX which gives in complete detail the full calculation for any induction motor.

CHAPTER XVI

CONNECTING TO CORRECT MAGNETIC SIDE PULL

Magnetic Conditions in Motor or Generator.—Broadly speaking, any dynamo-electric machine of the usual types, either alternating or direct current, may be considered as consisting of a magnet in two parts, one of which is stationary and the other movable. Since this is true, there exists between these two members a strong magnetic attraction or pull, and unless the rotating member is accurately centered and rigidly held, there is a tendency for the two members to approach one another and strike, with resulting damage to the machine. It is possible by a proper connection of the windings, on either the stator or the rotor, to assist the symmetry of the magnetic field and minimize any tendency toward this unbalanced side pull with its resulting mechanical vibration, noise and likelihood of damage. This corrective effect is produced by paralleling different parts of the windings or by the use of what are generally known as equalizing connections.

In Fig. 252 is shown a horseshoe magnet having suspended between its poles N and S a bar magnet, which does not quite close the gap between the magnetic poles but leaves an air gap or clearance at the points marked a and b. These gaps represent the clearance between the stationary and the rotating members of a dynamo-electric machine. The magnetic lines of force or flux are represented by the irregular dotted lines. The arrowheads on the flux do not indicate that the magnetic field is in motion, but merely that it is assumed as emanating from the north pole of the main magnet and entering the south pole. It is characteristic of these lines of magnetic force that they act like stretched rubber bands and constantly try to shorten their length while still remaining in the iron. This explains why the keeper of any horseshoe magnet is pulled up tight against the magnet, and also why there is a tendency in a dynamo-electric machine for the armature to pull over against the field, as mentioned in the first paragraph. Any magnetic pull may be con-

sidered as caused by this elasticity of the lines of force tending to shorten their length.

Stable Magnetic Equilibrium.—The system as shown in Fig. 252 is in comparatively stable magnetic equilibrium for the reason that when one of the poles of the main magnet tends to attract the bar magnet, thereby closing up the air gap *a* for instance, there is no change in the reluctance of the magnetic circuit, since the air gap *b*, which is in series with *a* is at the same time increased. Since there is no change in the magnetic reluctance of the circuit, there is no increase in the amount of magnetic

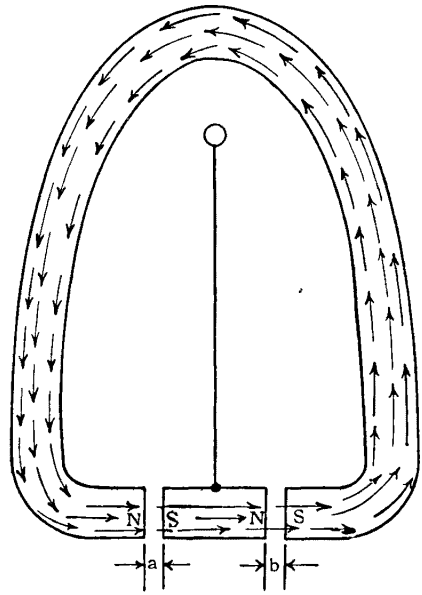


FIG. 252.—Horseshoe magnet with bar magnet suspended between the poles.

flux and, hence the amount of pull on the bar magnet does not increase appreciably as it approaches the one or the other pole of the horseshoe magnet. This condition is similar to that existing in a two-pole induction motor of which a cross section is shown in Fig. 253, showing in its simplest elements the stator laminations, the rotor laminations, and the air gap. The path of the magnetic flux or field is represented roughly by the dotted lines and the arrowheads, showing how this field branches in either half of the machine. If it be assumed in Fig. 253 that the rotor was moved out of center with the stator and consequently approached nearer to the stator bore at one point, there would

be no great increase on the side pull, tending to pull the two members together, for the reason that, as the air gap on one side was decreased, it would be increased diametrically opposite, and hence the total reluctance of the main magnetic path would be substantially unchanged. It is apparent from this that the magnetic condition is analogous to that of Fig. 252. It would be more nearly analogous if the faces of the poles were flat instead of cylindrical as shown in Fig. 254. In this case it is evident that a slight displacement to the right does not materially alter the

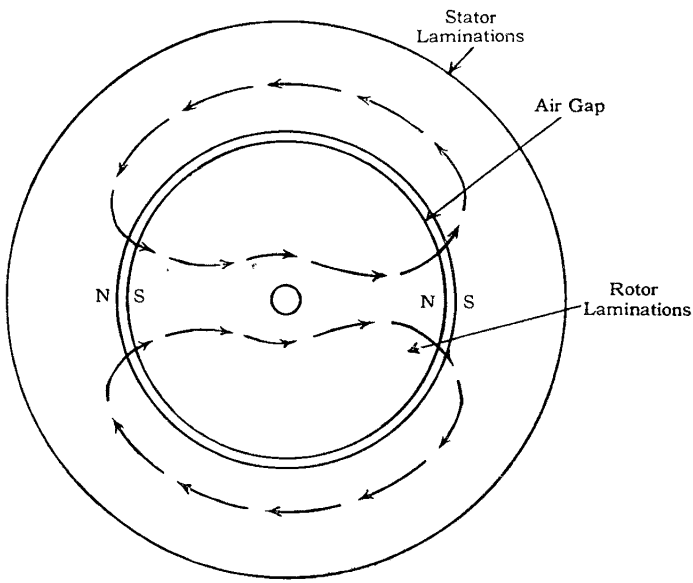


FIG. 253.—Cross section of induction motor with two-pole magnetic circuit.

total air gap, top and bottom, hence there is no appreciable change in the unbalanced magnetic side pull. With the usual form of cylindrical pole face, as shown in Fig. 255, a movement to the right has a tendency to increase the magnetic density under the pole tips *a-a* faster than it decreases under tips *b-b* and due to this localized action some unbalanced side pull may occur even in the case of a two-pole machine.

Unstable Magnetic Equilibrium.—In Fig. 256 is illustrated a different magnetic condition and one of extreme magnetic instability. This represents a bar magnet suspended between the poles of two adjacent horseshoe magnets and forming a keeper for either one of these magnets with which it might

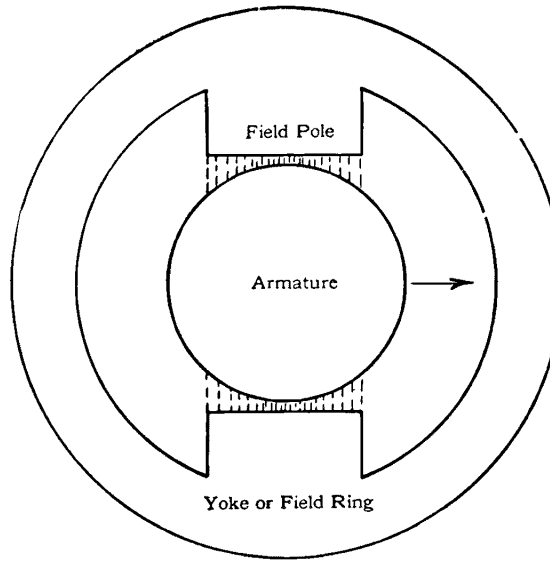


FIG. 254.—Two-pole induction motor with pole faces flat instead of having the normal cylindrical form.

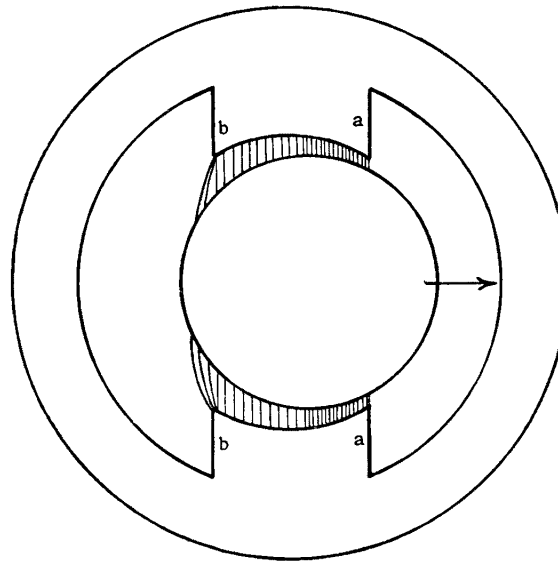


FIG. 255.—Two-pole motor with armature moved horizontally to right. The decrease of air gap at the pole tips *aa* more than offsets the increase at tips *bb* and some unbalanced side-pull results.

come in contact. It is possible that so delicate a magnetic balance might be struck that the bar magnet would not tend to come in contact with either horseshoe magnet, but would remain midway as shown in Fig. 256. However, the slightest change in the physical conditions, such as a breath of air blowing on the suspended magnet, would result in its being immediately and somewhat violently drawn up against one or the other horseshoe magnet. The reason for this is that as the bar magnet approaches one of the horseshoe magnets the pull of this magnet increases very rapidly as the air gap decreases, and, conversely,

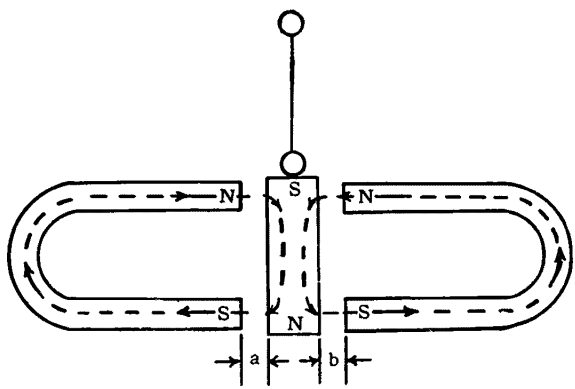


FIG. 256.—Bar magnet suspended between two horseshoe magnets.

the pull of the magnet on the other side restraining this motion decreases very rapidly as its corresponding air gap increases. Hence, as soon as there is any tendency for the very delicate magnetic balance to be destroyed, the suspended bar magnet will be drawn over to one or the other of the horseshoe magnets. This condition is similar to that existing in a four-pole induction motor, of which a cross section is shown in Fig. 257. Here it will be seen that the magnetic flux follows four distinct paths instead of two as shown in Fig. 253, and that it is possible by moving the rotor out of center with the stator to decrease the air gap in connection with one of these paths and to increase it at the same time in the path diametrically opposite. This tendency, if not resisted by great rigidity in the frame, shaft and bearings and by proper connection in the windings, might result in the rotor being drawn over against the stator for the same reason that the bar magnet in Fig. 256 is drawn over against one of the horseshoe magnets.

These six figures illustrate a comparatively stable and an unstable magnetic condition and illustrate how these same conditions may exist in rotating machinery and the necessity for some corrective tendency to preserve the mechanical line up between rotor and stator to insure an even magnetic pull at all points.

Series Connection Does Not Correct Unbalanced Pull.—Figure 258 shows diagrammatically the eighteen pole-phase groups in the primary winding of a three-phase six-pole induction motor. It also shows one of the phases as it would be arranged when connected for series star. Such a winding, aside from the

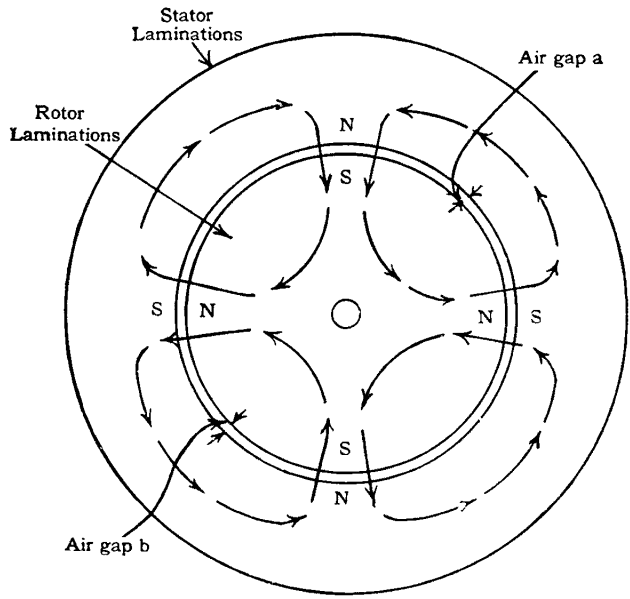


FIG. 257.—Cross section of four-pole induction motor showing four magnetic circuits.

fact that it is arranged symmetrically around the stator, would have no tendency toward correcting an unbalanced side pull due to mechanical inequality in the air gap or due to varying magnetic permeability of the iron. For example, if the rotor should be moved horizontally to the right towards the group marked 5 and 6, there would be an immediate decrease of the reluctance in the magnetic path of the flux lines which completed their circuit through groups 5 and 6. This decrease in reluctance would be followed by a consequent increase in the magnetic flux

in this local circuit and hence an increase in the pull in this same direction which, everything else being equal, would pull the rotor still farther over against the stator at this point. At the same time there would result a corresponding increase in the reluctance of the magnetic circuit through groups 2 and 3 and a consequent weakening of the pull in this direction. Since all these groups are in series and the same magnetizing current flows through them all, it is obvious that there is no chance to decrease the

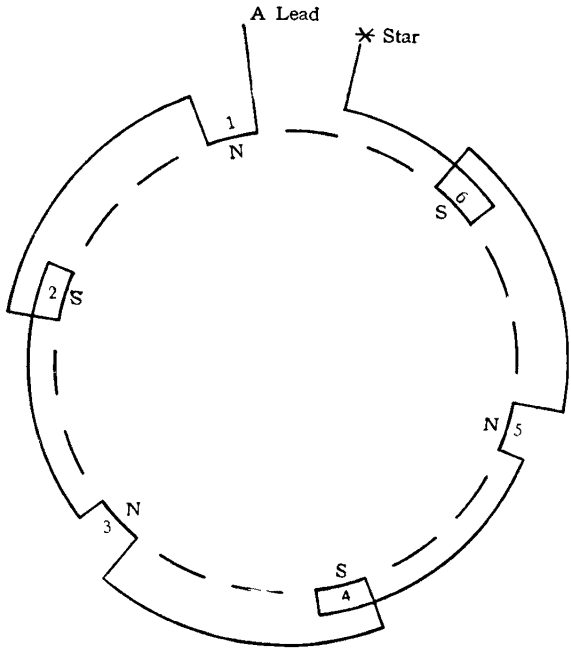


FIG. 258.—Connections for phase *A* of a three-phase, six-pole induction motor connected series star.

current in groups 5 and 6 and increase it in 2 and 3 for the sake of correcting the side pull. If it were possible to do this, the flux could be decreased through 5 and 6 and increased through 2 and 3 and a corrective tendency could be created. However, since they are in series, no adjustment of this kind is possible.

Usual Parallel Connection Helps, if "Top to Top."—Figure 259 shows a schematic diagram of a series star winding with the *A* phase connected as shown in Fig. 258 and corresponding groups similarly numbered from 1 to 6 inclusive. In Fig. 260 is shown a diagram of the pole-phase groups of a three-phase six-pole induction motor arranged to be connected in parallel

star instead of series, as shown in Fig. 258, but in which no particular attention has been paid to the question of magnetic balance and the maximum advantage has not been taken of the opportunity to improve conditions in this regard. A corresponding schematic diagram is shown in Fig. 261. The arrows on the outside of the winding, Fig. 261, marked x, x show the direction of the line or applied voltage and also the direction in which the magnetizing current flows through the winding. The smaller arrows marked y, y show the direction of the generated or

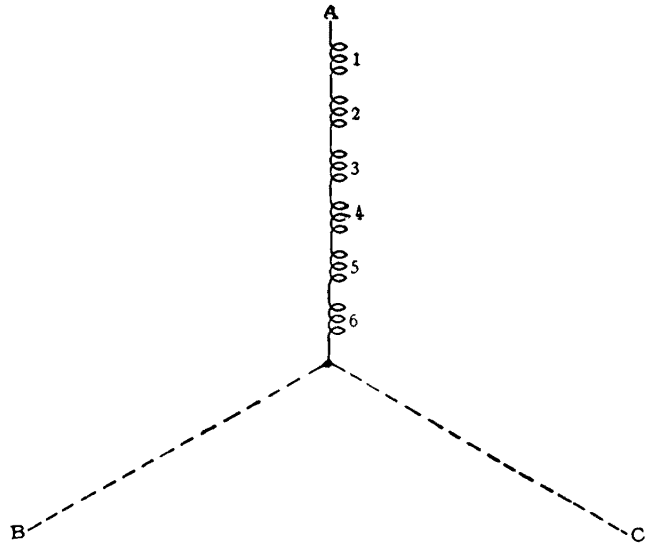


FIG. 259.—Schematic diagram of the winding shown in Fig. 258.

induced counter-electromotive force. To understand this fully, it must be borne in mind that at the same time that any induction motor is acting as a motor and delivering mechanical power, it is also acting as an alternating-current generator and generating a back or counter e.m.f. which is at all times nearly equal to the line or applied e.m.f. The slight difference between this counter e.m.f. and the line voltage, when applied to the ohmic resistance of the windings, is just enough to force the working current through these windings. Hence in studying the phenomenon to be described, it should be remembered that this generated counter e.m.f. is opposite in direction to the applied voltage and also opposite to the direction of flow of the magnetizing current.

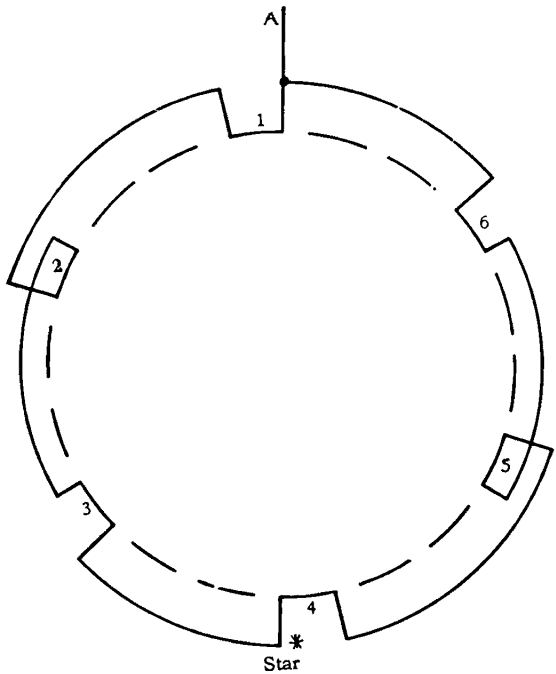


FIG. 260.—Connections for phase *A* of a three-phase, six-pole induction motor connected parallel star. Top to top connection of groups.

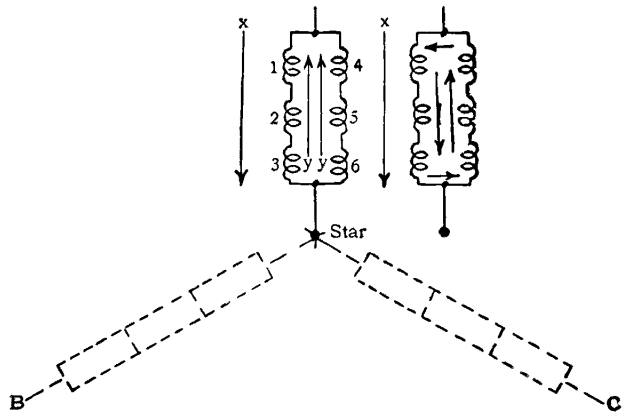


FIG. 261.—Schematic equivalent of the winding in Fig. 260. The arrows outside the figure at *xx* show the direction of the line or applied e.m.f. and also the magnetizing current. The arrows inside the figure at *yy* show the direction of the induced or generated counter-electromotive force in the primary windings.

Assume at this point that the rotor in Fig. 260 is again moved out of center with the stator and approaches the stator bore horizontally to the right, opposite groups 5 and 6. The electrical and magnetic effect in the order of its occurrence is as follows: First, the mechanical movement has decreased the clearance between stator and rotor, hence decreased the air gap and the magnetic reluctance of the local magnetic path around through groups 5 and 6. This decrease in the reluctance of the path results immediately in an increased flux, since the same magnetizing current is assumed to be flowing. This momentary increase in magnetic flux as it rotates past the conductors in groups 5 and 6 of the winding momentarily generates a higher back or counter e.m.f. in these groups, and since this is opposed to the line voltage, it has the effect of reducing the current through groups 5 and 6 and also group 4, which is in series with them. On the opposite side of the machine, as in groups 2 and 3, the opposite result takes place; *i.e.*, there is a momentary decrease of the magnetic flux through groups 2 and 3 and a consequent decrease in the generated e.m.f. and a consequent increase in the current through these groups, since there is momentarily less opposition offered to the line voltage which is forcing the current through these groups. Both of these changes are in the right direction to correct the side movement of the rotor. That is to say, we have momentarily a decrease of the magnetizing current on the strong side and a momentary increase of the magnetizing current on the weak side, which has a tendency to draw the rotor back to its central position and hold it there. While this represents the corrective effect of parallel windings, it is not used to the maximum possible degree, as illustrated in Figs. 265 and 267.

From the foregoing description of the decrease in the current in groups 4, 5 and 6 and the increase in 1, 2 and 3, it will be noted that the same result is accomplished as though a local current circulated in the closed path produced by the two parallels in the winding, and that this local circulating current was in the direction of the counter e.m.f. in groups 4, 5 and 6 and of the line voltage in groups 1, 2 and 3; in other words, that it flowed down through 1, 2 and 3 and up through 4, 5 and 6. This can be understood easily by examining Fig. 261.

Up to this point no attention has been called to the fact that while the magnetizing and the working currents both flow through the windings in the same direction, they do not do so at exactly the same time, or in other words that they are 90 deg. electrically out of phase with one another. It is mentioned here on account of the introduction of the local, corrective, circulating current. Since this corrective current is set up by the difference in the generated counter e.m.f. in the two legs of the parallel winding, it is 90 deg. out of phase with the counter e.m.f. and hence is in line with the main magnetizing current. For this reason it adds directly to the magnetizing current in one leg of the parallel and subtracts directly from it in the other, thus producing exactly the corrective effect desired. The relations of the various voltages and currents are given roughly in Fig. 262. A voltage *A* assumed to be rotating in a counterclockwise direction, is applied to the motor windings. On account of the inductance of these windings, a magnetizing current *I* flows, 90 electrical degress behind the applied e.m.f., which sets up a magnetic field *F* in phase with it. The magnetic field *F* induces or generates in the windings the counter e.m.f. *G* which is 90 electrical degress behind the field and hence directly opposed to the applied e.m.f. in direction but somewhat less in length or value. When the counter e.m.fs. in the two legs of the winding are not exactly equal, due to mechanical dissymmetry, as described in connection with Fig. 261, their difference causes a corrective current *C* to flow, which lags 90 electrical degrees behind the counter e.m.f. *G*. Thus *C* is directly in line with *I* and can add to or subtract from it to produce the corrective effect desired.

Top to Bottom Parallel Connection Has No Corrective Effect. Figure 263 shows a connection which electrically is equivalent to Fig. 260, but is slightly different mechanically, owing to practical winding considerations. In Fig. 263 it will be noted that all the north poles are in one parallel of the winding, as for example, poles 1, 3 and 5, and all the south poles are in the other parallel, as for example, 2, 4 and 6. This has practically no corrective magnetic effect, such as was described in connection with Fig. 260. If, for example, the rotor is moved upward towards group 1, and thus moved away from group 4, any corrective tendency should produce the result of decreasing the flux through group 1

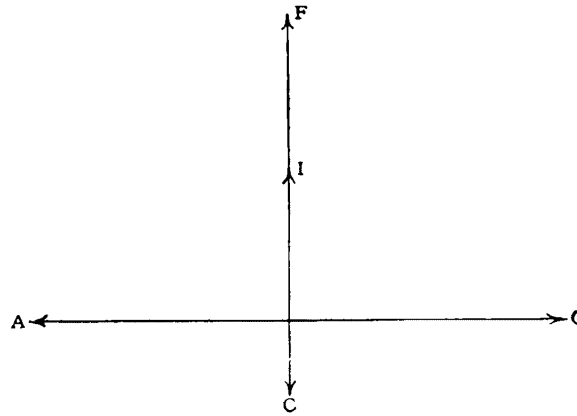


FIG. 262.—Vector diagram. Showing phase relations of magnetizing and corrective currents.

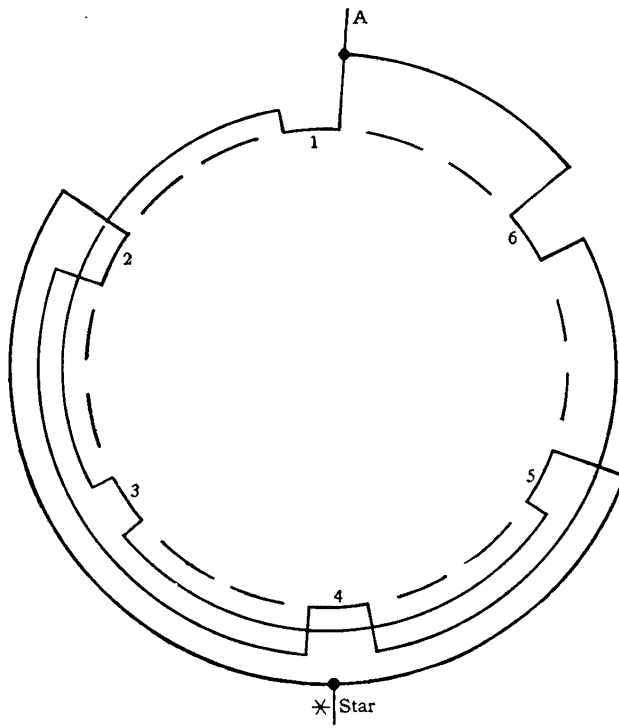


FIG. 263.—Three-phase, parallel star connection. Electrically this connection is the same as Fig. 260, but it has no effect in producing magnetic balance. It is merely what the winder knows as "top to bottom" connections of groups.

and increasing the flux through group 4. Since the magnetic flux through group 1 is interlinked with groups 2 and 6, which are electrically in series with group 4, and since the flux in group 4 is interlinked with groups 3 and 5, which are electrically in series with group 1, it will be seen that this electrical and magnetic interlinkage effectively prevents changing the magnetic conditions in two local magnetic circuits which are diametrically opposite in the machine, in opposite directions, and hence prevents any chance for corrective effect on the magnetic pull or balance.

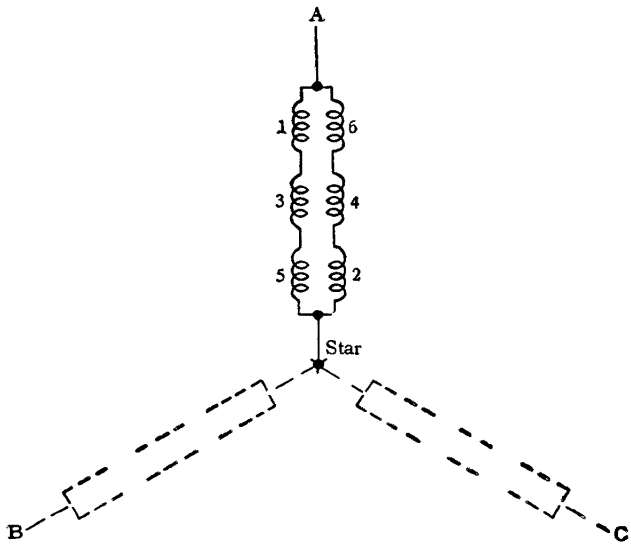


FIG. 264.—Schematic equivalent diagram of the winding in Fig. 263.

Best Parallel Arrangement for Corrective Effect.—In Fig. 265 is shown a diagram in which the maximum advantage is taken of the tendency for parallel windings properly connected to correct the unbalanced magnetic pull. The principle is the same as that explained in connection with Fig. 260, but in Fig. 265 the groups are paralleled in such a manner that when equalizer connections are added, the corrective effect is the same for all planes and there is no more tendency to correct a horizontal side pull than a vertical pull.

How it Corrects—Equalizer Connections.—A study of Fig. 265 and the corresponding schematic diagram in Fig. 266 illustrates how this corrective effect is produced by the equivalent of a local circulating current in the two branches of the parallel

circuit. This winding is still not so good as it can be made, for the reason that any corrective effect which takes place, for example, in groups 1 and 4 owing to vertical displacement of the rotor has also to affect the two other groups in the same parallel and to the same extent. In order to avoid this disadvantage and localize the corrective effect to exactly the point

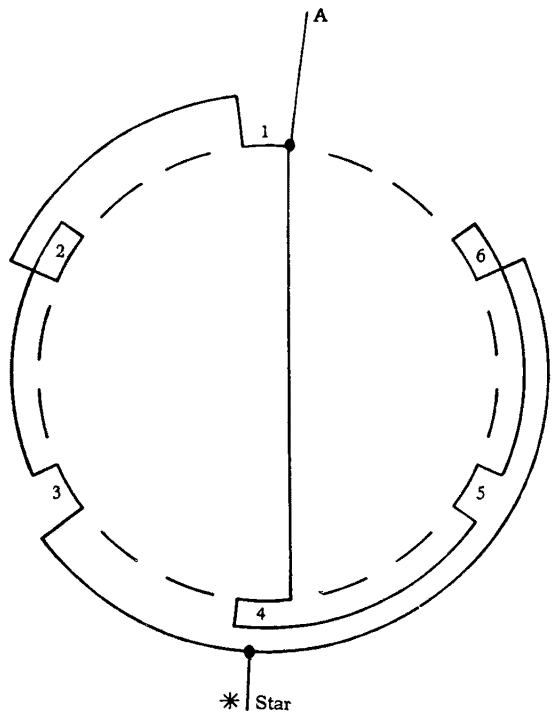


FIG. 265.—Connections for phase *A* of a three-phase, six-pole induction motor. Arranged to be connected parallel star so as to get the best condition for magnetic balance.

where it is required, additional cross connections or "equalizer" connections, as they are called, are placed in the winding at *m* and *n* as shown in Figs. 267 and 268. In Fig. 268, for example, it will be noted that as long as everything is symmetrical and the voltage perfectly balanced across the windings, there should be no difference in potential between the point *p*, lying between groups 1 and 2, and the point *q* lying between groups 4 and 5. Consequently, it would be possible to connect these two points by jumper or equalizer connections and there would be no effect;

in other words, no local current would flow. Assume, however, in Fig. 267 that the rotor is displaced vertically upward. Immediately the winding attempts to correct this condition by a local action in groups 1 and 4. If the winding were connected as shown in Fig. 265, this corrective action would have to take place in all the groups, since 2 and 3 are in series with 1, and 5 and 6 are in series with 4 and there is no other closed electrical path. In Fig. 267, by providing the jumper *pq*, we have provided a local circuit so that a local current can flow in groups 1 and 4 without

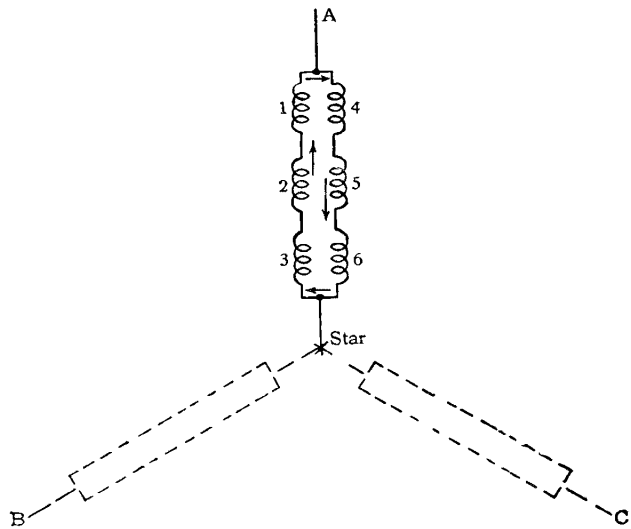


FIG. 266.—Schematic equivalent diagram of the winding in Fig. 265. Arrows show the direction of local corrective current.

regard to the other four groups, and hence the corrective of equalizing action is confined entirely to those groups where it is required. With the connections as made in Fig. 267 and the equalizer connections as there shown, the corrective possibility is the same at all points of the bore and can take place with a minimum effect on that portion of the winding in which action is not immediately required. This then represents the best possible combination. It is interesting to note in this connection that the squirrel-cage rotor winding on an induction motor of the type mentioned above exercises this corrective action almost perfectly, particularly when of comparatively low resistance, as is usually the case. Each bar has the possibility of acting in

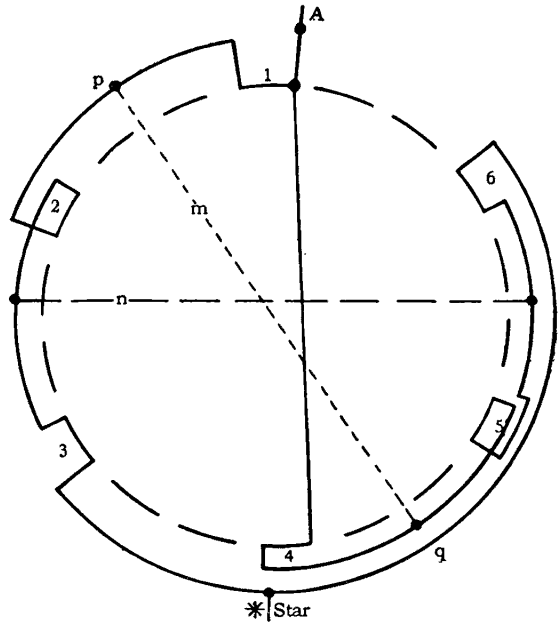


FIG. 267.—Parallel connections for magnetic balance as in Fig. 265. With the addition of equalizer connections *m* and *n* to improve magnetic conditions still further and provide local corrective effects at the points where required.

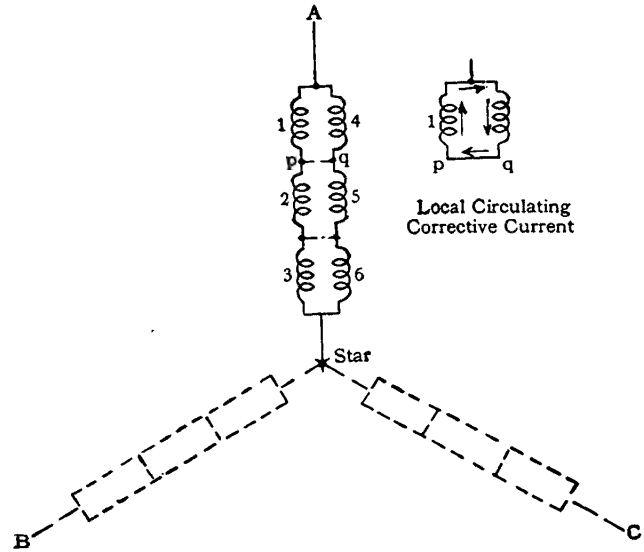


FIG. 268.—Schematic equivalent of diagram in Fig. 267. The small figure to the right illustrates the direction of the local corrective current.

combination with any bar and thus producing a local corrective current at any point around the field bore where it may be required at a given instant.

Equalizer Connections on Direct-current Armature.—In the foregoing an induction motor has been used for the purposes of illustration for the reason that no counter e.m.f. is generated in the shunt- or series-field coils of a direct-current machine. Consequently, in the case of Fig. 260, where the rotor was horizontally displaced as described, if the magnetizing current had been direct instead of alternating, no corrective effect would have taken place and the immediate result would have been an increased pull in the direction of the groups 5 and 6, and a decreased pull in the direction of groups 2 and 3. In order to secure this corrective balancing effect, advantage must be taken of the existence of an alternating magnetic flux somewhere in the circuit. Since it is generally known that there are equalizer connections in a direct-current armature, the question immediately arises, how it is possible to secure the same magnetic balancing effect that exists in the alternating-current motor. The answer to this is that the magnetic flux in the armature of a direct-current machine is alternating. In other words, as the armature iron passes first a north pole and then a south pole, an alternating magnetic flux is set up in the armature core, which is for this reason made laminated, as is the stator of an alternating-current motor.

Carrying this explanation through, if taps are taken from the coils of a direct-current armature and led to slip rings, alternating current could be taken through these slip rings. Therefore, if the form of the winding on the direct-current armature is in its nature parallel; that is to say of the "parallel" type as distinguished from the two-circuit or "wave" type, it will be possible to select points of normally equal potential and connect them with equalizer connections in exactly the same manner as illustrated in Fig. 268. Under ordinary operating conditions, when the air gap is symmetrical all around the armature, and when the magnetic field is also symmetrical, there is no difference of potential from one end of these equalizer connections to the other and consequently no corrective current flows. However, if for any reason, either mechanical or magnetic, the symmetry of the various magnetic circuits is disturbed, a cor-

rective current will flow in a portion of the armature winding in the same manner as illustrated in Fig. 268, and this local current will have a tendency to oppose or correct the cause which set up the magnetic disturbance. Going further, as the bearings wear and as there is a tendency for the armature to drop below the exact mechanical center, and consequently to approach the pole shoes on the lower half of the machine, if there were no corrective effect, the magnetic pull downward would be greater than the magnetic pull upward; this would have a cumulative effect in still further increasing the load on the bearing and consequently producing increasing wear until the armature finally struck on the pole shoes. However, since the corrective effect is present as the bearings wear, the equalizer connections preserve the magnetic symmetry so that the pull top and bottom is equalized to the same value that it had originally, and the bearing wear is limited to that produced by the weight of the armature and the pull of the belt or driving connection at all times.

Factors That Are Normally Corrective.—It should not be taken for granted from the foregoing that there is a constant and dangerous tendency in all machines toward instability of magnetic balance. The size of the air gap and the rigidity of the frame, shaft and bearings, also the exactness of the workmanship on most machines are sufficient in themselves to prevent any great disturbance due to this cause. Wherever actually required, direct-current armatures and the primary windings of alternating-current motors are provided with equalizer connections. Also in the case of squirrel-cage induction motors as previously noted, the rotor bars, acting in combination, each one with all the rest, form a multiplicity of equalizer connections or circuits so that it is practically impossible for any induction motor having a comparatively low-resistance squirrel-cage rotor winding to manifest signs of unbalanced magnetic pull. This is an additional reason why this form of simple and rugged motor has assumed its preeminent place in the field of converting electric to mechanical energy.

CHAPTER XVII

STANDARD GROUP DIAGRAMS FROM 2 TO 14 POLES

The form of diagram which is most often used in connecting induction motor windings is the so-called "group" diagram so often illustrated in the foregoing chapters where the coils are "stubbed" or grouped into pole-phase groups and then cross connected to form magnetic poles. This form of diagram is practically universally used for stators with open slots and because it is so often employed, it is considered desirable to give in this chapter a series of diagrams covering all possible combinations, both two and three phase, star and delta, from two to fourteen poles.

To attempt to show "developed" windings, that is a picture of the actual coils rolled out flat for all possible numbers of poles, phases, slots, coils per slot, etc. would require several hundred diagrams even for full-pitch windings and with the slots always an integral multiple of the phases times the poles, and if to this is added the possibility of chording and using a total number of slots not an even multiple of the phases times the poles, the number of pictures required to show all the connections would run into thousands. However, by the relatively simple scheme of considering the group of coils which forms one pole-phase group as a unit, the possible number of combinations becomes greatly limited, and as shown by the following diagrams all the combinations from two to fourteen poles can be indicated by means of diagrams shown in Figs. 269 to 349 inclusive.

From the nature of the diagrams here given it will be seen that they are not dependent on the total number of slots in the machine, nor upon the number of coils per group, nor upon the throw or pitch of the coils, but are general for all machines of the same number of phases and poles. Each one of the small arcs in the circle represents the ends of the coils in a single pole-phase group in the winding. In order to illustrate this, photographs have been taken of a machine in three stages. In Fig. 350 a machine is shown in which the coils have simply been placed in

(*Text continued on page 343.*)

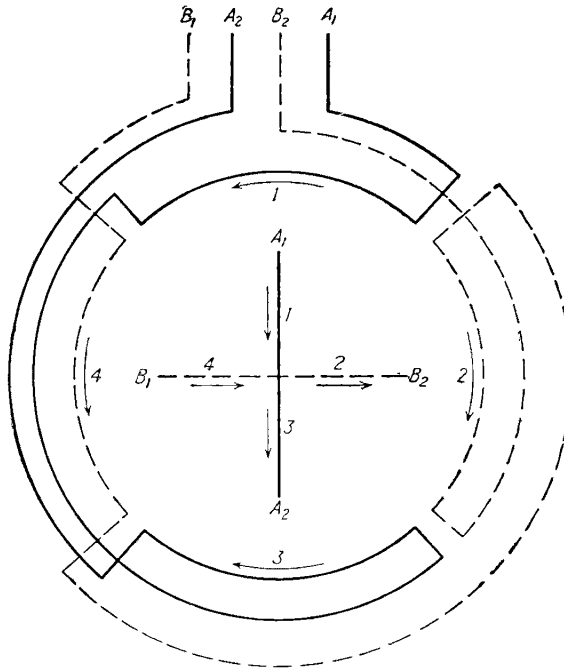


FIG. 269.—Two-pole, two-phase, series.

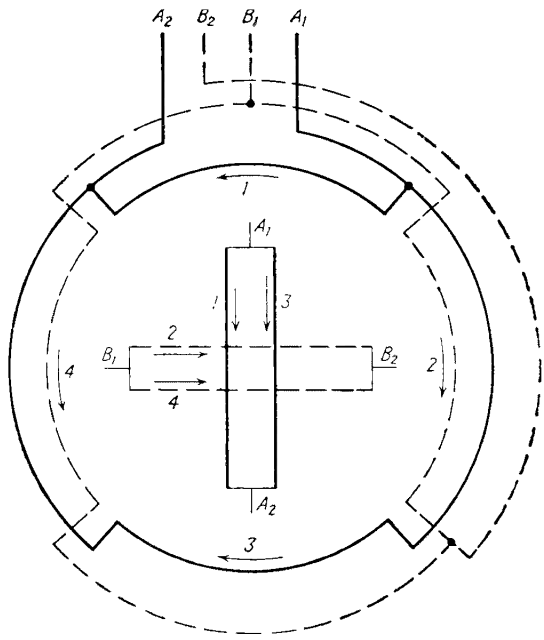


FIG. 270.—Two-pole, two-phase, parallel.

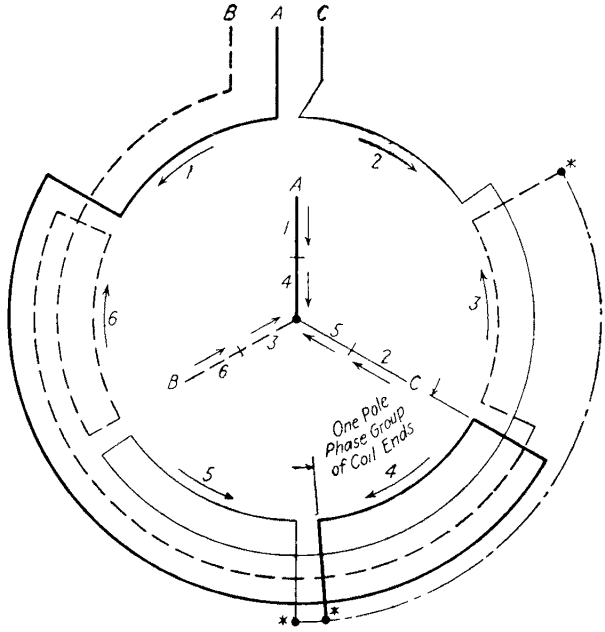


FIG. 271.—Two-pole, three-phase, series star.

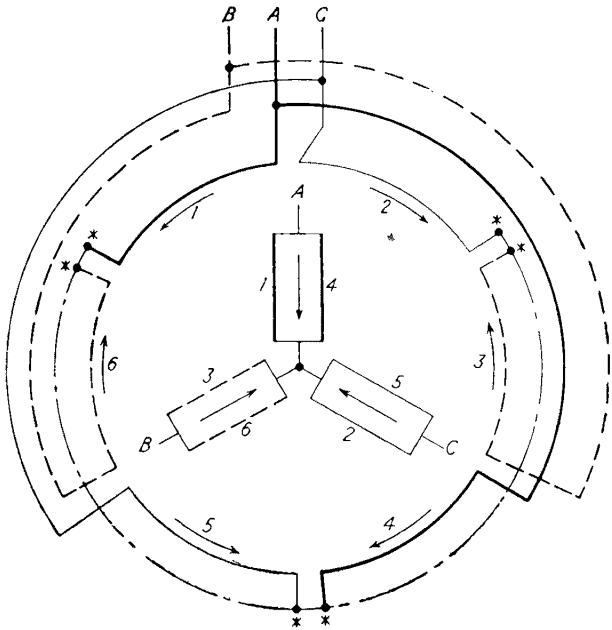


FIG. 272.—Two-pole, three-phase, parallel star.

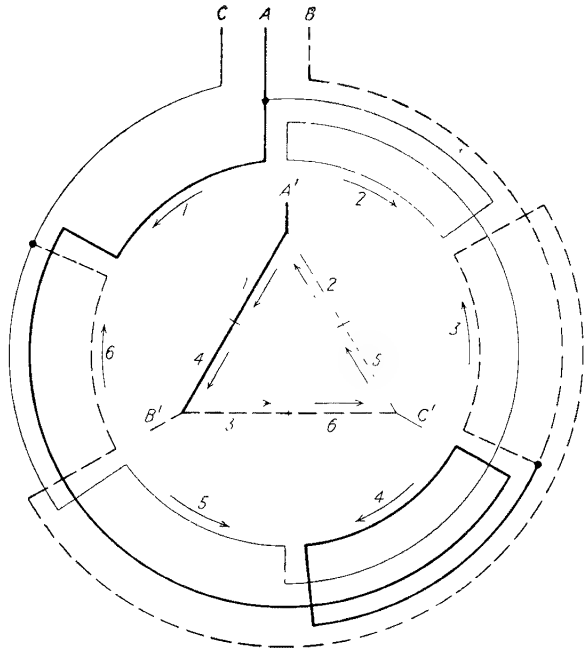


FIG. 273.—Two-pole, three-phase, series delta.

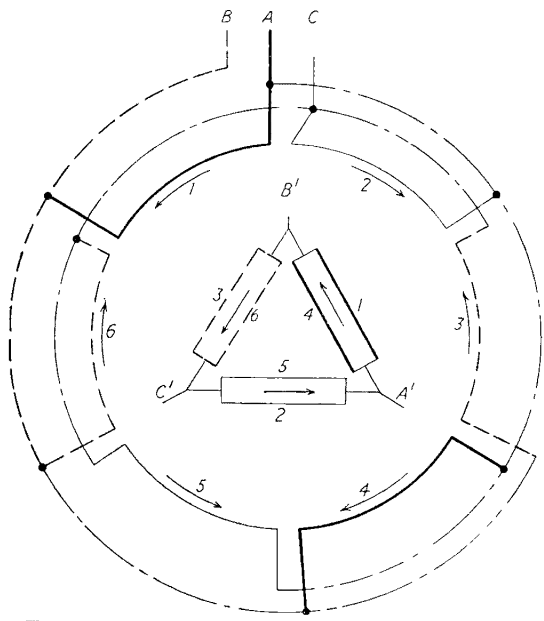


FIG. 274.—Two-pole, three-phase, parallel delta.

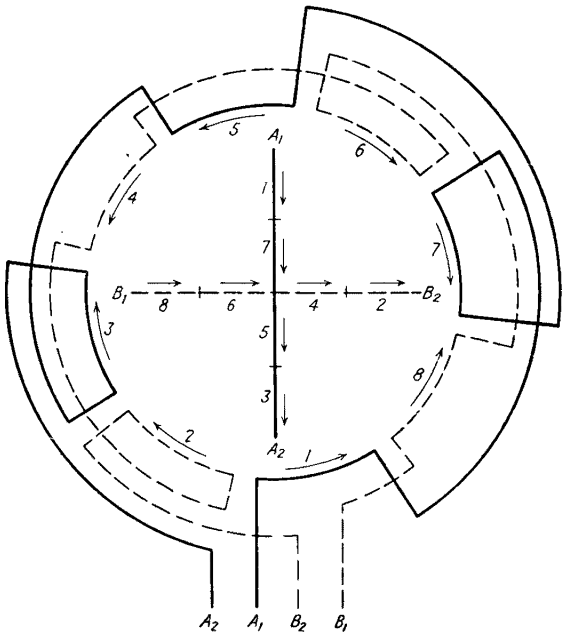


FIG. 275.—Four-pole, two-phase, series.

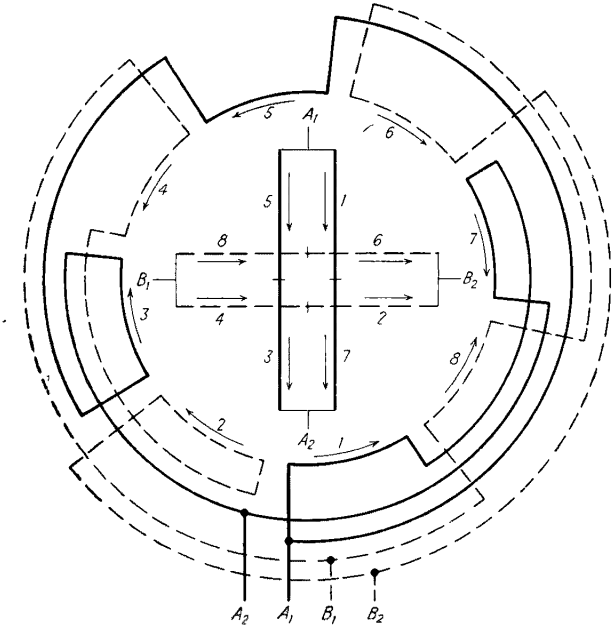


FIG. 276.—Four-pole, two-phase, two-parallel.

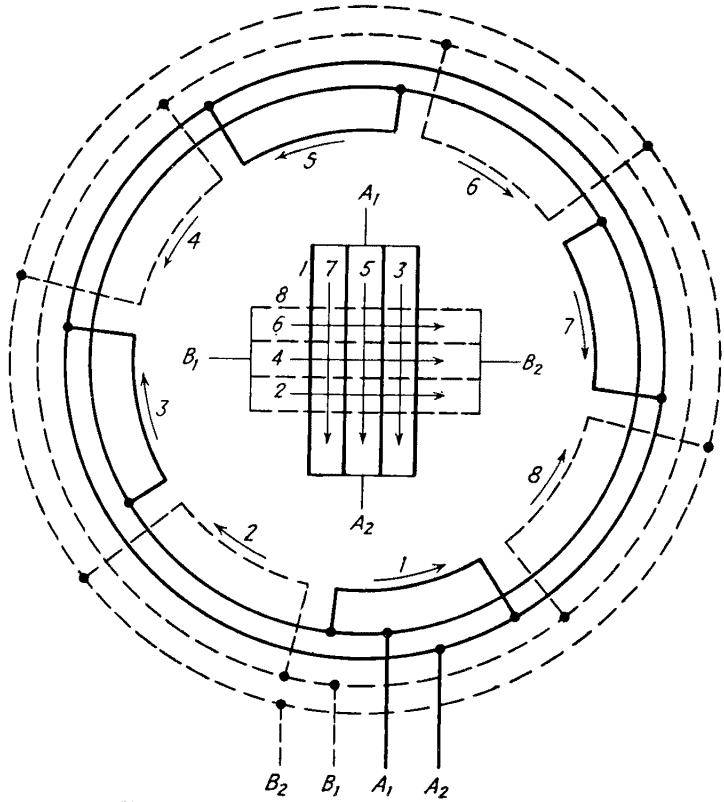


FIG. 277.—Four-pole, two-phase, four-parallel.

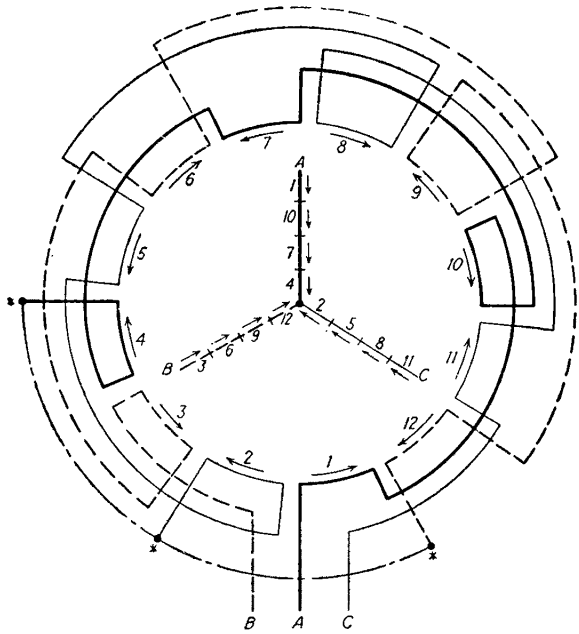


FIG. 278.—Four-pole, three-phase, series star.

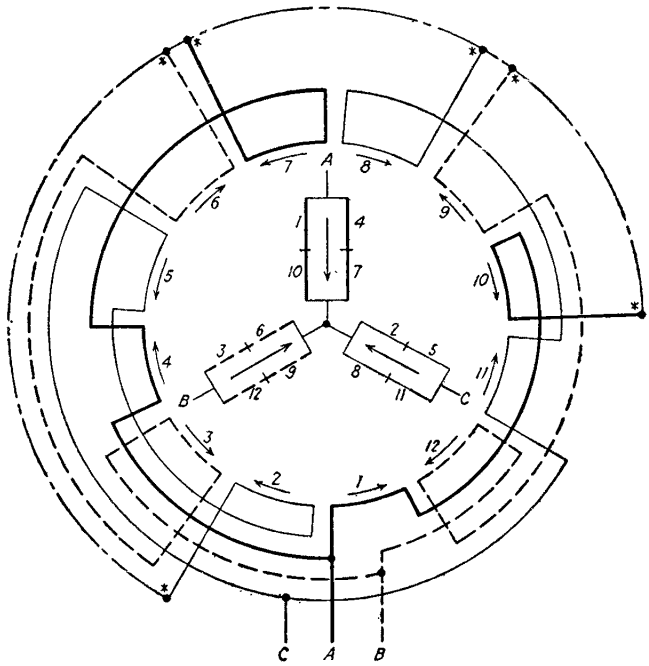


FIG. 279.—Four-pole, three-phase, two-parallel star.

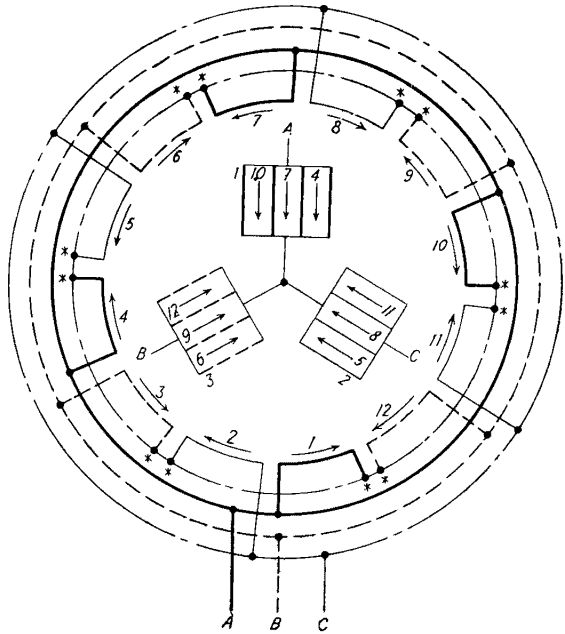


FIG. 280.—Four-pole, three-phase, four-parallel star.

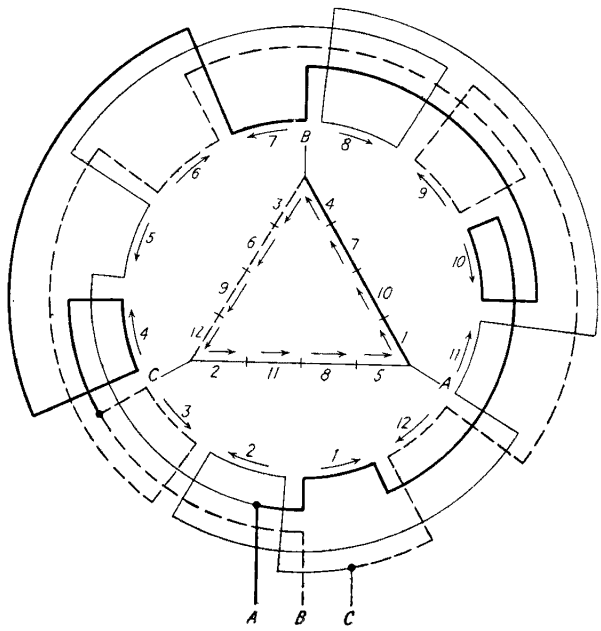


FIG. 281.—Four-pole, three-phase, series delta.

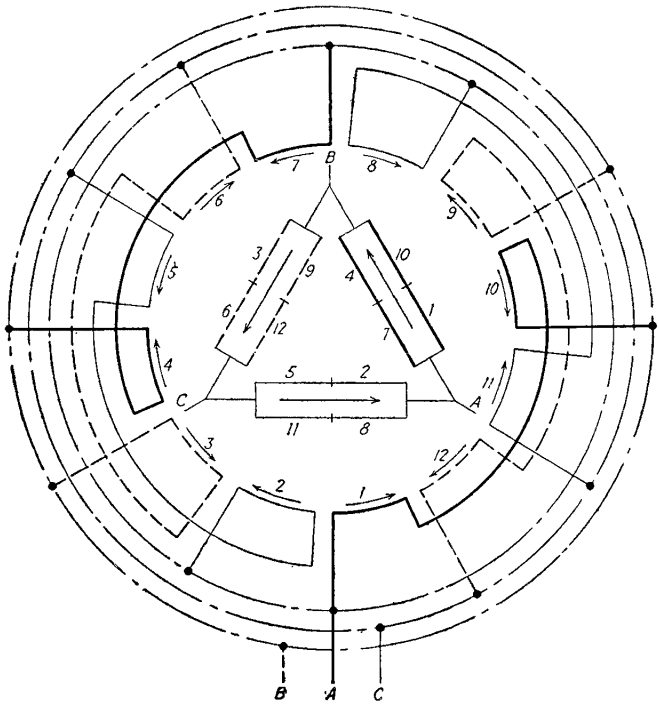


FIG. 282.—Four-pole, three-phase, two-parallel delta.

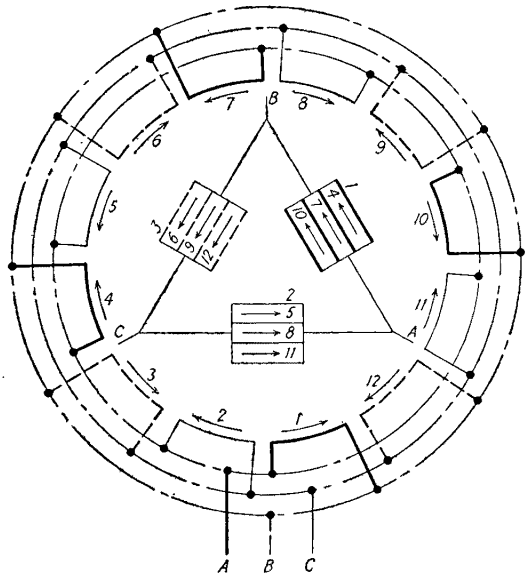


FIG. 283.—Four-pole, three-phase, four-parallel delta.

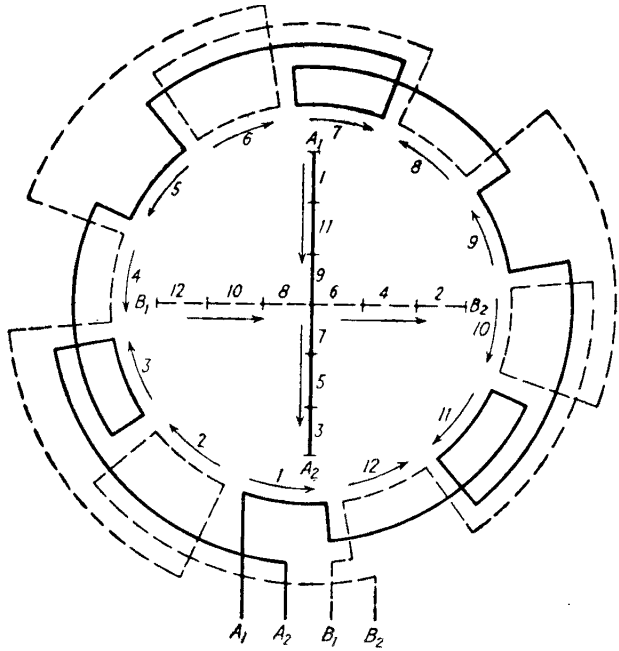


FIG. 284.—Six-pole, two-phase, series.

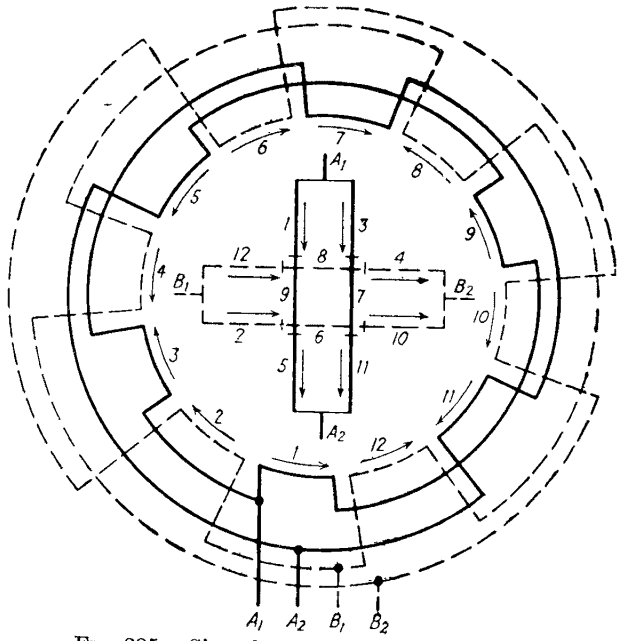


FIG. 285.—Six-pole, two-phase, two-parallel.

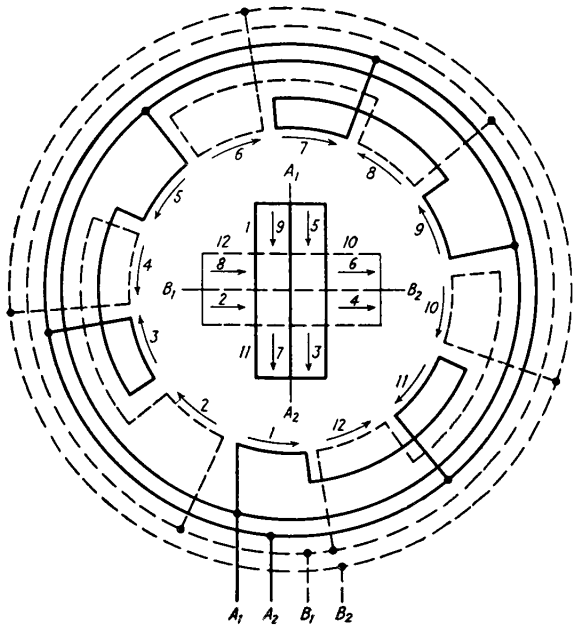


FIG. 286.—Six-pole, two-phase, three-parallel.

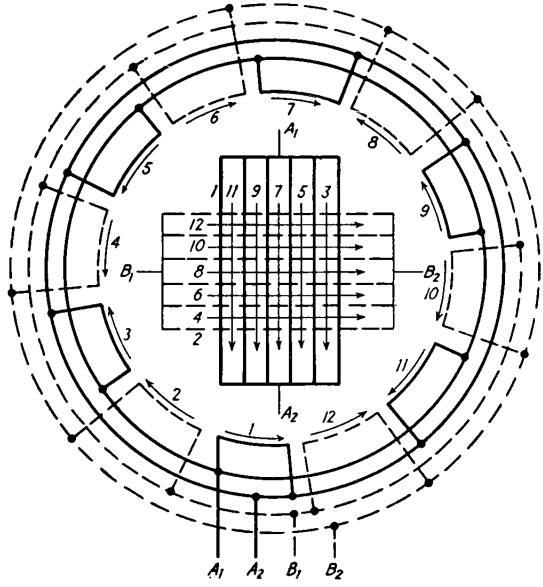


FIG. 287.—Six-pole, two-phase, six-parallel.

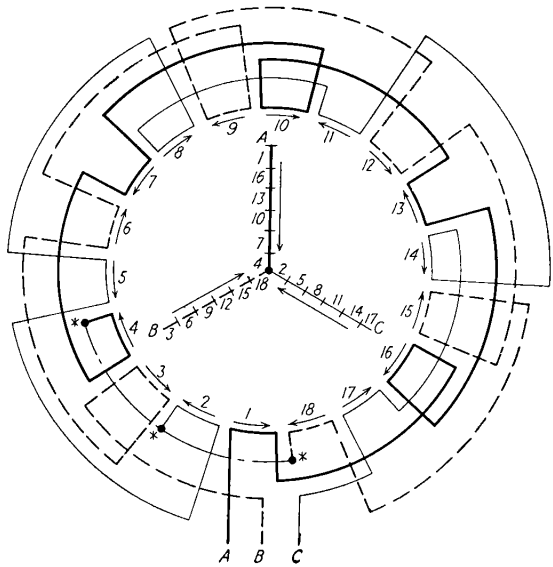


FIG. 288.—Six-pole, three-phase, series star.

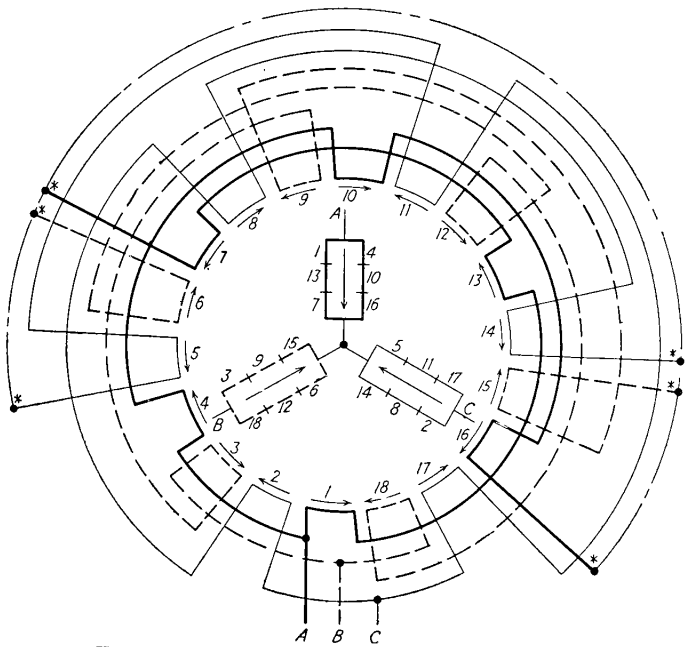


FIG. 289.—Six-pole, three-phase, two-parallel star.

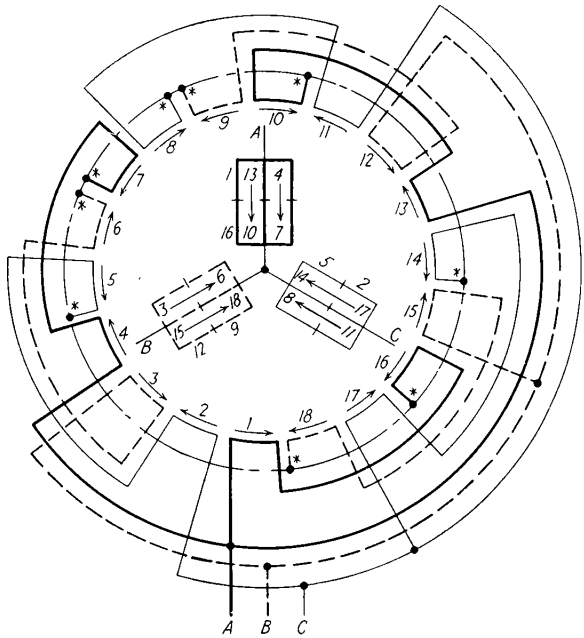


FIG. 290.—Six-pole, three-phase, three-parallel star.

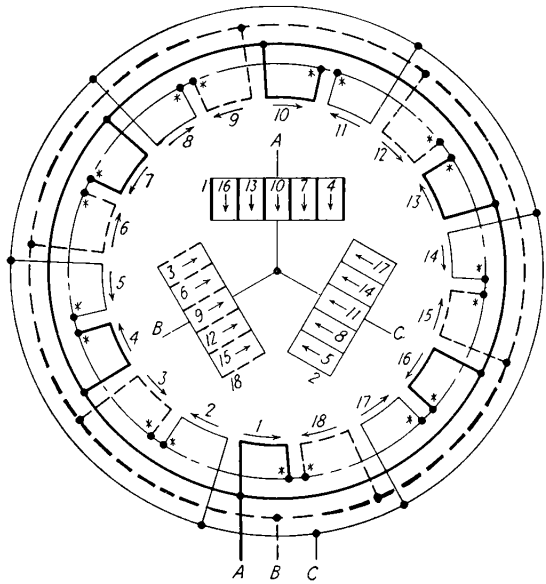


FIG. 291.—Six-pole, three-phase, six-parallel star.

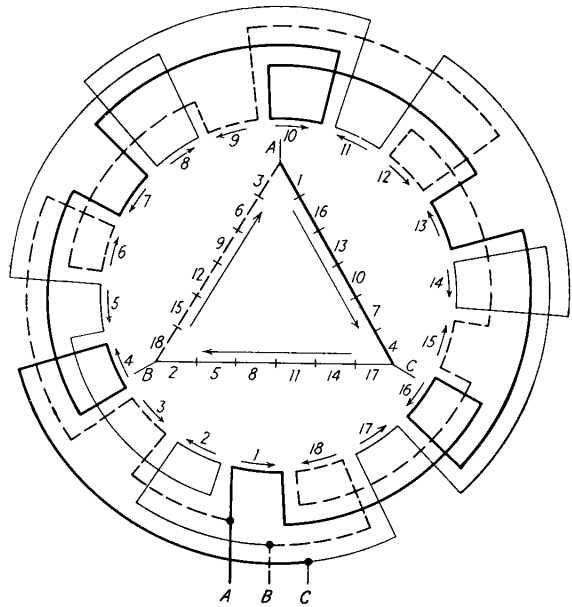


FIG. 292.—Six-pole, three-phase, series delta.

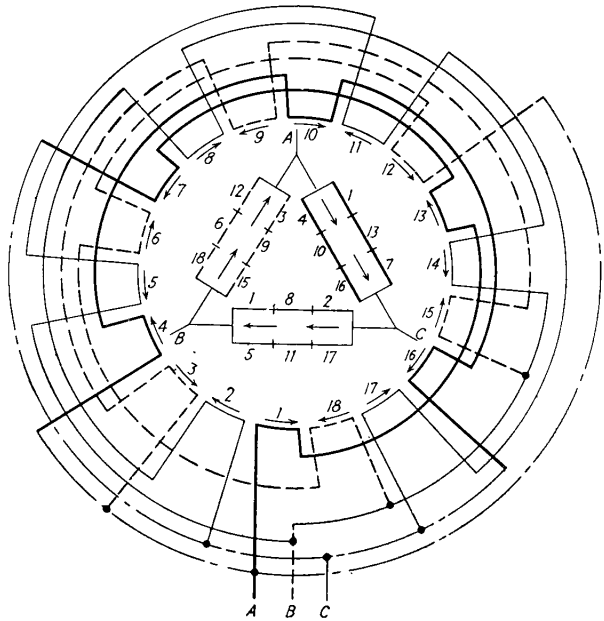


FIG. 293.—Six-pole, three-phase, two-parallel delta.

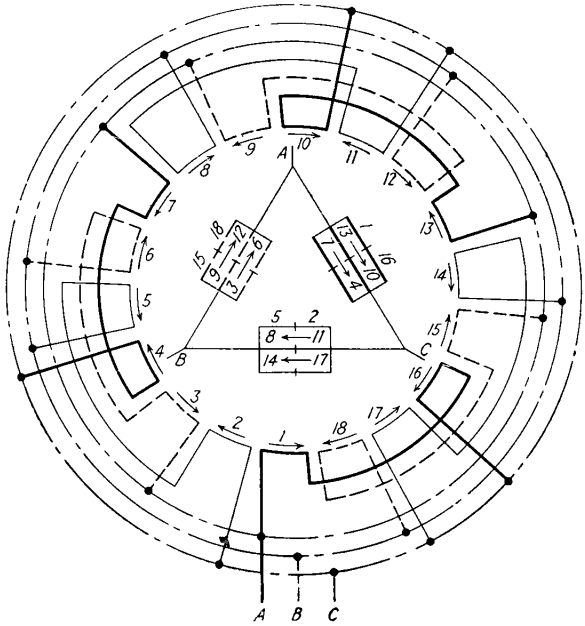


FIG. 294.—Six-pole, three-phase, three-parallel delta.

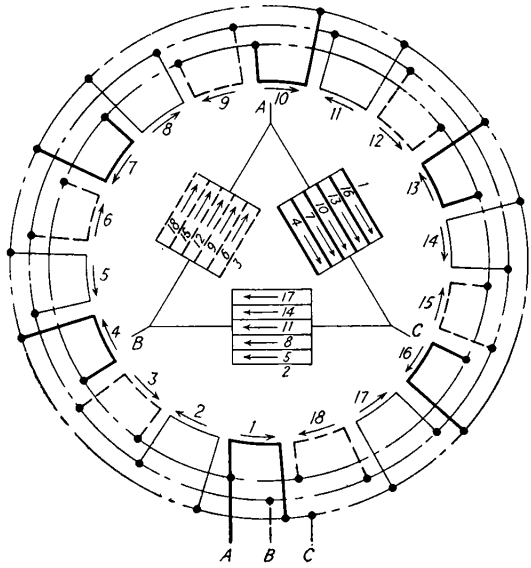


FIG. 295.—Six-pole, three-phase, six-parallel delta.

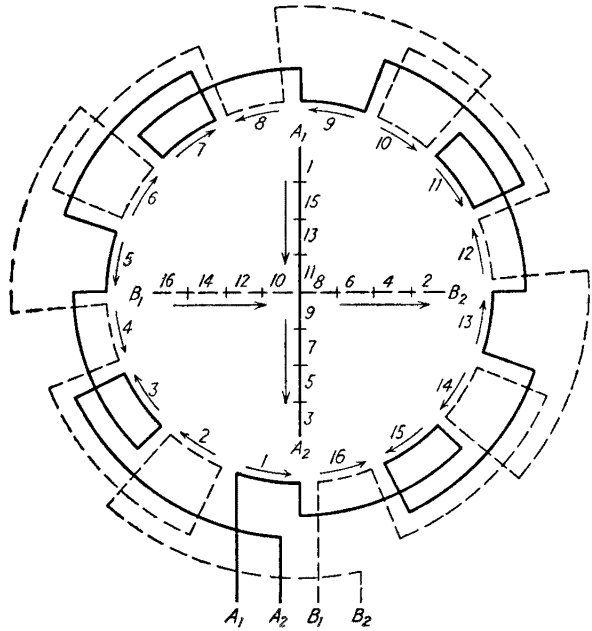


FIG. 296.—Eight-pole, two-phase, series.

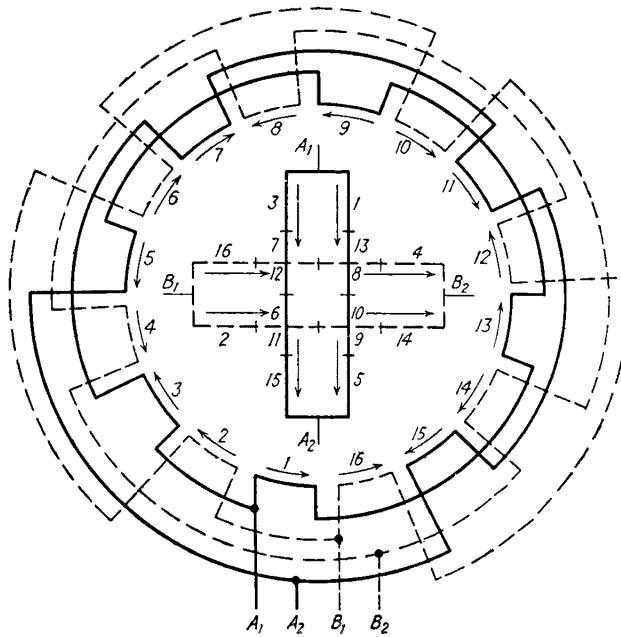


FIG. 297.—Eight-pole, two-phase, two-parallel.

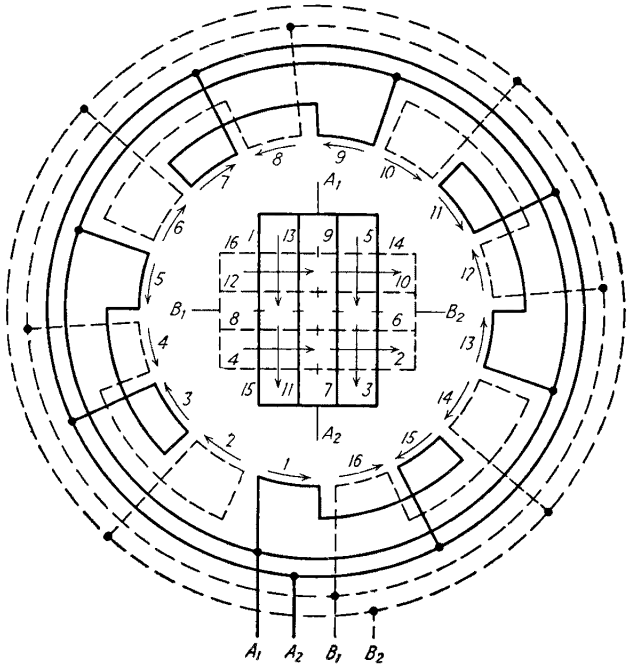


FIG. 298.—Eight-pole, two-phase, four-parallel.

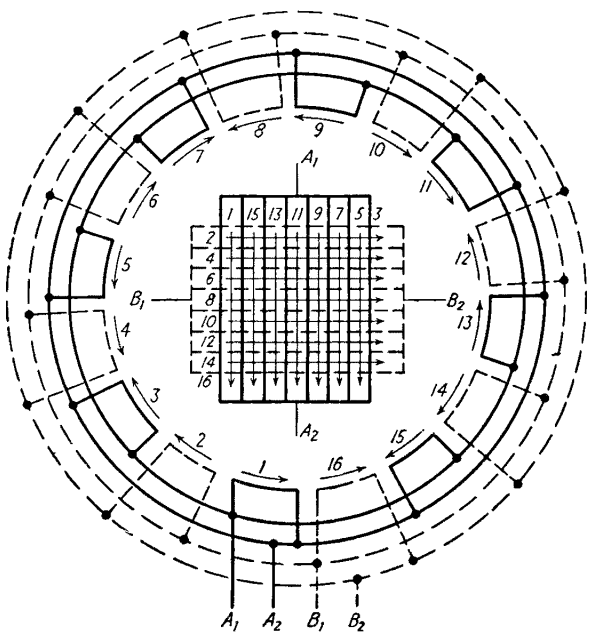


FIG. 299.—Eight-pole, two-phase, eight-parallel.

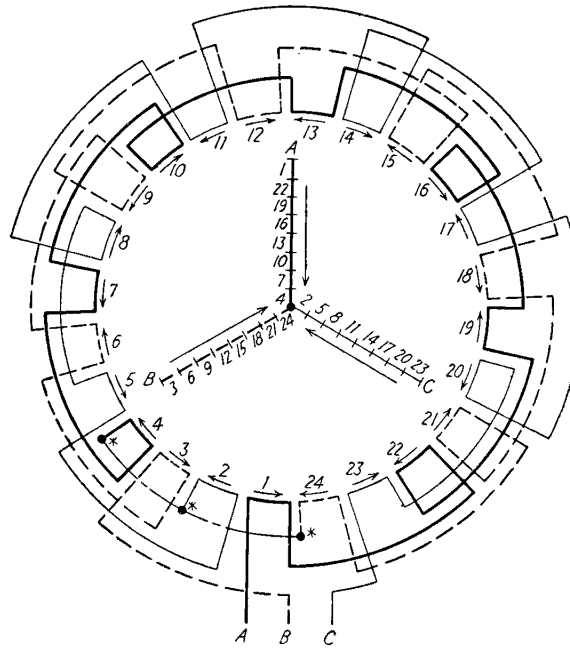


FIG. 300.—Eight-pole, three-phase, series star.

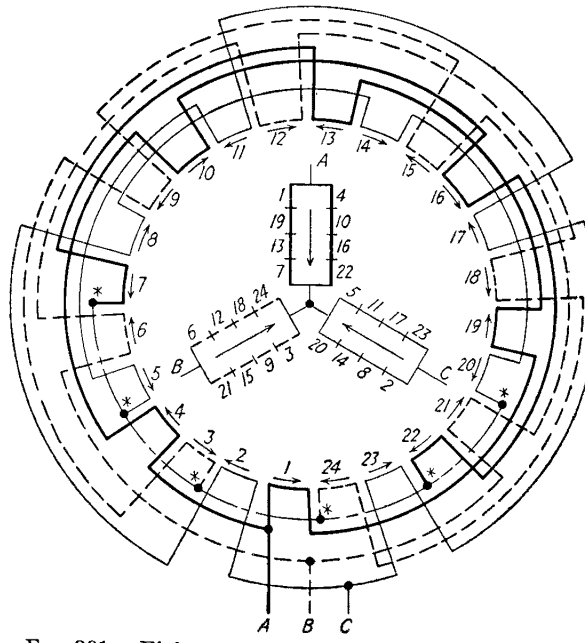


FIG. 301.—Eight-pole, three-phase, two-parallel star.

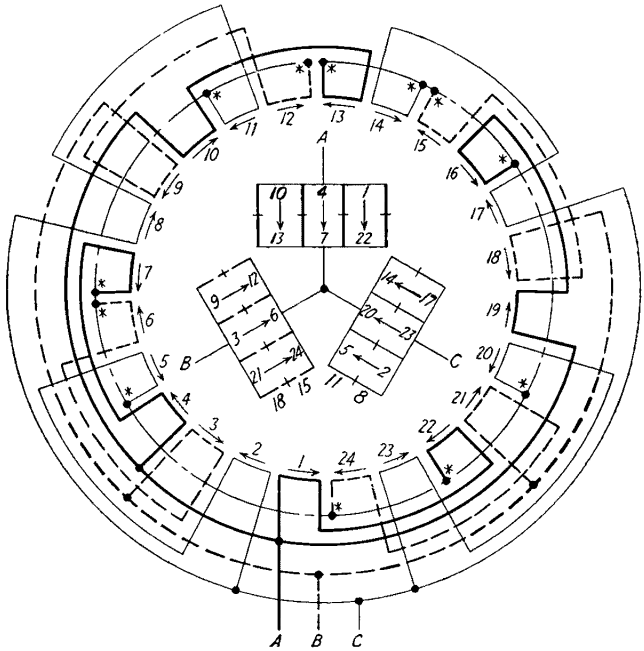


FIG. 302.—Eight-pole, three-phase, four-parallel star.

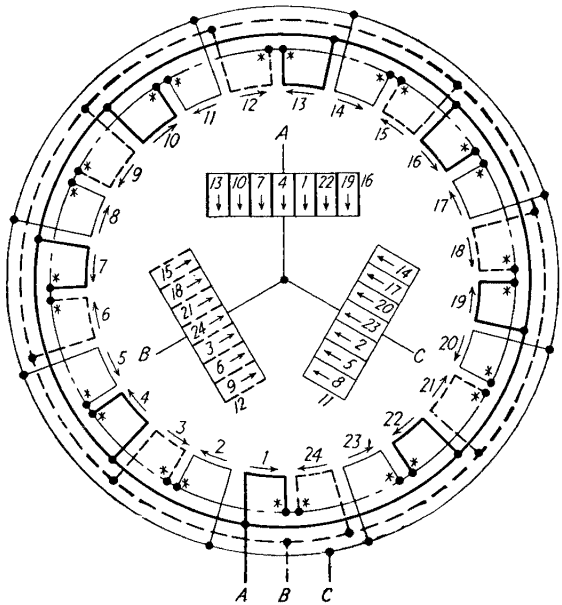


FIG. 303.—Eight-pole, three-phase, eight-parallel star.

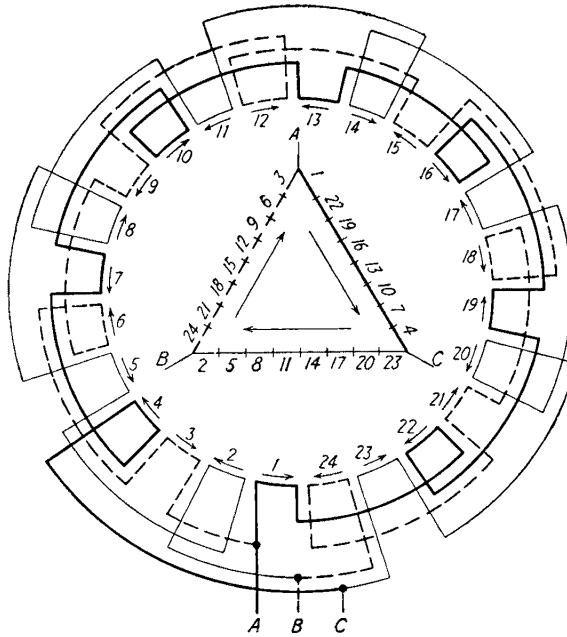


FIG. 304.—Eight-pole, three-phase, series delta.

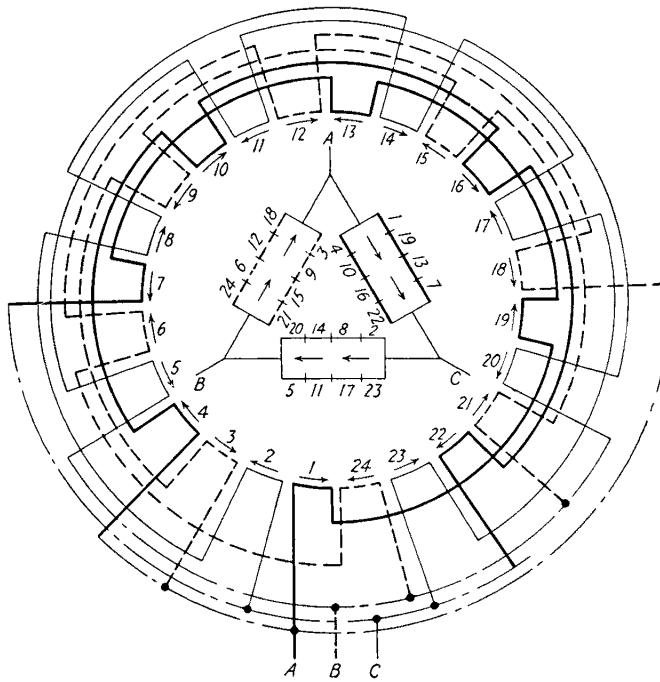


FIG. 305.—Eight-pole, three-phase, two-parallel delta.

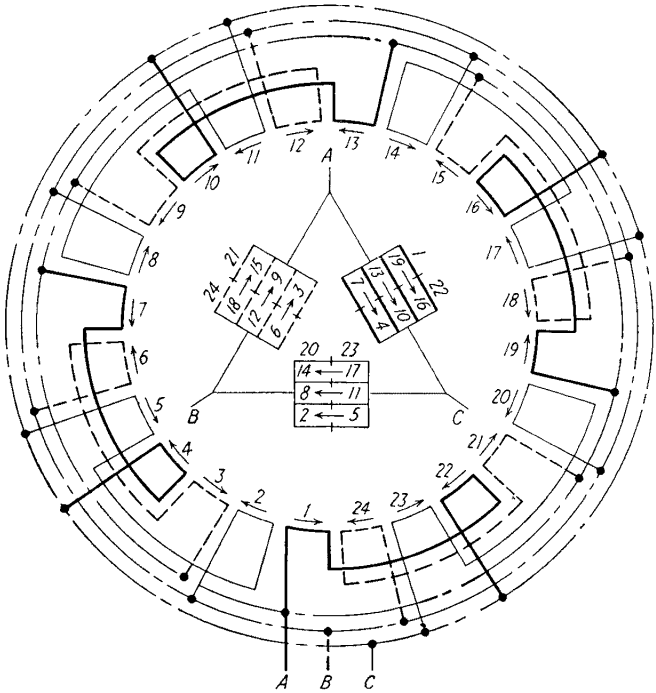


FIG. 306.—Eight-pole, three-phase, four-parallel delta.

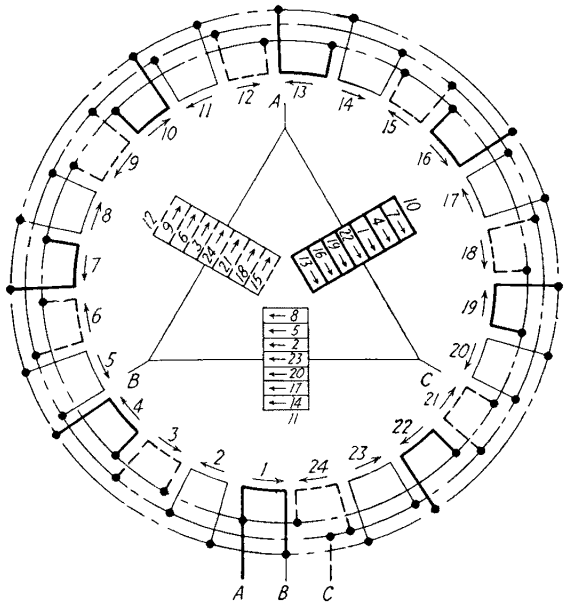


FIG. 307.—Eight-pole, three-phase, eight-parallel delta.

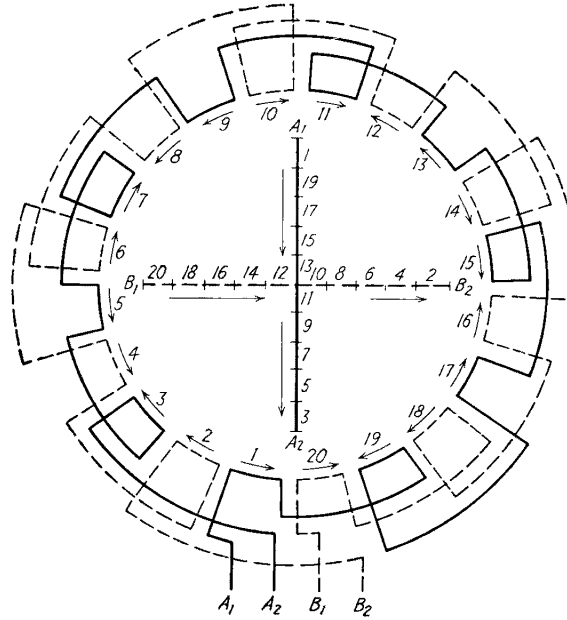


FIG. 308.—Ten-pole, two-phase, series.

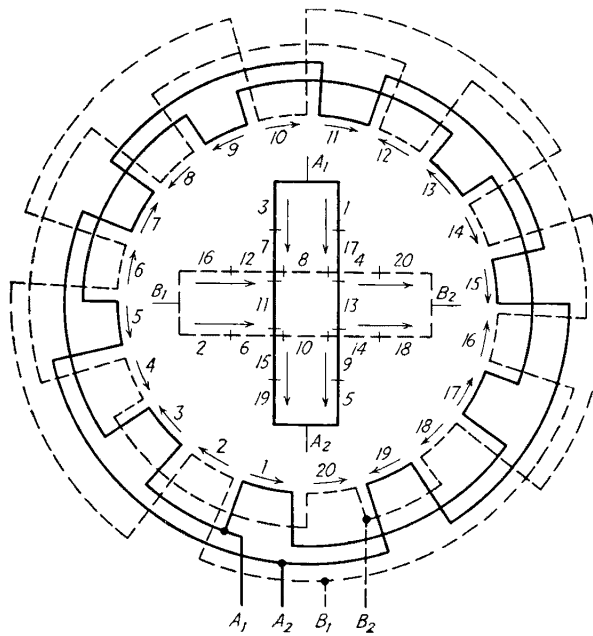


FIG. 309.—Ten-pole, two-phase, two-parallel.

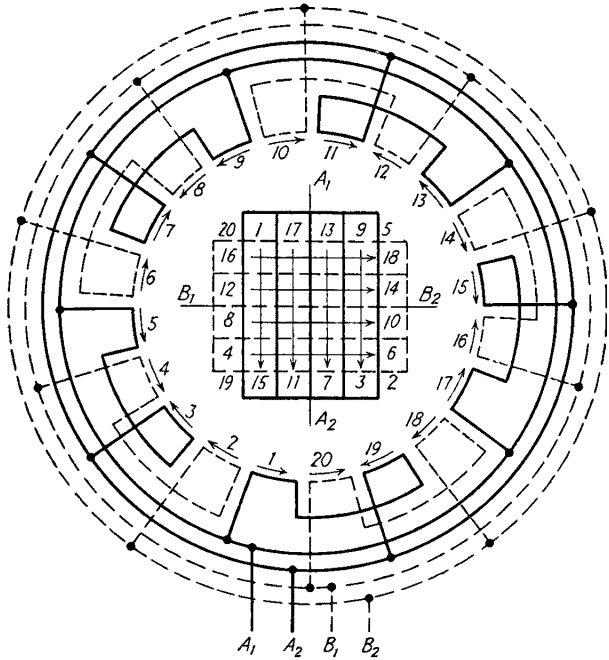


FIG. 310. Ten-pole, two-phase, five-parallel.

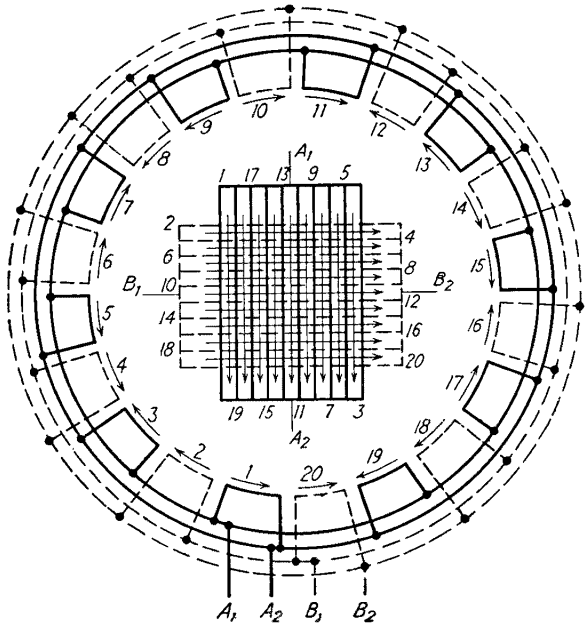


FIG. 311.—Ten-pole, two-phase, ten-parallel.

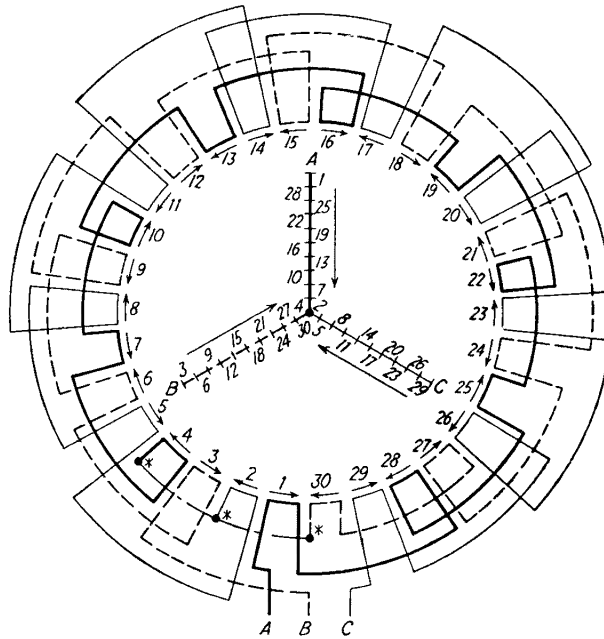


FIG. 312.—Ten-pole, three-phase, series star.

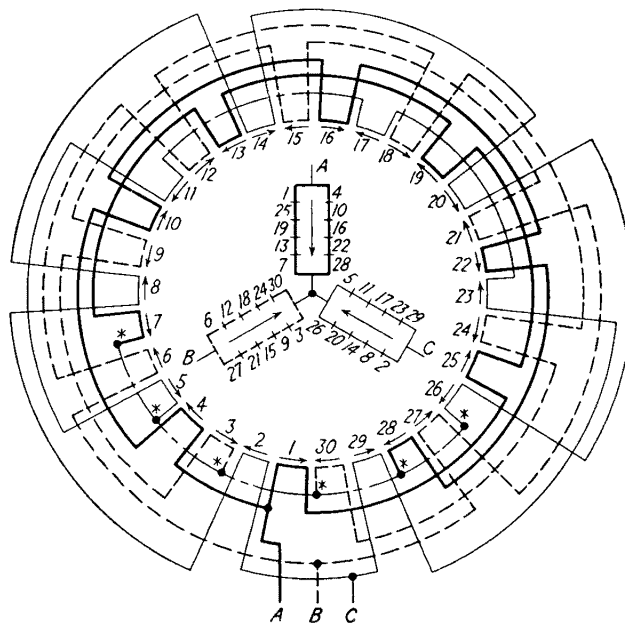


FIG. 313.—Ten-pole, three-phase, two-parallel star.

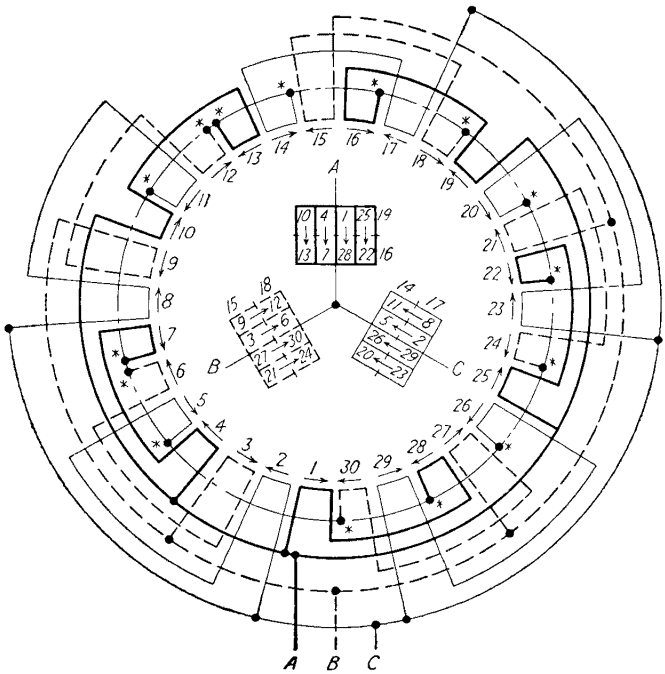


FIG. 314.—Ten-pole, three-phase, five-parallel star.

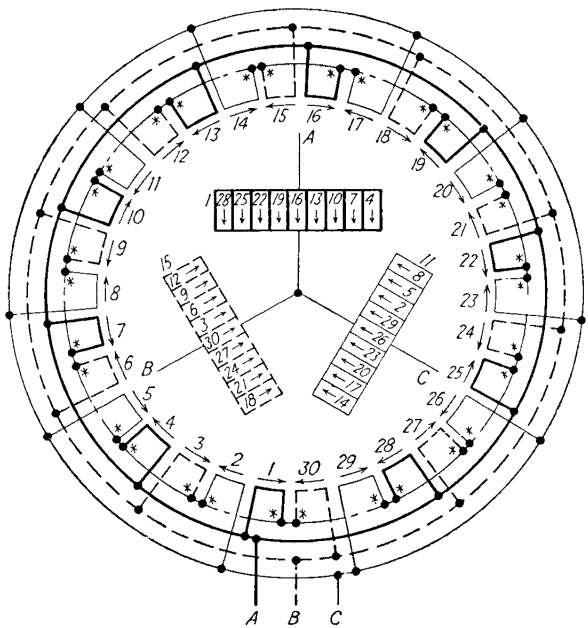


FIG. 315.—Ten-pole, three-phase, ten-parallel star.

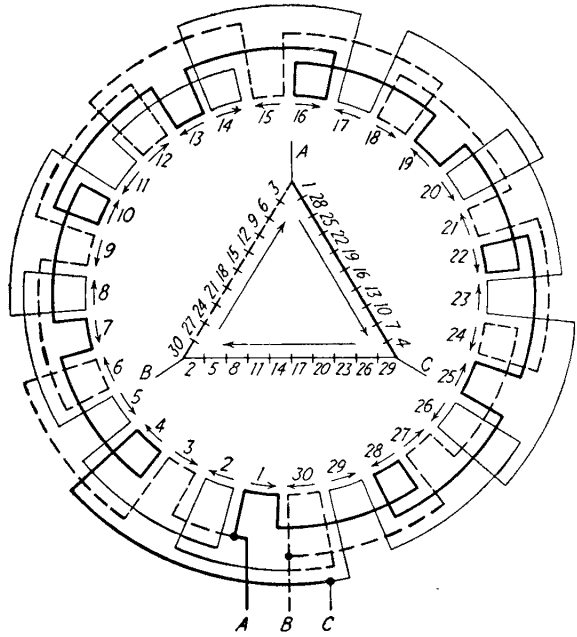


FIG. 316.—Ten-pole, three-phase, series delta.

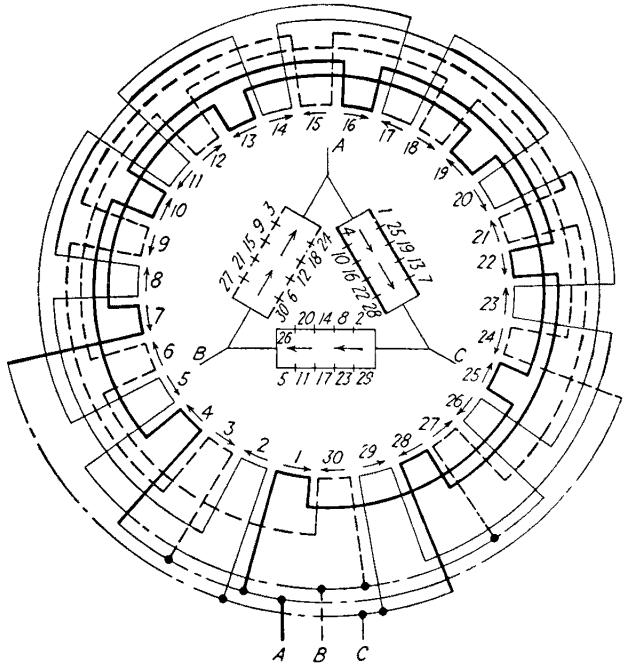


FIG. 317.—Ten-pole, three-phase, two-parallel delta.

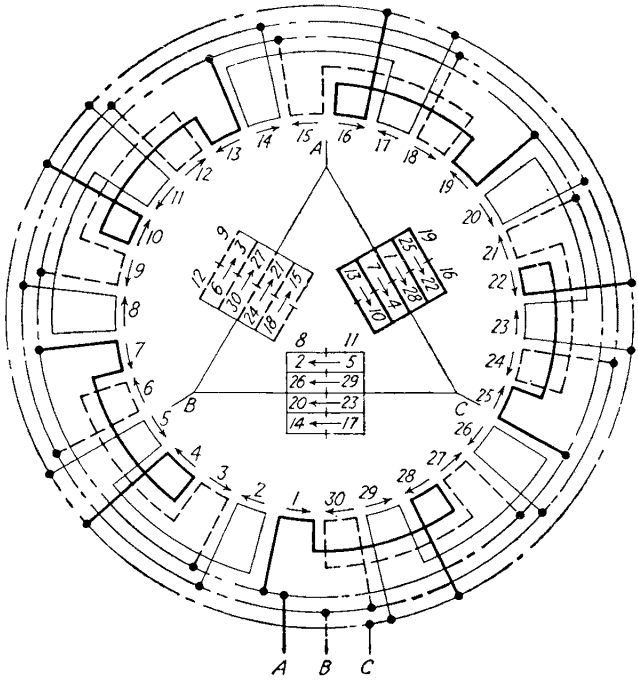


FIG. 318.—Ten-pole, three-phase, five-parallel delta.

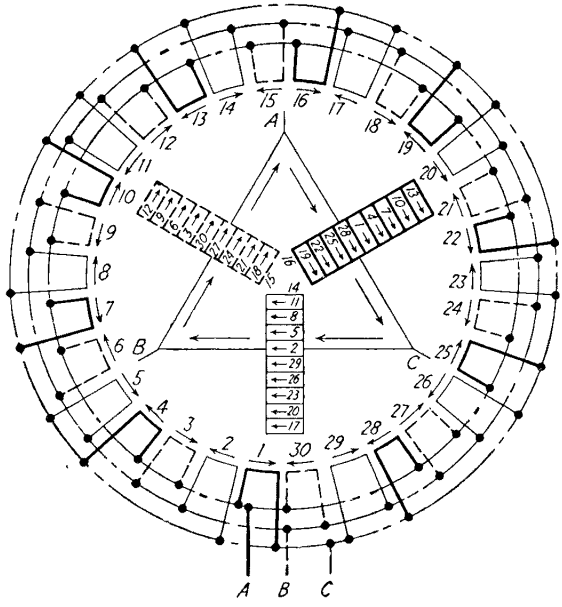


FIG. 319.—Ten-pole, three-phase, ten-parallel delta.

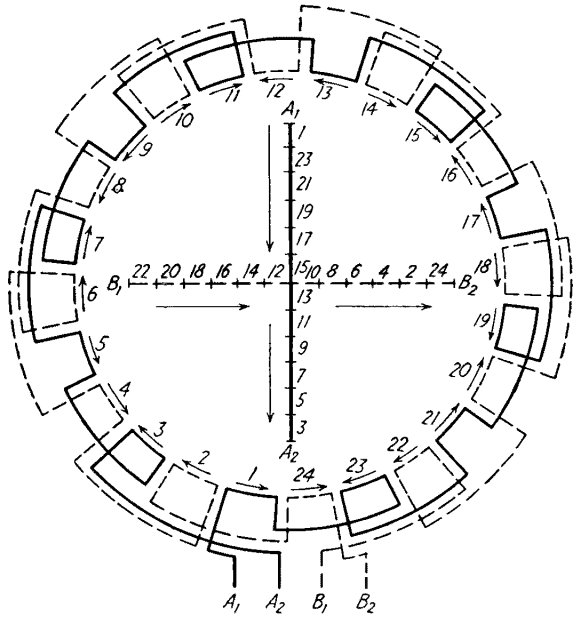


FIG. 320. –Twelve-pole, two-phase, series.

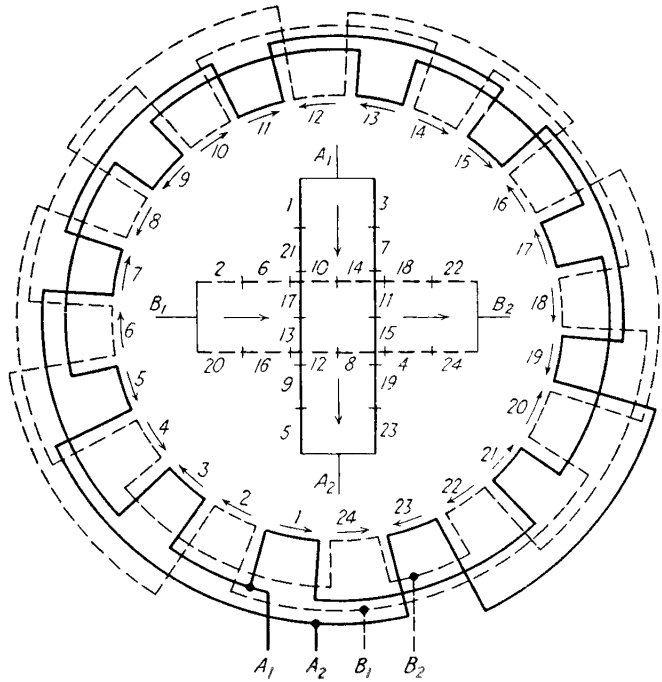


FIG. 321.—Twelve-pole, two-phase, two-parallel.

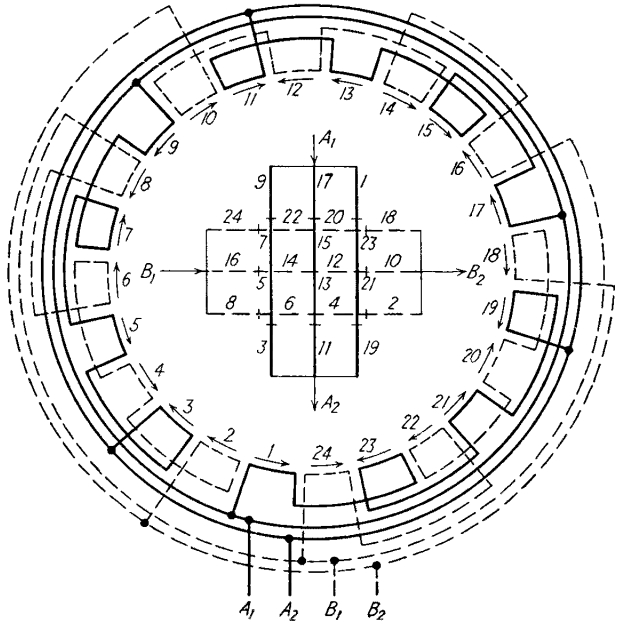


FIG. 322.—Twelve-pole, two-phase, three-parallel.

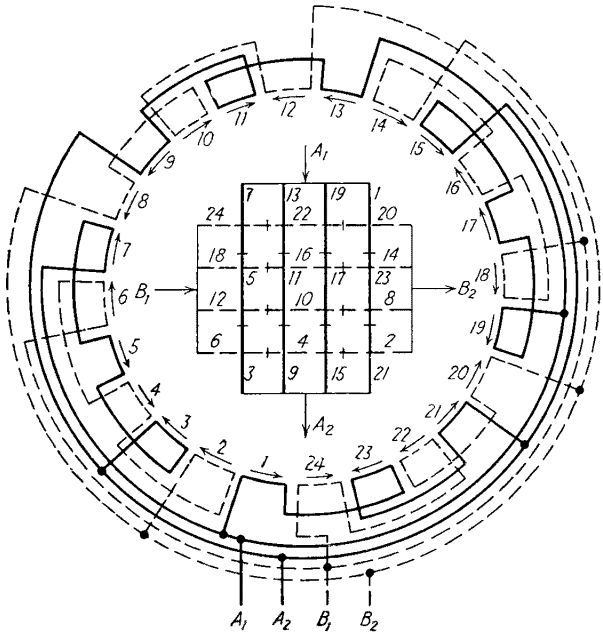


FIG. 323.—Twelve-pole, two-phase, four-parallel.

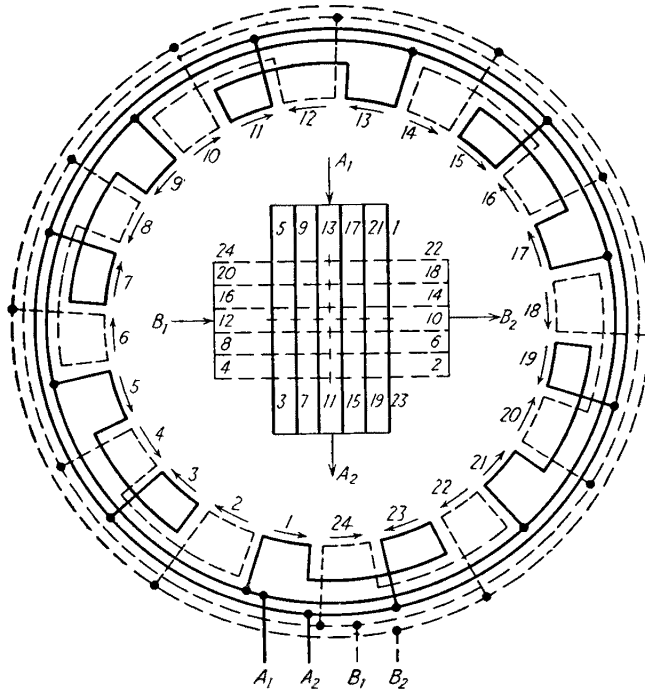


FIG. 324.—Twelve-pole, two-phase, six-parallel.

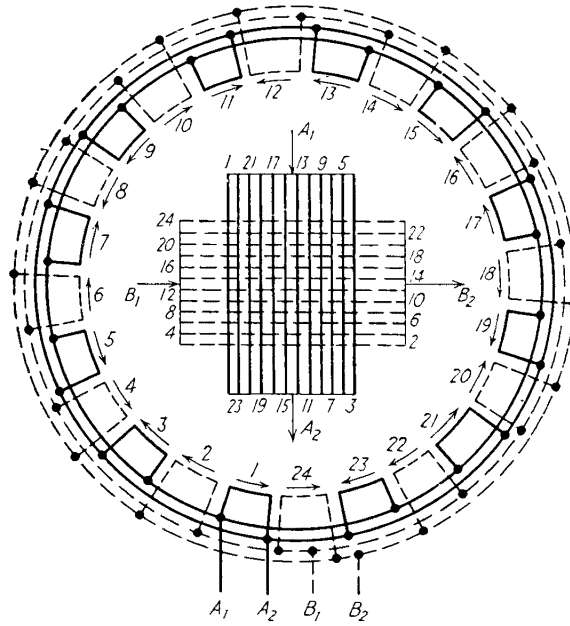


FIG. 325.—Twelve-pole, two-phase, twelve-parallel.

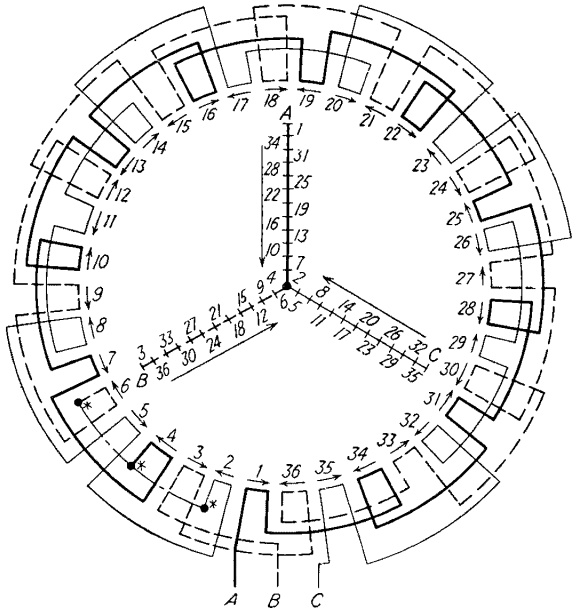


FIG. 326.—Twelve-pole, three-phase, series star.

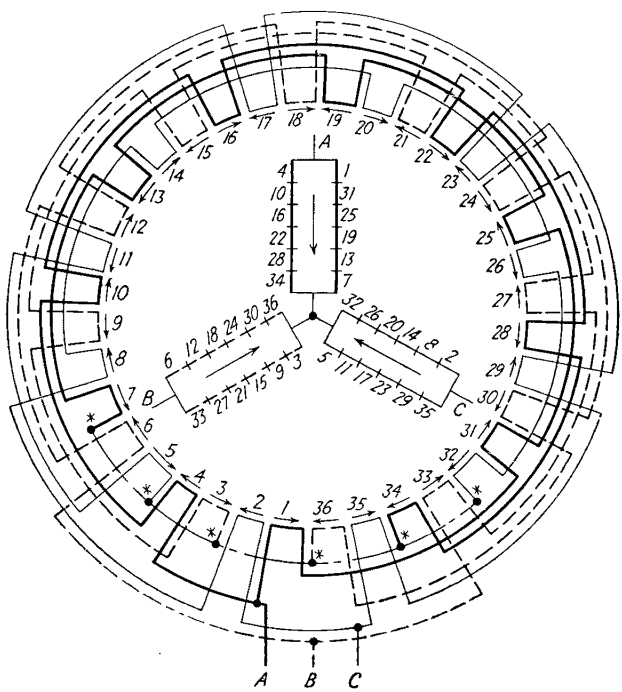


FIG. 327.—Twelve-pole, three-phase, two-parallel star.

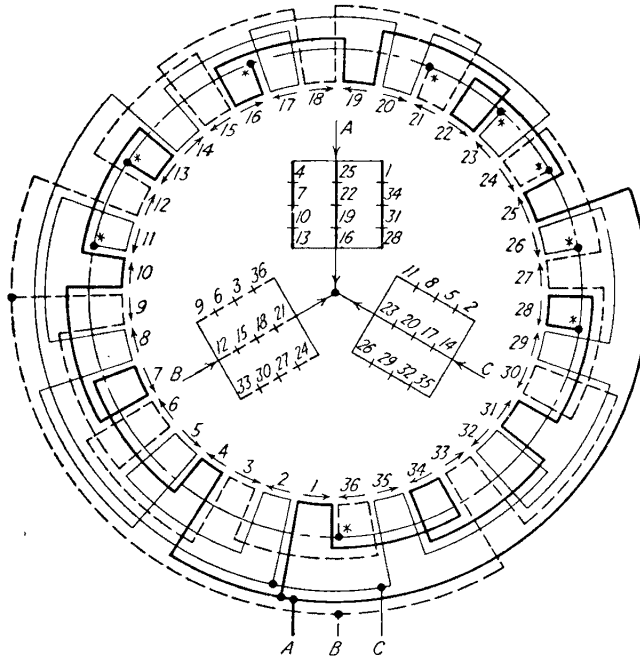


FIG. 328.—Twelve-pole, three-phase, three-parallel star.

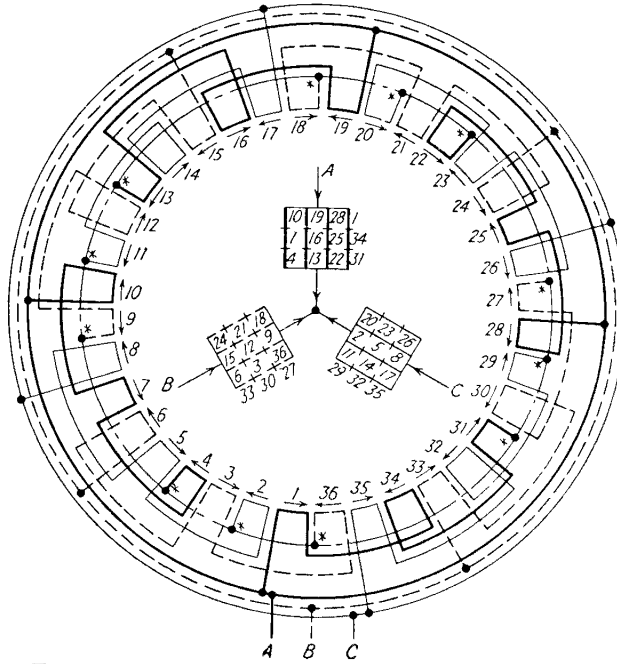


FIG. 329.—Twelve-pole, three-phase, four-parallel star.

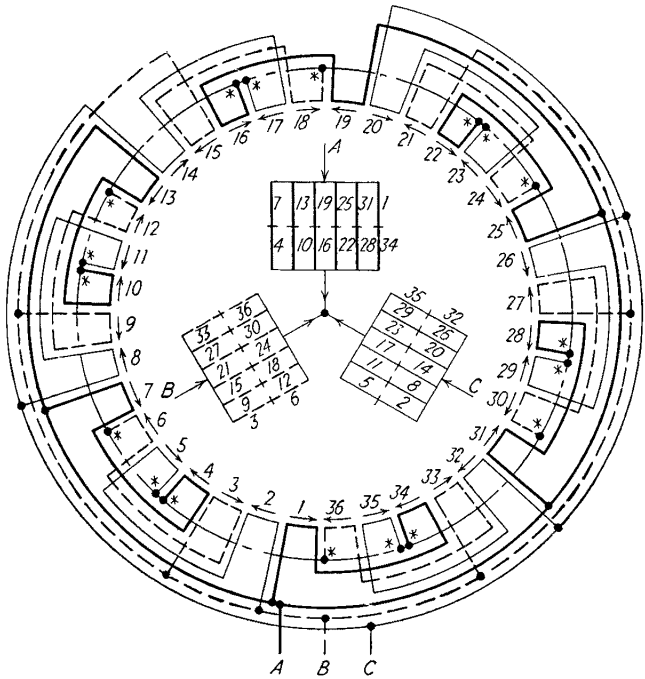


FIG. 330.—Twelve-pole, three-phase, six-parallel star.

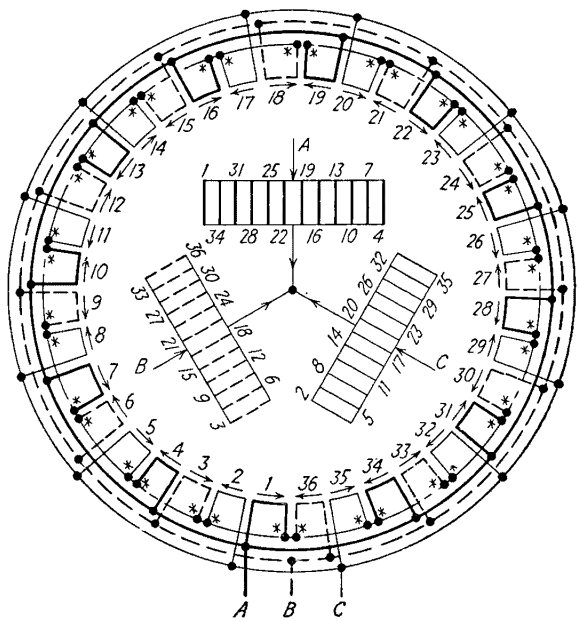


FIG. 331.—Twelve-pole, three-phase, twelve-parallel star.

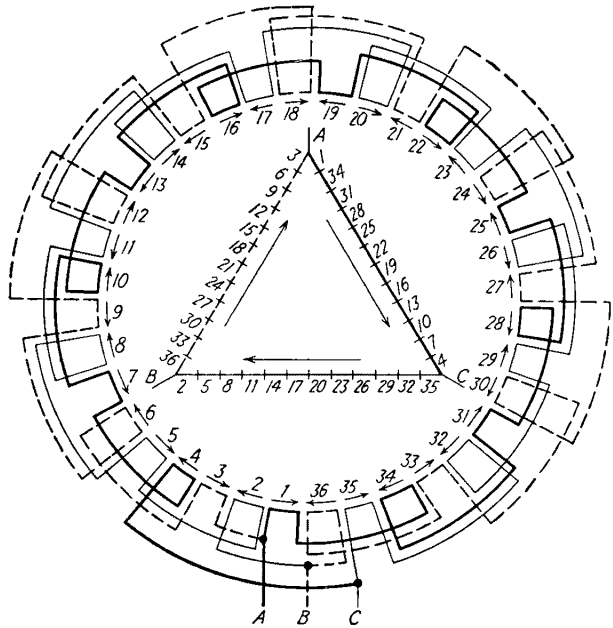


FIG. 332.—Twelve-pole, three-phase, series delta.

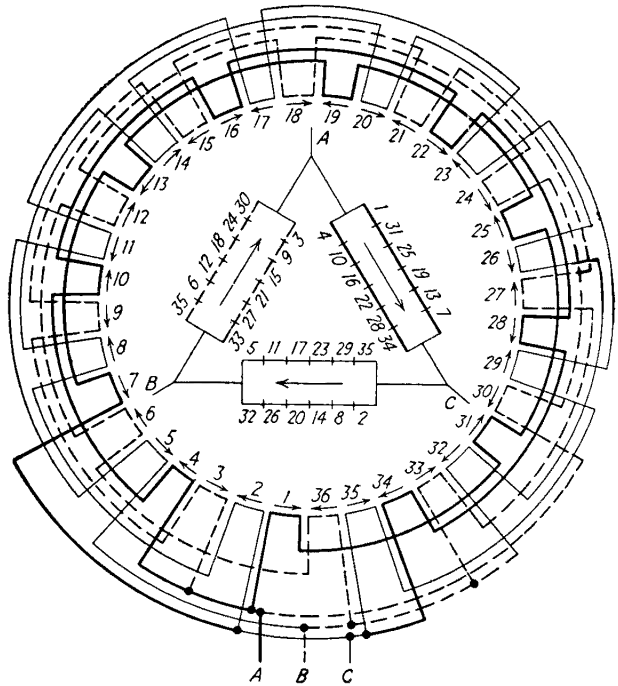


FIG. 333.—Twelve-pole, three-phase, two-parallel delta.

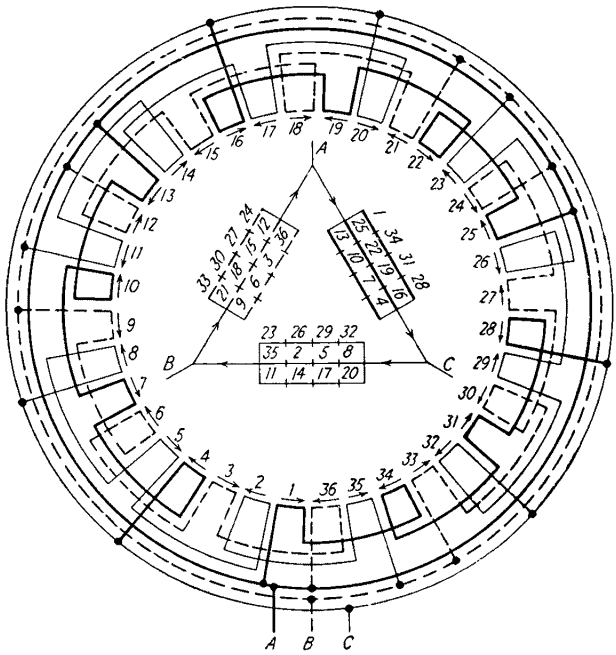


FIG. 334.—Twelve-pole, three-phase, three-parallel delta.

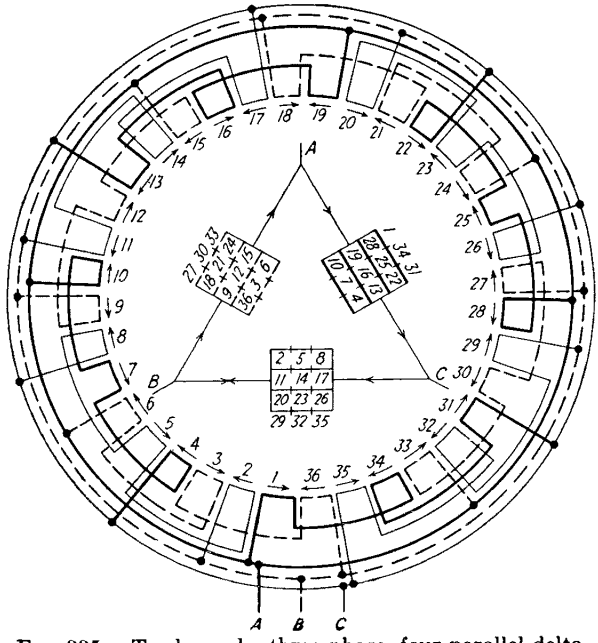


FIG. 335.—Twelve-pole, three-phase, four-parallel delta.

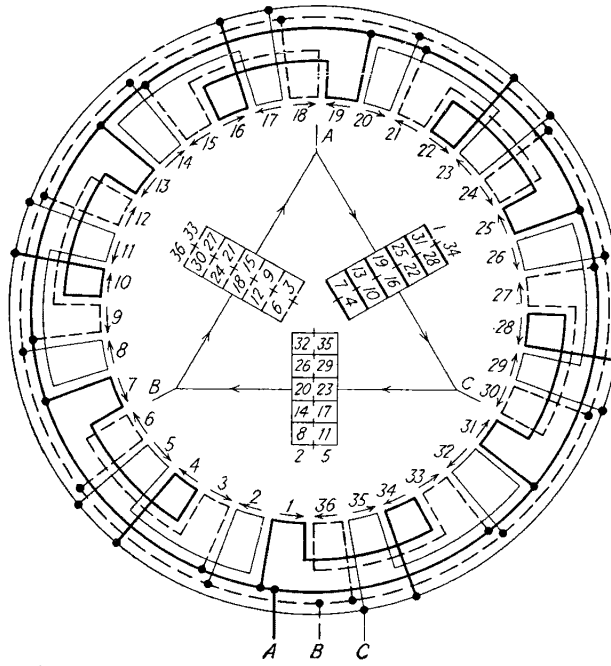


FIG. 336.—Twelve-pole, three-phase, six-parallel delta.

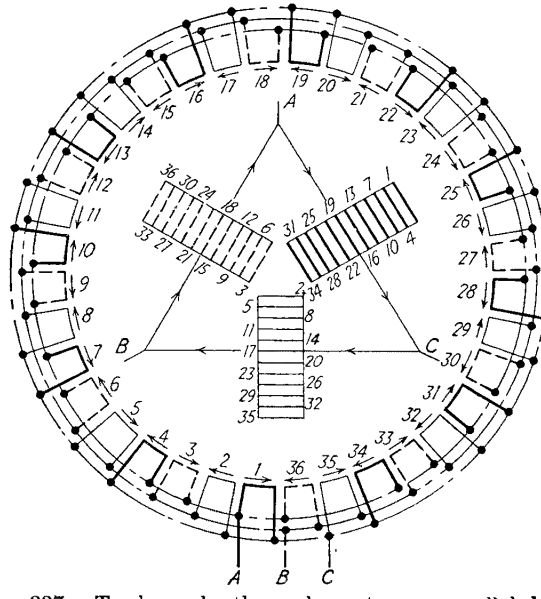


FIG. 337.—Twelve-pole, three-phase, twelve-parallel delta.

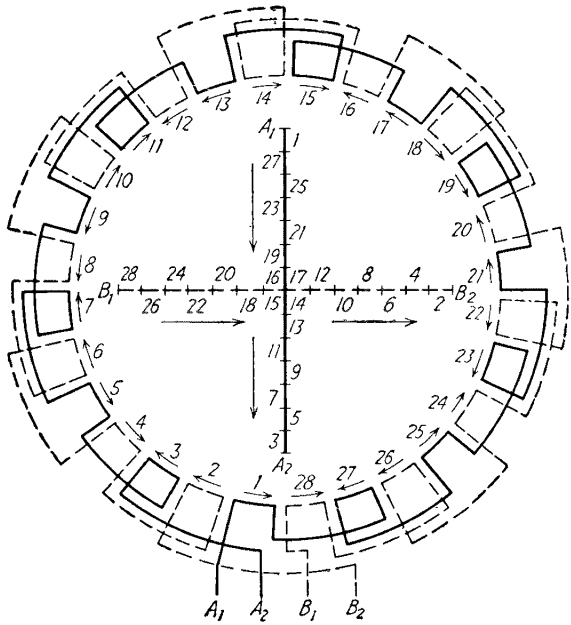


FIG. 338.—Fourteen-pole, two-phase, series.

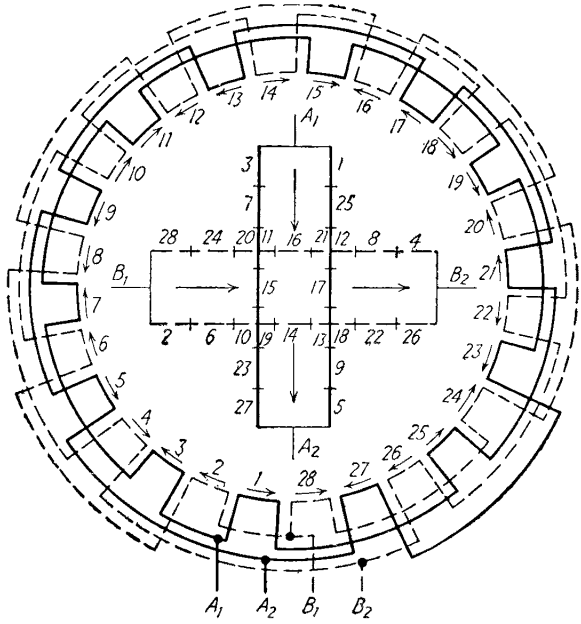


FIG. 339.—Fourteen-pole, two-phase, two-parallel.

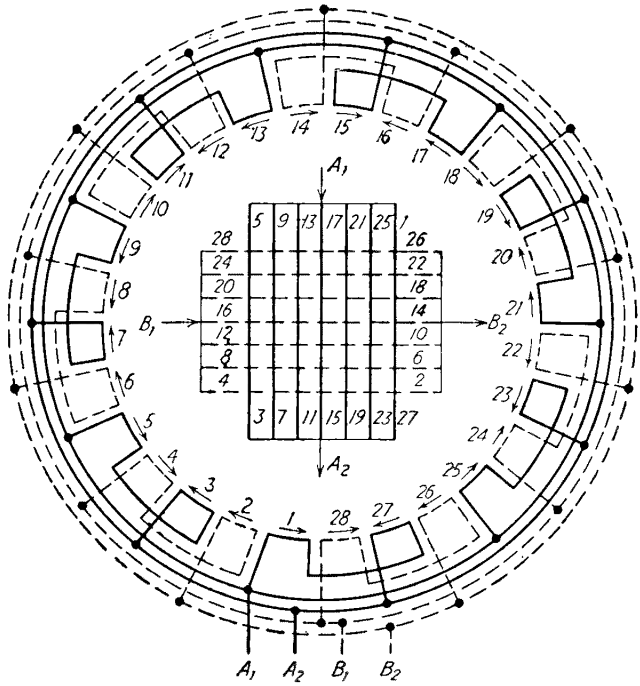


FIG. 340.—Fourteen-pole, two-phase, seven-parallel.

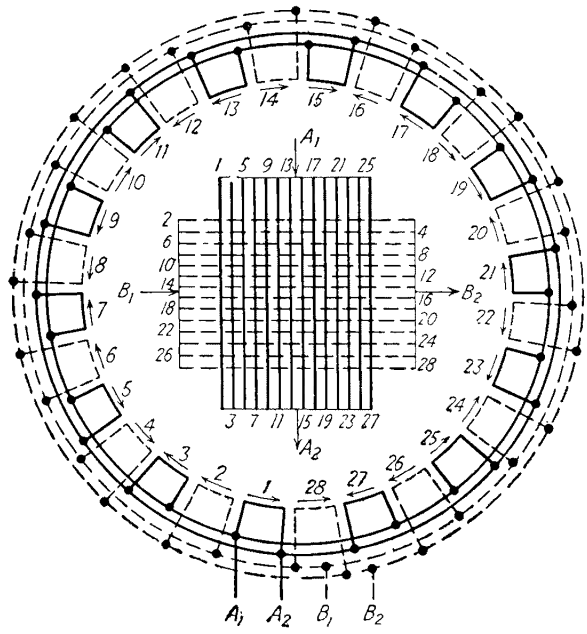


FIG. 341.—Fourteen-pole, two-phase, fourteen-parallel.

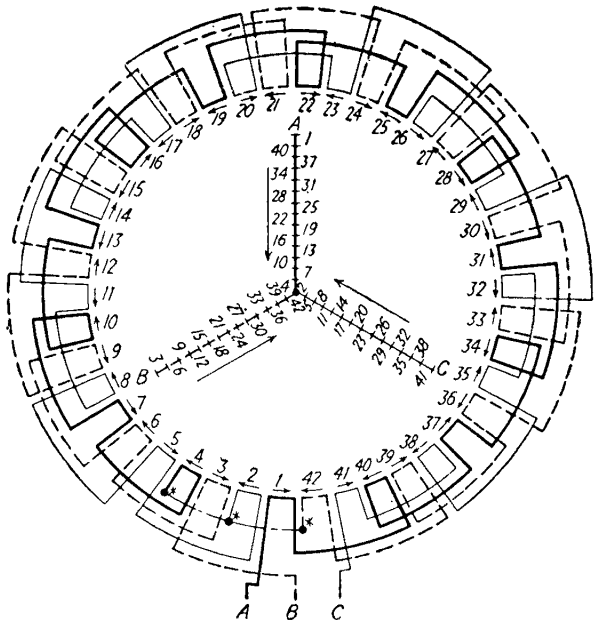


FIG. 342.—Fourteen-pole, three-phase, series star.

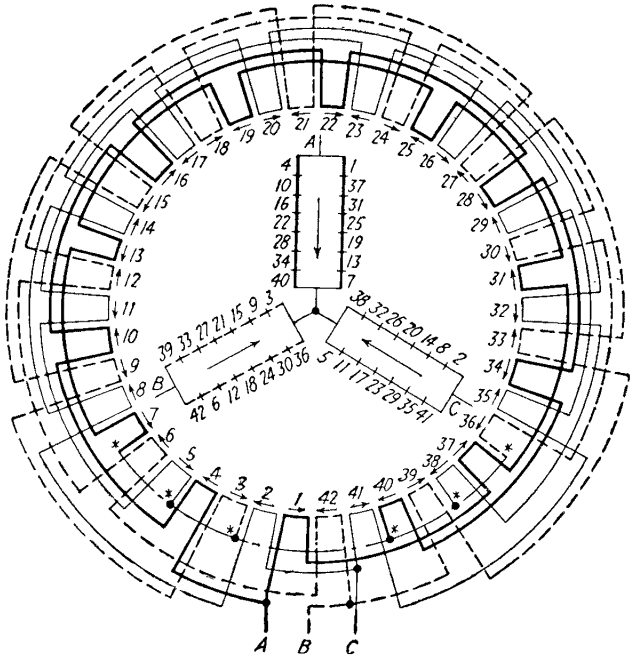


FIG. 343.—Fourteen-pole, three-phase, two-parallel star.

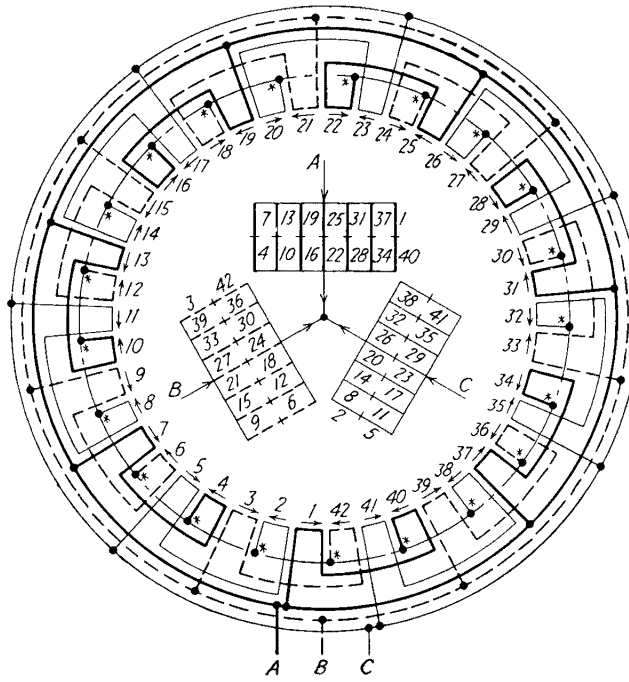


FIG. 344.—Fourteen-pole, three-phase, seven-parallel star.

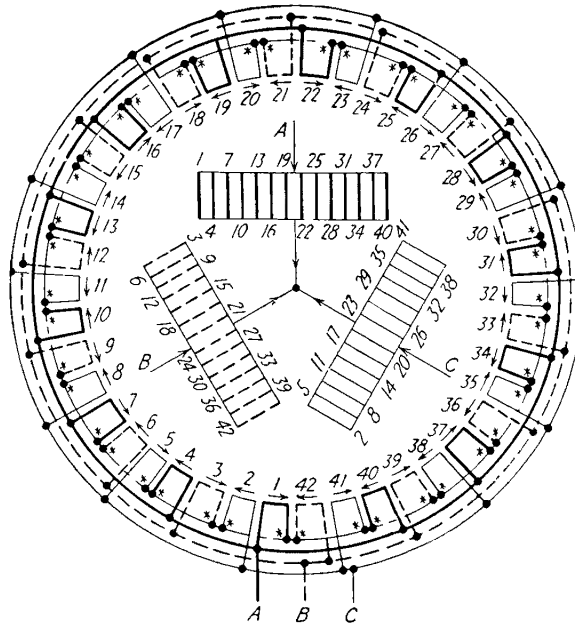


FIG. 345.—Fourteen-pole, three-phase, fourteen-parallel star.

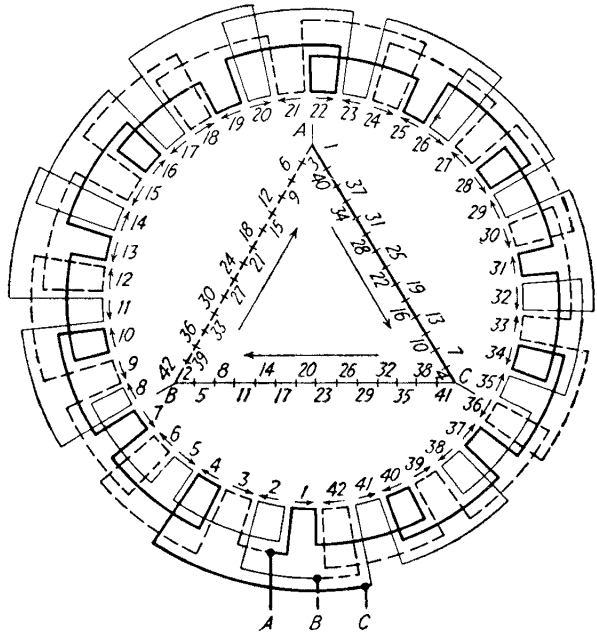


FIG. 346.—Fourteen-pole, three-phase, series delta.

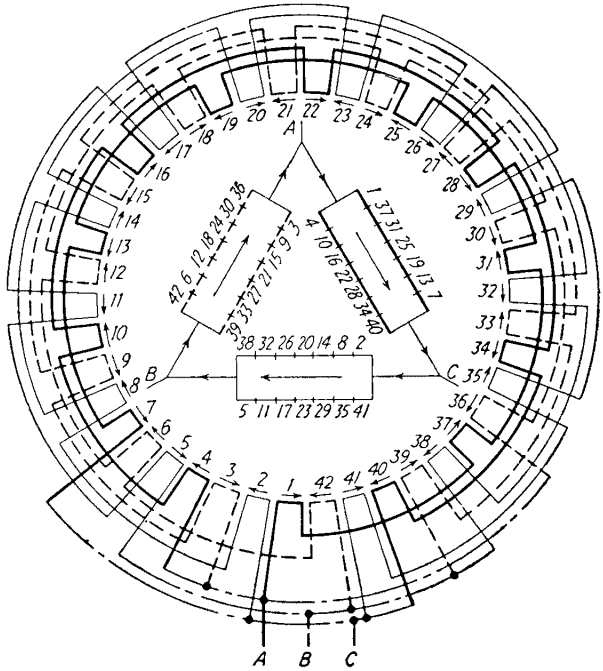


FIG. 347.—Fourteen-pole, three-phase, two-parallel delta.

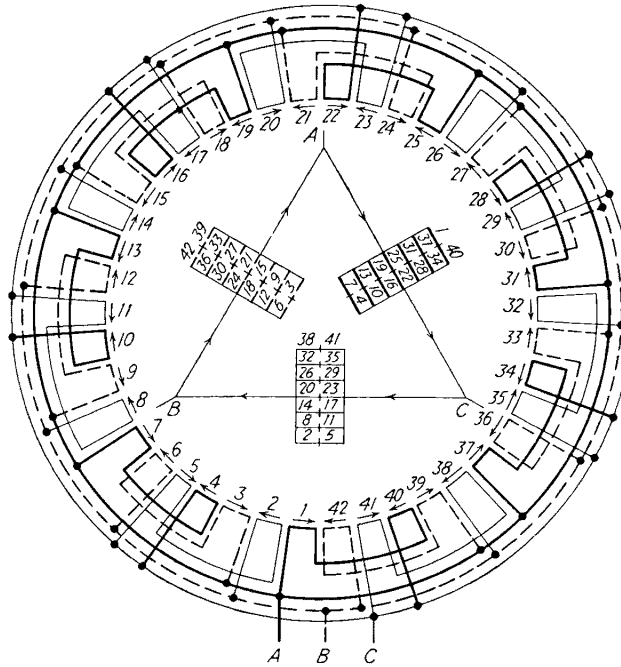


FIG. 348.—Fourteen-pole, three-phase, seven-parallel delta.

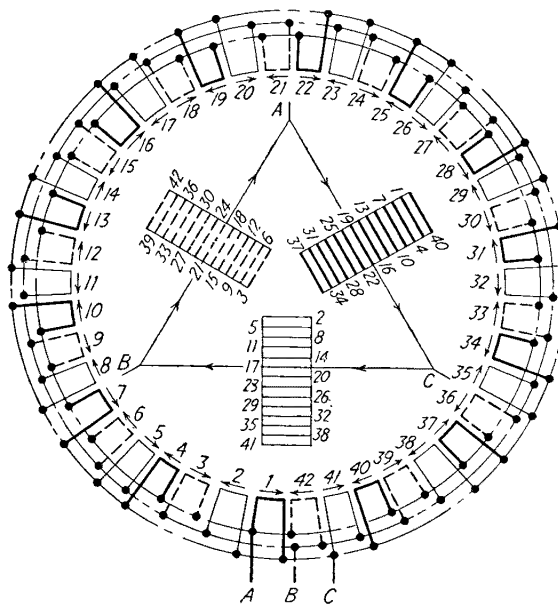


FIG. 349.—Fourteen-pole, three-phase, fourteen-parallel delta.

the slots by the winder and no connections have been made. The wires, which are the beginnings and endings of the coils, are sticking out at random. In Fig. 351 the coils have been connected into several distinct groups, and the remaining wires, which protrude radially toward and away from the center of the

FIG. 350.—Coils wound but unconnected.

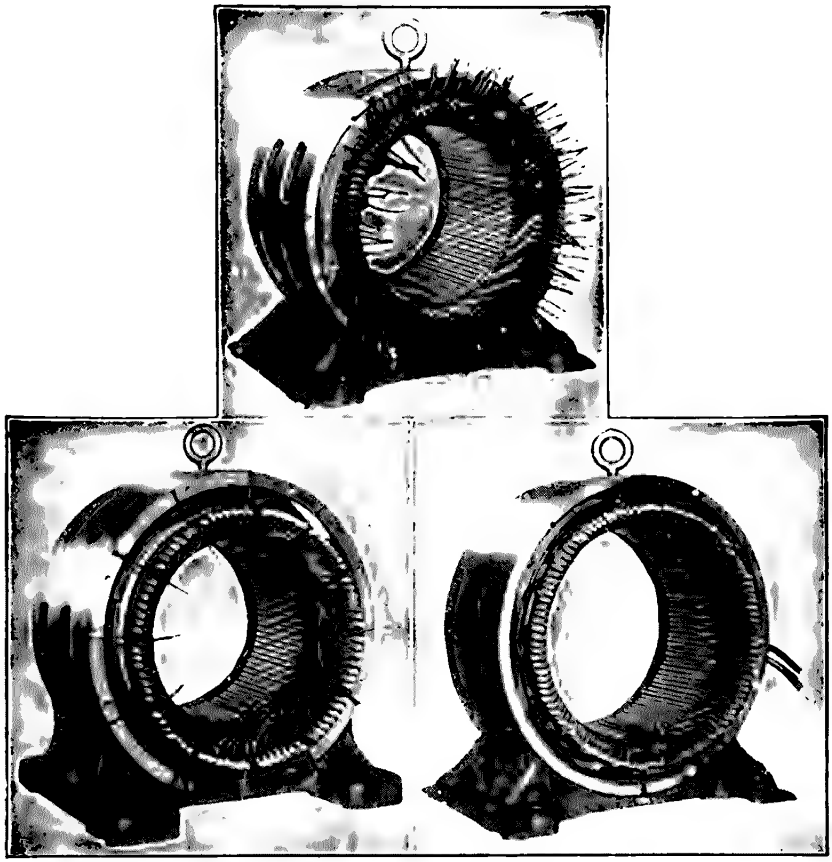


FIG. 351.—Coils "stubbed" or connected with pole-phase groups.

FIG. 352.—The completed connection.

machine, form the beginning and the end of each pole-phase group. The operation which has been performed between Fig. 350 and Fig. 351 can be described in this way: Suppose, for example, that there are 96 total coils in the winding and that it is to be connected for three phases and four poles. There will then be $3 \times 4 = 12$ pole-phase groups, and $96 \div 12 = 8$

coils in each group. Starting at any arbitrary point, the winder connects the first eight coils in series by connecting the end of coil 1 to the beginning of coil 2, and the end of coil 2 to the beginning of coil 3, etc., until eight coils are in series. The beginning of coil 1 is then bent outward and left long and the end of coil 8 is bent inward and left long. Between these two are seven short "stubs" or coil-to-coil connections, which are shown taped up in Fig. 351. The winder then proceeds to connect coils 9 to 16 in series, in the same manner, to form pole-phase group 2, and so on around the machine until he has completed 12 pole-phase groups and used all the coils. The winding should look as shown in Fig. 351.

In case the winding has certain coils provided with heavier insulation on the end turns to take the strain of the full voltage of the machine where different phases are adjacent, the operation is slightly different. Then, the number of coils per pole-phase group must be checked before the windings are inserted in the slots, and specially insulated phase coils placed on both ends of each group. In this case the location of the pole-phase groups is definitely determined by the winder before he starts connecting the coils together.

The next step is to mark the pole-phase groups *A-B-C-A-B-C*, etc., around the machine and then to connect all the groups together in the proper manner to form a three-phase winding by means of a diagram of the same form as those shown in this chapter. The completed winding will then appear as shown in Fig. 352.

While it is intended to reproduce here only the standard diagrams over a wide range of speeds, it is useful to review the general theory of their construction and the simple methods by which any winding may be checked for phase polarity. This is shown in Figs. 353 to 356, inclusive. In Fig. 353, a winding chosen at random is shown "stubbed" into pole-phase groups for a two-phase connection, and in Fig. 355, stubbed for a three-phase connection. To determine the proper connections for the pole-phase groups in a two-phase winding, the rule is to mark on the groups, arrows alternating in direction in pairs, *i.e.*, on two successive groups the arrows are clockwise and on the two immediately adjacent the arrows are counterclockwise. Such arrows, for example, are shown in Fig. 353 just above the windings.

If now one end of any group in a phase is chosen as a lead and all the groups are followed through and connected as indicated by the arrows, the connection will be correct. Such a connection

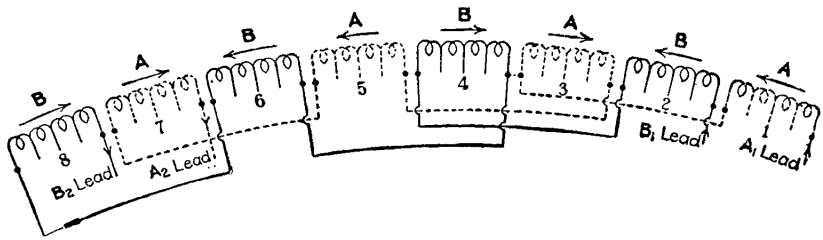


FIG. 353.—Checking a two-phase connection.

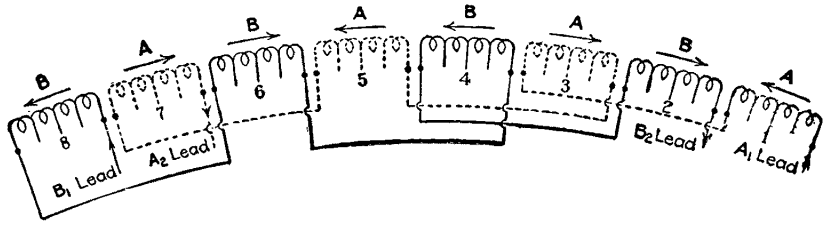


FIG. 354.—Similar to Fig. 353, but "B" phase reversed.

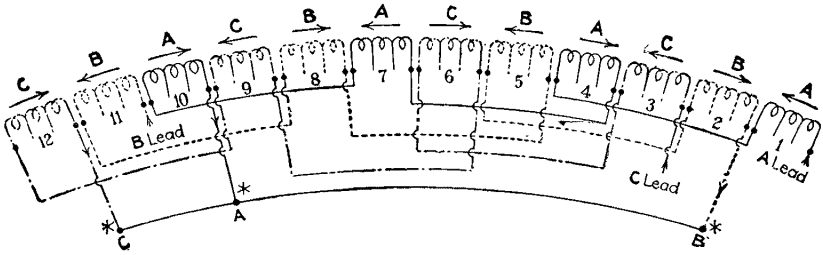


FIG. 355.—Checking a three-phase connection.

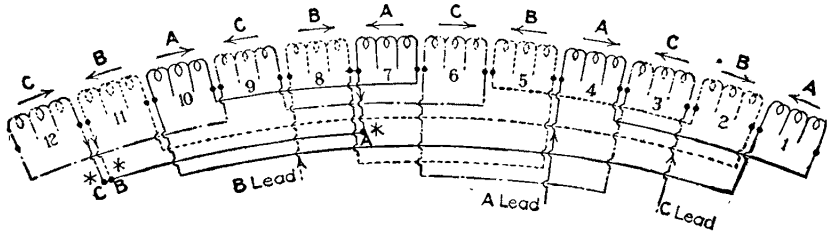


FIG. 356.—Similar to Fig. 355, but leads taken off different groups.

is shown in Fig. 353. However, suppose the arrows had alternated in pairs, but started with a different group, as shown just above the windings in Fig. 354. The result is shown in Fig. 354,

which is just as correct as Fig. 353, except that the motor would run with the opposite direction of rotation. Since the rotation can be changed by reversing the two leads of either phase outside of the motor, it is evident that the rule using the arrows alternating in pairs is correct in all cases. It should also be noted that it makes no difference from what group the lead is taken, provided all the groups are followed through with the arrows.

In the three-phase machine it is even simpler. The rule in that case is to put arrows on the groups alternating in direction from group to group, as shown in Fig. 355. Any group may then be chosen as a "lead" group or a "star" group so long as the arrows are followed in passing *from the lead to the star* in each phase. Figure 355 shows one arrangement and Fig. 356 another, equally correct; there might be an indefinite number more, simply by choosing the lead from another group and following the arrows through to the star in each phase. Although shown for a developed four-pole winding only, these diagrams may be considered as strictly general, as additional groups may be added to make six, eight, or any other number of poles, and the current passed through them in any order, so long as the phases are kept in the correct rotation, and the current in the right direction, as indicated by the arrows.

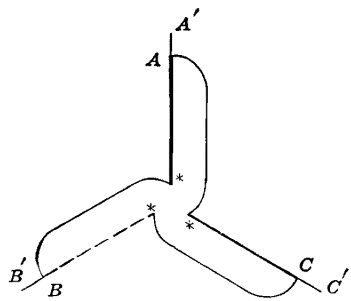


FIG. 357.—Changing from star to delta.

In case a delta connection is wanted instead of a star, check the connections through as for a star and then connect the A star to the B lead, the B star to the C lead, and the C star to the A lead, as shown in Fig. 357; or connect the A lead to the B star, the B lead to the C star, and the C lead to the A star. The three new leads will be taken from the corners of the delta so formed.

CHAPTER XVIII

LAP WINDINGS WITH UNEQUAL COIL GROUPINGS

Possibilities of Windings with Unequal Coil Grouping.—As long as the number of slots in which a lap winding is to be placed is an integral multiple of the product of the number of phases times the number of poles, there are an equal number of coils in each pole-phase group of that winding. For example, given 72 slots in which to place a three-phase six-pole lap winding, it appears at once that there will be four coils in each pole-phase group, since $\frac{72}{3 \times 6} = 4$. However, it is often possible to place a winding perfectly balanced as to voltage and phase relation in a number of slots not evenly divisible by the number of phases times the number of poles. Such a case, for example, would occur in winding 90 slots for a three-phase eight-pole connection. By inspection, it appears that in case all coils are used and none cut out or left dead there will be on an average $\frac{90}{3 \times 8} = 3\frac{3}{4}$ coils per pole-phase group. Expressed another way, six of the eight groups in each phase will contain four coils, and there will be two groups having only three coils each. For all practical purposes this odd grouping can often be done by inspection by putting in or leaving out the odd coils in a manner that seems to give reasonable symmetry. In most cases this would work out satisfactorily, particularly if the operator has had previous experience on similar odd groupings.

"Least Common Multiple" Scheme.—As distinguished from this more or less guess work, there is a helpful method known as the "least common multiple scheme," which permits these odd groupings to be laid out so that the voltage is exactly the same in all phases and the phase angle between phases is mathematically correct. In other words, so that it is theoretically a perfectly balanced winding.

This method was first published, so far as the author is aware, by E. M. Tingley in the *Electrical Review* and *Western Electrician*

for Jan. 23, 1915. The method is graphical, and when once thoroughly familiarized, forms a most useful tool for the practical armature winder or the designing engineer. It is true that the method does not cover all cases where a fairly good practical working connection may be made, but it has the advantage that where it does apply and works out, it gives a theoretically correct connection and insures the maximum satisfaction from that particular combination.

How to Lay Out a Winding by the Least Common Multiple Scheme.—The method is as follows: First, consider two numbers (*a*) the total number of slots in which the winding is to be placed, and (*b*) the product of the phase times the number of poles. Second, find the least common multiple of these two numbers. Assume as a concrete case the one mentioned above of 90 slots and a three-phase eight-pole winding. The two basic numbers in this case are 90 and 24, since $3 \times 8 = 24$. Find the least common multiple as follows:

$$90 = 3 \times 3 \times 5 \times 2$$
$$24 = 2 \times 2 \times 2 \times 3.$$

Use each factor the maximum number of times it occurs in either number. Thus:

$$\textit{Least common multiple} = 3 \times 3 \times 5 \times 2 \times 2 \times 2 = 360$$

Now in the graphical scheme that is to follow one slot will be represented by $360/90 = 4$ dots or spaces and one-pole phase group will be represented by $360/24 = 15$ dots or spaces. Now set down 360 dots arranged as in Fig. 358, which has 45 dots in a row horizontally, representing the three pole-phase groups that make up one pole and having eight vertical rows to represent all eight poles.

Then starting at the upper left-hand corner, place a check mark over the first dot. This check mark represents the first coil in the first *A* phase group. As stated above, four dots represent one slot, so that four dots are counted to the right and a second check placed over the fifth dot. Count four again and place a check over the ninth dot, and so on through the entire 360 dots, placing a check over each fourth one consecutively. This means that dots 1, 5, 9, 13, 17, 21, 25 and so on up to 357 have a check over them. The next step is to count the check marks between

Poles	Phase *A*	Phase *B*	Phase *C*
1	4	4	4
2	3	4	4
3	4	3	4
4	4	4	3
5	4	4	4
6	3	4	4
7	4	3	4
8	4	4	3

Number of coils in each pole-phase group.
Phase *A* 4, 3, 4, 4, 4, 3, 4, 4.
Phase *B* 4, 4, 3, 4, 4, 4, 3, 4.
Phase *C* 4, 4, 4, 3, 4, 4, 4, 3.

One side of each coil is represented in the diagram by a "⌐." The other side of the coil could be located if desired by allowing for the pitch or span of the coil.

The above diagram shows that this winding may be connected in two parallels or two multiple circuits since the diagram forms two similar series for each phase.

FIG. 358.—Sample "least common multiple" diagram for determining number of coils in each pole-phase group.

the vertical lines which divide the dots into groups of fifteen. That is, taking the top line opposite pole 1, there are 4 checks under phase *A*, 4 under phase *B* and 4 under phase *C*. In the second line, there are 3 under *A*, 4 under *B*, and four under *C*. Third line, 4 under *A*, 3 under *B*, 4 under *C* and so on. The next step is to write these numbers down in this order, thus:

4-4-4, 3-4-4, 4-3-4, 4-4-3, 4-4-4, 3-4-4, 4-3-4, 4-4-3 and they represent the proper number of coils in each phase group passing around the winding through the 24 pole-phase groups *A*-*B*-*C*, *A*-*B*-*C*, etc. or setting down the vertical columns as in Fig. 358 shows all the phase *A* groups in order.

Will Such a Winding Parallel?—A second important fact that can be noted from Fig. 358 is whether or not a winding so laid out can be paralleled or whether it is good only in series. This is shown by noting whether the check marks repeat after half the dots have been passed over. A check was placed over dot 1 in the first line and it is noted that opposite pole 5 the check mark again falls on dot 1 in the fifth line, or dot 181 counting all the way through. This means, in turn, that the second half of the grouping exactly repeats and duplicates the first half. Since this is true, the grouping is perfectly proper for connecting in parallel, but a "top to top" diagram should be used as in Fig. 369 or 370 for paralleling such unequal groupings in all cases rather than a "top to bottom" diagram.

From another standpoint it might have been supposed that the winding would parallel, and that is because the total number of coils (90) is divisible by 6, which is the total number of paths. This can be appreciated by reference to Fig. 359, which shows a three-phase eight-pole two-parallel star, "top to top" diagram. Since there are three phases and two paths in each phase, there are six total paths, and since there are 90 coils in all, there must be 15 coils in each path.

From Fig. 359, which is laid out from Fig. 358, it is noted that groups 1, 22, 19, 16 are in one path of the *A* phase and that groups 4, 7, 10, 13 are in the other path. Counting the number of coils in these groups gives $4 + 4 + 4 + 3 = 15$ in one path, and $3 + 4 + 4 + 4 = 15$ in the other path, which is correct. Were it otherwise, for example, $4 + 4 + 4 + 4 = 16$ in one path and $3 + 4 + 3 + 4 = 14$ in the other path, the two halves of the *A* phase would be unequal in voltage, and there would be

constantly flowing around the closed loop in the *A* phase a local circulating current, heating up the winding even though the motor might be running light and carrying no load. Hence, it is always necessary to check the parallel paths of an unequal coil-grouped winding to be sure all paths in the same phase have the same number of coils.

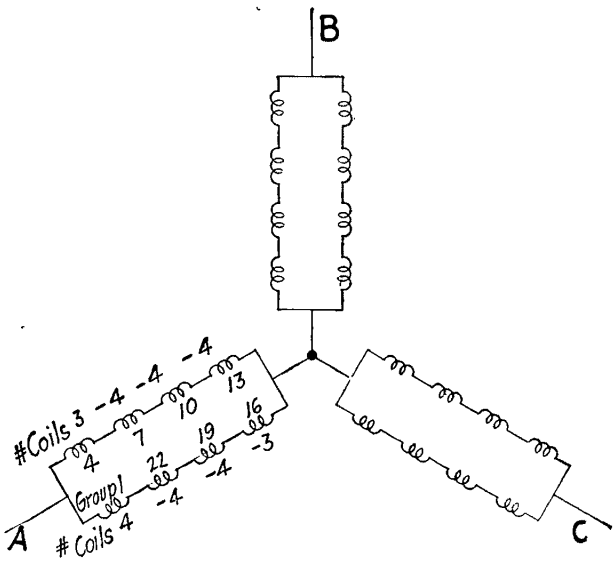
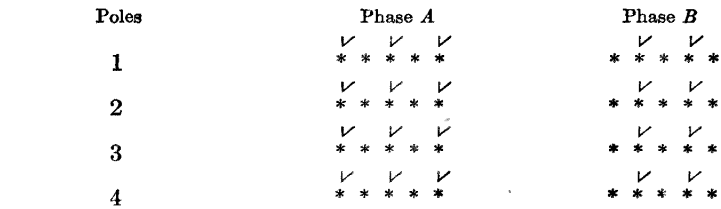


FIG. 359.—Eight-pole, three-phase, two-parallel star, schematic diagram showing that total number of coils in each branch of each phase must be the same to avoid local circulating currents.

Number of Coils in Each Path.—While it is better that all paths in all phases be the same, this is not so essential as that all the paths in the same phase have the same number of coils. For example, in Fig. 359, the two paths in the *A* phase and the *C* phase might have 15 coils each and the two paths in the *B* phase have but 14 and the motor operate fairly well, whereas if one of the paths in the *A* and *B* phases had 15 coils and the other path in parallel with it had only 14 coils, the local circulating currents in each phase might produce destructive heating.

Two-phase Condition to Look Out For.—There is one main precaution that must be used with this least common multiple scheme, and that is the tendency shown in Fig. 360. This shows, for example, a two-phase four-pole winding placed in 20 slots. Here the least common multiple of 8 ($= 2 \times 4$) and

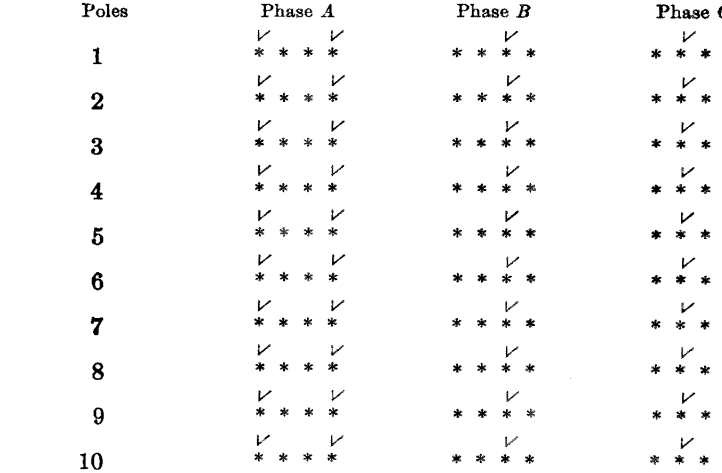
20 is 40. Hence 5 dots represent a pole-phase group and 2 dots represent a slot. Laying out the table as in Fig. 360, the coil groups come 3, 2, 3, 2, 3, 2, 3, 2. Obviously, this would put all



Coil groups 3, 2, 3, 2, 3, 2, 3, 2.
Use instead 3, 3, 2, 2, 3, 3, 2, 2.

FIG. 360.

the groups with three coils in the *A* phase and all those with two coils in the *B* phase. The proper remedy or correction for this is to place the coils 3, 3, 2, 2, 3, 3, 2, 2, in which case it is apparent that each phase has two coil groups of three and two coil groups of two; the winding will parallel if desired, since there would be four total paths and five coils (= 2 + 3) in each path.



Coils run 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, etc.
Use instead 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1.

FIG. 361.

Three-phase Condition to Look Out For.—A similar warning is necessary for three-phase windings, as, for example, in a case represented by Fig. 361, where a three-phase 10-pole winding

is laid out for 40 slots. In this case the two basic numbers are 30 ($= 3 \times 10$) and 40. The least common multiple is 120, so there is a total of 120 dots. Four dots represent a pole-phase group and three dots a slot. It will be noted that the coils per group work out 2, 1, 1, 2, 1, 1, 2, 1, 1, etc., which would put all the groups with two coils in the *A* phase. These must, therefore, be arranged so that the group with two coils falls successively in the *A*, *B* and *C* phase as shown under Fig. 361. In doing this try to avoid having two adjacent groups with two coils as would be the case if they were arranged 2, 1, 1, 1, 2, 1, 1, 2, 2, 1, 1, etc.[1]

From this it will be seen that, while the least common multiple scheme does not always produce a solution and sometimes produces a wrong solution, nevertheless it is easily checked and forms a very simple and convenient starting point from which to lay out a winding having unequal coil groups.

Explanation of Following Tables.—With the idea of giving suggestions and offering possible solutions, there follow in this chapter tables showing windings with unequal coil groupings for both two- and three-phase, also star- and delta-connected, windings for speeds from 2 to 14 poles inclusive. These tables are similar to those designed by A. C. Roe for the *Electrical World* and used in connection with the large charts which he originated for taking care of windings with unequal coil groupings.

In the tables as given the coil groups are numbered and these numbers correspond to those on the groups in the diagram referred to. Hence, if coil group 12, for example, in the tables has four coils and the table says use diagram *C*, then group 12 in diagram *C* should have four coils, etc.

[1] There are some excellent suggestions along this line in Terrell Croft's book, "Alternating-current Armature Winding," McGraw-Hill Book Company, Inc.

TABLE XIII.—TWO-PHASE—TWO POLES

Number of slots	Pole-phase groups				Possible connections as grouped
	1	2	3	4	
18	4	4	5	5	Series
18	4	4	4*X*	4*X*	Parallel
30	7	7	8	8	Series
30	7	7	7*X*	7*X*	Parallel
42	10	10	11	11	Series
42	10	10	10*X*	10*X*	Parallel
54	13	13	14	14	Series
54	13	13	13*X*	13*X*	Parallel
90	22	22	23	23	Series
90	22	22	22*X*	22*X*	Parallel
126	31	31	32	32	Series
126	31	31	31*X*	31*X*	Parallel
150	37	37	38	38	Series
150	37	37	37*X*	37*X*	Parallel

X indicates dead coil in location shown. Numbers indicate coils per group.

Diagrams for connecting:

Fig. 269 for series connection.

Fig. 270 for parallel connection.

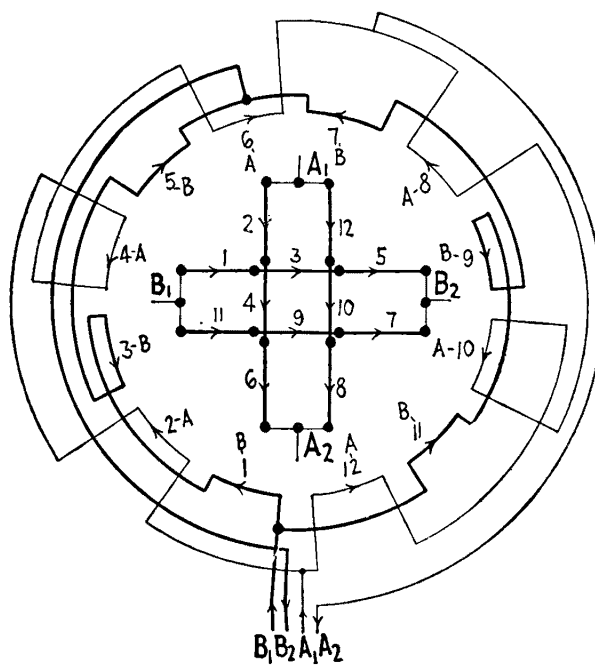


FIG. 362.—Six-pole, two-phase, two-parallel, top to top connection.

TABLE XIV.—TWO-PHASE—FOUR POLES

Number of slots	Pole-phase groups								Possible connections as grouped
	1	2	3	4	5	6	7	8	
12	1	1	2	2	1	1	2	2	Series and two parallel
18	2*X*	2	2	2	2*X*	2	2	2	Series, two parallel and four parallel
20	2	2	3	3	2	2	3	3	Series and two parallel
28	3	3	4	4	3	3	4	4	Series and two parallel
30	4	4	3	4	4	3	4	4	Series
30	3	3*X*	4	4	3	3*X*	4	4	Series and two parallel
36	4	4	5	5	4	4	5	5	Series and two parallel
42	5*X*	5	5	5	5*X*	5	5	5	Series two parallel and four parallel
54	7	7	7	6	7	7	6	7	Series
54	7	7	6*X*	6	7	7	6	6*X*	Series and two parallel
84	11	11	10	10	11	11	10	10	Series and two parallel
90	11	11*X*	11	11	11	11*X*	11	11	Series, two parallel and four parallel
108	13	13	14	14	13	13	14	14	Series and two parallel
126	15	15	16	16	15	15	16	16	Series and two parallel
140	17	17	18	18	17	17	18	18	Series and two parallel

X indicates dead coil in location shown. Numbers indicate coils per group.
Diagrams for connecting:
Fig. 275 for series connection.
Fig. 276 for two-parallel connection.
Fig. 277 for four-parallel connection.

TABLE XV.—TWO-PHASE—SIX POLES

Number of slots	Pole-phase groups												Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	
18	1	1	2	2	1	1	2	2	1	1	2	2	Series and three parallel
20	2	2	1	2	2	1	2	2	1	2	2	1	Series and two parallel
28	2	2	3	2	2	3	2	2	3	2	2	3	Series and two parallel
30	2	2	3	3	2	2	3	3	2	2	3	3	Series and three parallel
32	3	3	2	3	3	2	3	3	2	3	3	2	Series and two parallel
42	3	3	4	4	3	3	4	4	3	3	4	4	Series and three parallel
54	4	4	5	5	4	4	5	5	4	4	5	5	Series and three parallel
64	6	5	5	6	5	5	6	5	5	6	5	5	Series and two parallel
80	6	7	7	6	7	7	6	7	7	6	7	7	Series and two parallel
90	7	7	8	8	7	7	8	8	7	7	8	8	Series and three parallel
104	8	9	9	8	9	9	8	9	9	8	9	9	Series and two parallel
112	10	9	9	10	9	9	10	9	9	10	9	9	Series and two parallel
128	10	11	11	10	11	11	10	11	11	10	11	11	Series and two parallel
140	11	12	12	11	12	12	11	12	12	11	12	12	Series and two parallel
150	12	12	13	13	12	12	13	13	12	12	13	13	Series and three parallel
160	14	13	13	14	13	13	14	13	13	14	13	13	Series and three parallel

Numbers indicate coils per group.

Diagrams for connecting:

Fig. 284 for series connection.

Fig. 362 for two-parallel connection.

Fig. 286 for three-parallel connection.

TABLE XVI.—TWO-PHASE—EIGHT POLES

Number of slots	Pole-phase groups																Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
20	2	1	1	2	1	1	1	1	2	1	1	2	1	1	1	1	Series and two parallel
24	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	Series, two and four parallel
36	3	2	2	2	2	3	2	2	3	2	2	2	2	3	2	2	Series and two parallel
36	2*X*	2	2	2	2	2*X*	2	2	2*X*	2	2	2	2	2*X*	2	2	Four parallel and eight parallel
42	3	3	2	2*X*	3	3	2	2	3	3	2	2*X*	3	3	2	2	Series, four parallel
54	4	4	3	3	3	3*X*	3	3	4	4	3	3	3*X*	3	3	3	Series and two parallel
60	3	4	4	4	4	3	4	4	3	4	4	4	4	3	4	4	Series and two parallel
72	5	5	4	4	5	5	4	4	5	5	4	4	5	5	4	4	Series, two parallel and four parallel
84	6	5	5	5	5	6	5	5	6	5	5	5	5	6	5	5	Series and two parallel
90	6	6	5*X*	5	6	6	5	5	6	6	5	5*X*	6	6	5	5	Series, two parallel and four parallel
104	7	7	6	6	7	7	6	6	7	7	6	6	7	7	6	6	Series, two parallel and four parallel
108	6	7	7	7	7	6	7	7	6	7	7	7	7	6	7	7	Series and two parallel
120	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	Series, two and four parallel
150	10	9	9*X*	9	9	10	9	9	10	9	9	9*X*	9	10	9	9	Series and two parallel
150	9*X*	9	9*X*	9	9	9*X*	9	9	9*X*	9	9	9*X*	9	9*X*	9	9	Four and eight parallel
168	11	11	10	10	11	11	10	10	11	11	10	10	11	11	10	10	Series two and four parallel
180	12	11	11	11	11	12	11	11	12	11	11	11	11	12	11	11	Series and two parallel
216	14	14	13	13	14	14	13	13	14	14	13	13	14	14	13	13	Series, two and four parallel

X indicates dead coil in location shown. Numbers indicate coils per group.

Diagrams for connecting:

Fig. 296 for series connection.

Fig. 363 (new) for two-parallel connection.

Fig. 298 for four-parallel connection.

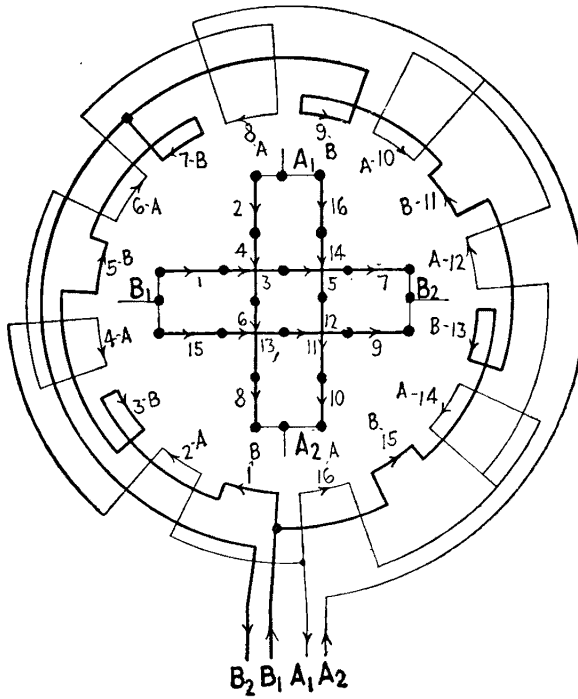


FIG. 363.—Eight-pole, two-phase, two-parallel, top to top connection.

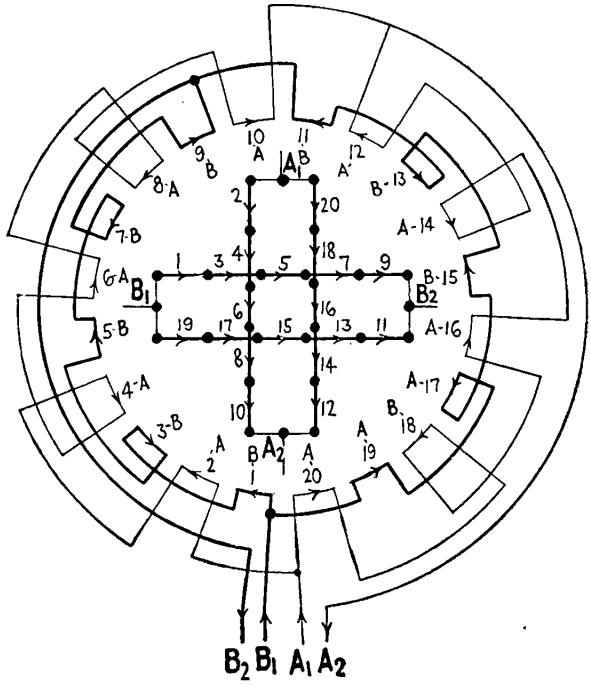


FIG. 364.—Ten-pole, two-phase, two-parallel, top to top connection.

TABLE XVII.—TWO-PHASE—10 POLES

Number of slots	Pole-phase groups																				Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
30	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	Series and five parallel
36	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	Series and two parallel
48	3	2	3	2	2	3	2	3	2	2	3	2	3	2	2	3	2	3	2	2	Series and two parallel
54	2	3	3	2	3	3	2	3	3	3	2	3	3	2	3	3	2	3	3	3	Series
72	4	4	3	4	3	4	4	3	4	3	4	4	3	4	3	4	4	3	4	3	Series and two parallel
84	5	4	4	4	4	5	4	4	4	4	5	4	4	4	4	5	4	4	4	4	Series and two parallel
90	5	5	4	4	5	5	4	4	5	5	4	4	5	5	4	4	5	5	4	4	Series and five parallel
90	4*X*	5	4	4	5	5	4	4	5	4*X*	4	4	5	5	4	4	5	5	4	4	Two parallel
96	4	5	5	5	5	4	5	5	5	5	4	5	5	5	5	4	5	5	5	5	Series and two parallel
108	6	5	6	5	5	6	5	6	5	5	6	5	6	5	5	6	5	6	5	5	Series and two parallel
128	7	6	7	6	6	7	6	7	6	6	7	6	7	6	6	7	6	7	6	6	Series and two parallel
144	8	7	7	7	7	8	7	7	7	7	8	7	7	7	7	8	7	7	7	7	Series and two parallel
150	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	Series and five parallel
150	7	7	8	8	7	7	8	8	7	7	8	7*X*	7	7	8	8	7	7	7*X*	8	Two parallel
168	9	8	9	8	8	9	8	9	8	8	9	8	9	8	8	9	8	9	8	8	Series and two parallel
216	10	11	11	11	11	10	11	11	11	11	10	11	11	11	11	10	11	11	11	11	Series and two parallel

X indicates dead coil in location shown. Numbers indicate coils per group.

Diagrams for connecting:

Fig. 308 for series connection.

Fig. 364 for two-parallel connection.

Fig. 310 for five-parallel connection.

TABLE XVIII.—TWO-PHASE—12 POLES

Number of slots	Pole-phase groups																								Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
30	1	1	2	1	1	1	1	2	1	1	2	1	1	1	1	2	1	1	2	1	1	1	1	2	Series and three parallel
36	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	Series, two parallel, three parallel and six parallel
42	2	2	1	2	2	2	2	1	2	2	1	2	2	2	2	1	2	2	1	2	2	2	2	1	Series and three parallel
54	2	2	3	2	2	2	2	3	2	2	3	2	2	2	2	3	2	2	3	2	2	2	2	3	Series and three parallel
60	2	2	3	3	2	2	3	3	2	2	3	3	2	2	3	3	2	2	3	3	2	2	3	3	Series, two parallel, three parallel and six parallel
80	4	3	3	4	3	3	4	3	3	4	3	3	4	3	3	4	3	3	4	3	3	4	3	3	Series, two parallel and four parallel
90	4	4	3	4	4	4	4	3	4	4	3	4	4	4	4	3	4	4	3	4	4	4	4	3	Series and three parallel
104	5	4	4	5	4	4	5	4	4	5	4	4	5	4	4	5	4	4	5	4	4	5	4	4	Series, two parallel and four parallel
128	6	5	5	6	5	5	6	5	5	6	5	5	6	5	5	6	5	5	6	5	5	6	5	5	Series, two parallel and four parallel
150	6	6	7	6	6	6	6	7	6	6	7	6	6	6	6	7	6	6	7	6	6	6	6	7	Series and three parallel
180	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	Series, two parallel, three parallel and six parallel
210	9	9	9	8	9	9	8	9	9	9	9	8	9	9	8	9	9	9	9	8	9	9	8	9	Series and three parallel
252	10	10	11	11	10	10	11	11	10	10	11	11	10	10	11	11	10	10	11	11	10	10	11	11	Series, two parallel, three parallel and six parallel

Numbers indicate coils per group.

Diagrams for connecting:

Fig. 320 for series connection.

Fig. 365 for two-parallel connection.

Fig. 322 for three-parallel connection.

Fig. 323 for four-parallel connection.

Fig. 324 for six-parallel connection.

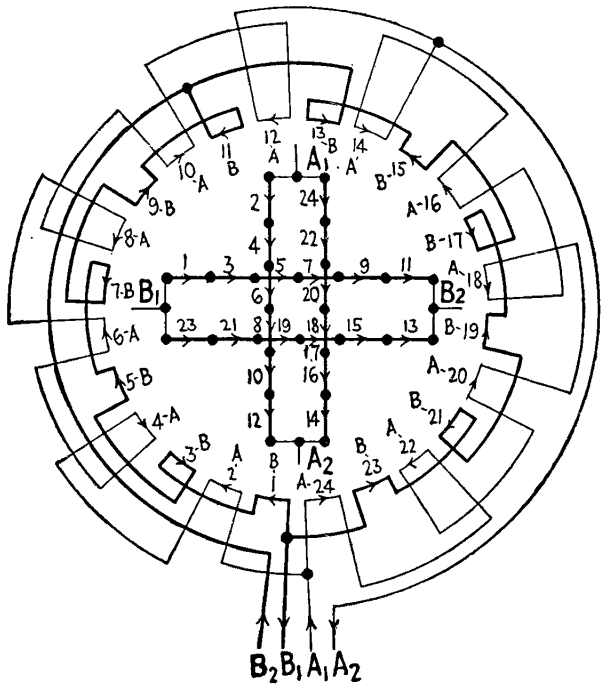


FIG. 365.—Twelve-pole, two-phase, two-parallel, top to top connection.

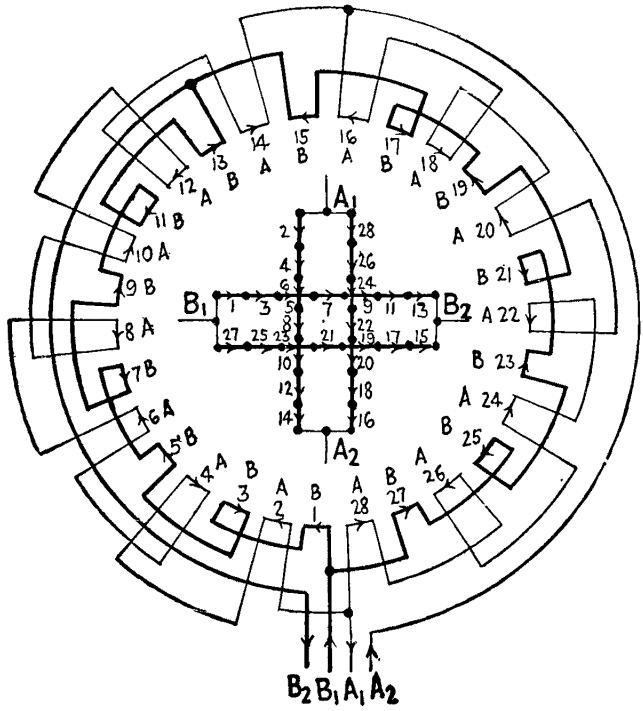


FIG. 366.—Fourteen-pole, two-phase, two-parallel, top to top connection.

TABLE XIX.—TWO-PHASE—14 POLES

Number of slots	Pole-phase groups																												Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	
36	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	2	1	1	2	1	1	1	Series and two parallel
42	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2	Series and seven parallel
54	1	2	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	Series
60	3	2	2	2	2	2	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	3	2	2	2	2	2	2	Series and two parallel
72	3	3	2	2	3	3	2	3	3	2	2	3	2	2	3	3	2	2	3	3	2	3	3	2	2	3	3	2	Series and two parallel
80	2	3	3	3	3	3	3	2	3	3	3	3	3	3	2	3	3	3	3	3	3	2	3	3	3	3	3	3	Series and two parallel
96	4	4	3	3	4	3	3	4	4	3	3	4	3	3	4	4	3	3	4	3	3	4	4	3	3	4	3	3	Series and two parallel
108	3	4	4	4	4	4	4	3	4	4	4	4	4	4	3	4	4	4	4	4	4	3	4	4	4	4	4	4	Series and two parallel
120	4	4	5	4	4	5	4	4	4	5	4	4	5	4	4	4	5	4	4	5	4	4	4	5	4	4	5	4	Series and two parallel
144	6	5	5	5	5	5	5	6	5	5	5	5	5	5	6	5	5	5	5	5	5	6	5	5	5	5	5	5	Series and two parallel
180	6	6	7	7	6	6	7	6	6	7	7	6	6	7	6	6	7	7	6	7	7	6	6	7	7	6	7	7	Series and two parallel
210	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	7	7	8	8	Series, two parallel and seven parallel
240	9	9	8	8	9	9	8	9	9	8	8	9	9	8	9	9	8	8	9	9	8	9	9	8	8	9	9	8	Series and two parallel

Numbers indicate coils per group.

Diagrams for connecting:

Fig. 238 for series connection.

Fig. 366 for two-parallel connection.

Fig. 340 for seven-parallel connection.

TABLE XX.—THREE-PHASE—TWO POLES

Number of slots	Pole-phase groups						Possible connections as grouped
	1	2	3	4	5	6	
16	2*X*	3	2	3	2	3	Series
20	3*X*	3	3	3*X*	3	3	Series and two parallel
28	4*X*	5	4	5	4	5	Series
32	5*X*	5	5	5*X*	5	5	Series and two parallel
40	6*X*	7	6	7	6	7	Series
56	9*X*	9	9	9*X*	9	9	Series and two parallel
64	10*X*	11	10	11	10	11	Series
80	13*X*	13	13	13*X*	13	13	Series and two parallel
100	16*X*	17	16	17	16	17	Series
112	18*X*	19	18	19	18	19	Series
128	21*X*	21	21	21*X*	21	21	Series and two parallel
140	23*X*	23	23	23*X*	23	23	Series and two parallel
160	26*X*	27	26	27	26	27	Series

X indicates dead coil in location as shown. Numbers indicate coils per group. Connections indicated are proper for either star or delta.

Diagrams for connecting:

Fig. 271 for series star.

Fig. 272 for two-parallel star.

Fig. 273 for series delta.

Fig. 274 for two-parallel delta.

TABLE XXI.—THREE-PHASE—FOUR POLES

Number of slots	Pole-phase groups												Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	
16	1*X*	1	1	1	2	1	2	1	1	1	1	2	Series only
18	1	2	1	2	1	2	1	2	1	2	1	2	Series and two parallel
20	1*X*	2	1	2	1	2	1*X*	2	1	2	1	2	Series and two parallel
28	2*X*	2	2	3	2	2	2	3	2	2	2	3	Series only
30	2	3	2	3	2	3	2	3	2	3	2	3	Series and two parallel
32	2*X*	3	2	3	2	3	2*X*	3	2	3	2	3	Series and two parallel
40	3*X*	3	3	4	3	3	3	4	3	3	3	4	Series only
42	3	4	3	4	3	4	3	4	3	4	3	4	Series and two parallel
54	4	5	4	5	4	5	4	5	4	5	4	5	Series and two parallel
56	4*X*	5	4	5	4	5	4*X*	5	4	5	4	5	Series and two parallel
64	5*X*	5	5	6	5	5	5	6	5	5	5	6	Series only
80	6*X*	7	6	7	6	7	6*X*	7	6	7	6	7	Series and two parallel
90	7	8	7	8	7	8	7	8	7	8	7	8	Series and two parallel
100	9	8	8	8	9	8	8	8	9	8*X*	8	8	Series only
112	10	9	9	9	10	9	9	9	10	9*X*	9	9	Series only
126	10	11	10	11	10	11	10	11	10	11	10	11	Series and two parallel
128	10*X*	11	10	11	10	11	10*X*	11	10	11	10	11	Series and two parallel
140	11*X*	12	11	12	11	12	11*X*	12	11	12	11	12	Series and two parallel
150	12	13	12	13	12	13	12	13	12	13	12	13	Series and two parallel
160	13*X*	13	13	14	13	13	13	14	13	13	13	14	Series only

X indicates dead coil in location as shown. Numbers indicate coils per group. Connections shown are proper for either star or delta.

Diagrams for connecting:

Fig. 278 for series star.

Fig. 281 for series delta.

Fig. 279 for two-parallel star.

Fig. 282 for two-parallel delta.

TABLE XXII.—THREE-PHASE—SIX POLES

Number of slots	Pole-phase groups																		Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
20	1*X*	1	1	1	1	1	1	1	1	1*X*	1	1	1	1	1	1	1	1	Series, two parallel, three parallel and six parallel
28	1*X*	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	Series only
30	2	2	1	2	1	2	1	2	2	2	2	1	2	1	2	1	2	2	Series and two parallel
32	2*X*	2	1	2	1	2	1	2	2*X*	2	2	1	2	1	2	1	2	2	Series and two parallel
40	2*X*	2	3	2	2	2	2	3	2	2	2	2	3	2	2	2	2	2	Series only
42	2	2	3	2	3	2	3	2	2	2	2	3	2	3	2	3	2	2	Series and two parallel
56	3*X*	3	3	3	3	3	3	3	3*X*	3	3	3	3	3	3	3	3	3	Series, two parallel, three parallel and six parallel
64	3*X*	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	Series and three parallel
80	4*X*	4	5	4	5	4	5	4	4	4*X*	4	5	4	5	4	5	4	4	Series and two parallel
100	5*X*	6	5	6	5	6	5	6	5	6	5	6	5	6	5	6	5	6	Series and three parallel
112	6*X*	6	6	6	7	6	6	6	6	7	6	6	6	6	7	6	6	6	Series only
112	6*X*	6	6	6	6*X*	6	6	6	6	6*X*	6	6	6	6	6*X*	6	6	6	Series, two parallel and three parallel
128	7*X*	7	7	7	7	7	7	7*X*	7	7	7	7	7	7	7	7	7	7	Series, two parallel and three parallel
140	7*X*	8	7	8*X*	7	8	7	8*X*	7	8	7	8*X*	7	8	7*X*	8	7	8	Series and three parallel
150	8	8	9	8	9	8	9	8	8	8	8	9	8	9	8	9	8	8	Series and two parallel
160	8*X*	9	9	9	8*X*	9	9	9	8	8*X*	9	9	9	8*X*	9	9	9	8	Series and two parallel

X indicates dead coil in location shown. Numbers indicate coils per group. Connections shown are proper for either star or delta.

Diagrams for connecting:

Fig. 288 for series star.
Fig. 367 for two-parallel star.
Fig. 290 for three-parallel star.
Fig. 291 for six-parallel star.
Fig. 292 for series delta.
Fig. 368 for two-parallel delta.
Fig. 294 for three-parallel delta.
Fig. 295 for six-parallel delta.

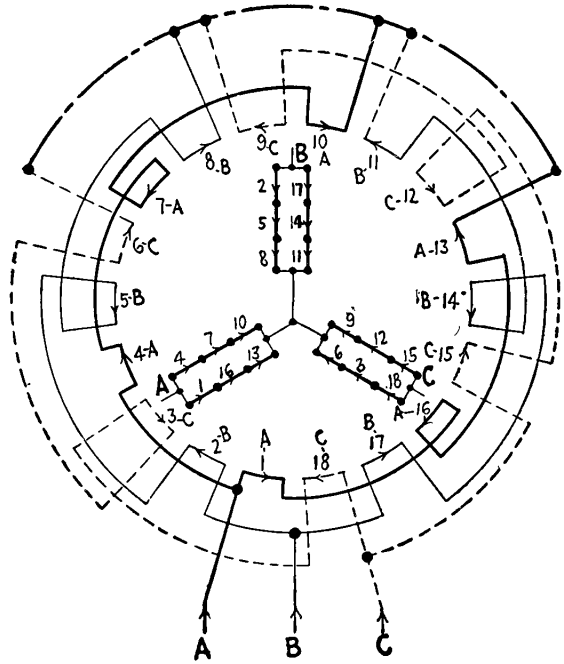


FIG. 367.—Six-pole, three-phase, two-parallel star, top to top connection.

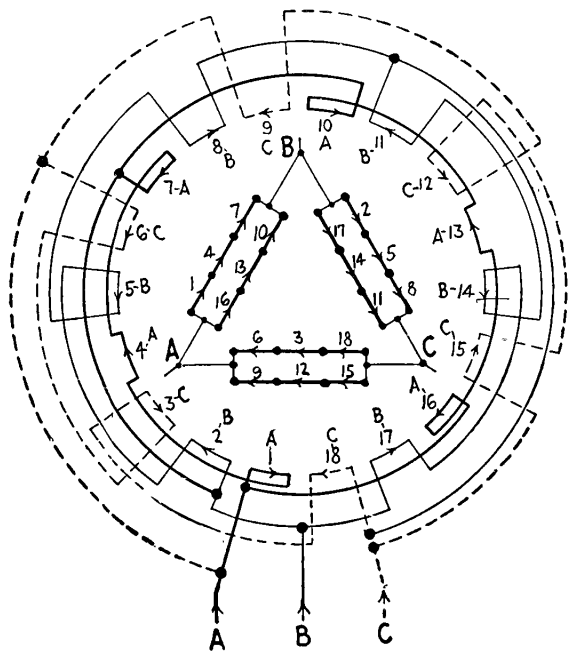


FIG. 368.—Six-pole, three-phase, two-parallel delta, top to top connection.

TABLE XXIII.—THREE-PHASE—EIGHT POLES

Number of slots	Pole-phase groups																							Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
28	1*X*	1	1	1	1	1	1*X*	1	1	1	1	1	1*X*	1	1	1	1	1	1*X*	1	1	1	1	1	Series, two parallel and four parallel
30	1	1	1	2	1	1	1	2	1	1	1	2	1	1	1	2	1	1	1	2	1	1	1	2	Series and two parallel
32	2	1	1*X*	1	2	1	1	1	2	1	1	1	2	1	1*X*	1	2	1	1	1	2	1	1	1	Series and two parallel
36	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	Series, two parallel and four parallel
40	2	2	2	1	2	1	2	1*X*	2	1	2	2	2	1	2	1	2	1	2	1	2	2	2	1	Series only
42	1	2	2	2	1	2	2	2	1	2	2	2	1	2	2	2	1	2	2	2	1	2	2	2	Series and two parallel
54	3	2	2	2	3	2	2	2	3	2	2	2	3	2	2	2	3	2	2	2	3	2	2	2	Series and two parallel
60	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	Series, two parallel and four parallel
64	3	2*X*	3	2	3	2	3	3	3	2	3	2	3	2	3	3	3	2	3	2	3	2	3	3	Series only
80	4	3	3*X*	3	4	3	3	3	4	3	3	3	4	3	3*X*	3	4	3	3	3	4	3	3	3	Series and two parallel
90	3	4	4	4	3	4	4	4	3	4	4	4	3	4	4	4	3	4	4	4	3	4	4	4	Series and two parallel
100	4*X*	4	4	4	4	4	4*X*	4	4	4	4	4	4*X*	4	4	4	4	4	4*X*	4	4	4	4	4	Series, two parallel and four parallel
108	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	Series, two parallel and four parallel
112	4	5*X*	4	5	4	5	4	5*X*	4	5	4	5	4	5*X*	4	5	4	5	4	5*X*	4	5	4	5	Series, two parallel and four parallel
126	6	5	5	5	6	5	5	5	6	5	5	5	6	5	5	5	6	5	5	5	6	5	5	5	Series and two parallel
128	6	5	5	5*X*	6	5	5	5	6	5	5	5	6	5	5*X*	5	6	5	5	5	6	5	5	5	Series and two parallel
140	6	6*X*	6	5	6	6	6	5	6	6	6	5	6	6*X*	6	5	6	6	6	5	6	6	6	5	Series and two parallel
150	6	6	6	7	6	6	6	7	6	6	6	7	6	6	6	7	6	6	6	7	6	6	6	7	Series and two parallel
160	6*X*	7	6	7	6	7	6*X*	7	6	6	7	7	7*X*	6	6	7	7	6	6*X*	6	7	7	6	7	Series, two parallel and four parallel
180	7	8	7	8	7	8	7	8	7	8	7	8	7	8	7	8	7	8	7	8	7	8	7	8	Series, two parallel and four parallel

X indicates dead coil in location shown. Numbers indicate coils per group. Connections indicated are proper for star or delta.

Diagrams for connecting:

Fig. 300 series star.
Fig. 369 two-parallel star.
Fig. 302 four-parallel star.
Fig. 304 series delta.
Fig. 370 two-parallel delta.
Fig. 306 four-parallel delta.

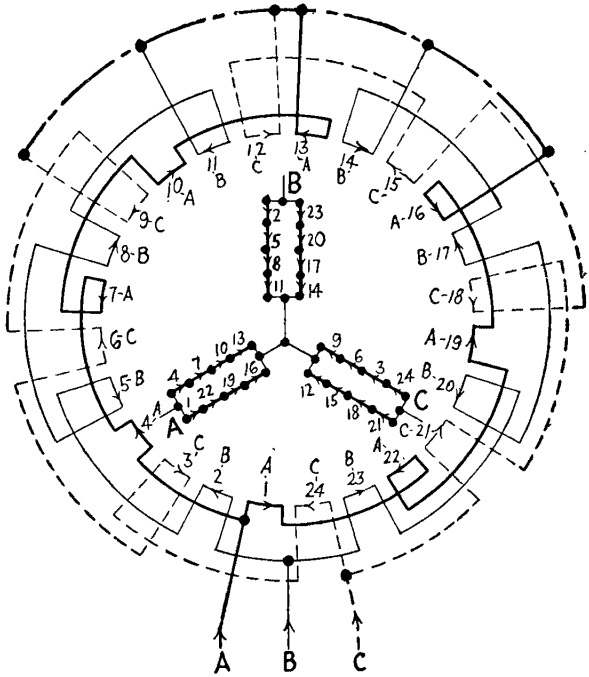


FIG. 369.—Eight-pole, three-phase, two-parallel star, top to top connection.

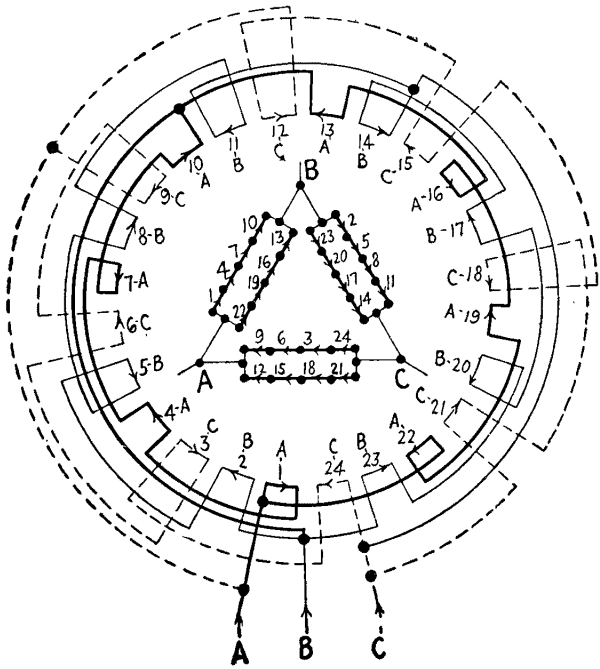


FIG. 370.—Eight-pole, three-phase, two-parallel delta, top to top connection.

TABLE XXIV.—THREE-PHASE—10 POLES

Number of slots	Pole-phase groups																												Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
36	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1	Series and two parallel
40	1	2*X*	1	2	1	1	1	1	2	1	2	1	2	1	1	1	1	2	1	2	1	2	1	1	1	1	2	1	2	1	Series
42	2	1	2	1	1	2	1	2	1	1	2	1	2	1	1	2	1	2	1	1	2	1	2	1	1	2	1	2	1	1	Series and two parallel
54	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	2	2	2	2	1	Series and two parallel
72	3	2	3	2	2	3	2	3	2	2	3	2	3	2	2	3	2	3	2	2	3	2	3	2	2	3	2	3	2	2	Series and two parallel
75	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	Series and five parallel
80	3	3	2	3	2	3	2	3	3	3	3	2	3*X*	2	3	2	3	3	3	3	2	3	2	3	2	3	3	2*X*	2	2	Series and two parallel
96	4	3	3	3	3	4	3	3	3	3	4	3	3	3	3	4	3	3	3	3	4	3	3	3	3	4	3	3	3	3	Series and two parallel
108	4	4	3	4	3	4	4	3	4	3	4	4	3	4	3	4	4	3	4	3	4	4	3	4	3	4	4	3	4	3	Series and two parallel
112	4	4*X*	3	4	3	4	4	3	4*X*	3	4	4	3	4	3	4	4*X*	3	4	3	4	4	3	4*X*	3	4	4	3	4	3	Series and two parallel
128	5	4	4*X*	4	4	5	4	4	4	4	5	4	4	4	4	5	4	4*X*	4	4	5	4	4	4	4	5	4	4	4	4	Series and two parallel
135	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	4	5	Series and five parallel
144	4	5	5	5	5	4	5	5	5	5	4	5	5	5	5	4	5	5	5	5	4	5	5	5	5	4	5	5	5	5	Series and two parallel
168	6	6	6	5	5	6	6	6	5	5	6	6	6	5	5	6	6	6	5	5	6	6	6	5	5	6	6	6	5	5	Series and two parallel
192	7	6	7	6	6	7	6	7	6	6	7	6	7	6	6	7	6	7	6	6	7	6	7	6	6	7	6	7	6	6	Series and two parallel
216	8	7	7	7	7	8	7	7	7	7	8	7	7	7	7	8	7	7	7	7	8	7	7	7	7	8	7	7	7	7	Series and two parallel
252	9	8	9	8	8	9	8	9	8	8	9	8	9	8	8	9	8	9	8	8	9	8	9	8	8	9	8	9	8	8	Series and two parallel

X indicates dead coil in location shown. Numbers indicate coils per group. Connections indicated are proper for either star or delta.

Diagrams for connecting:

Fig. 312 for series star.

Fig. 371 for two-parallel star.

Fig. 314 for five-parallel star.

Fig. 316 for series delta.

Fig. 372 for two-parallel delta.

Fig. 318 for five-parallel delta.

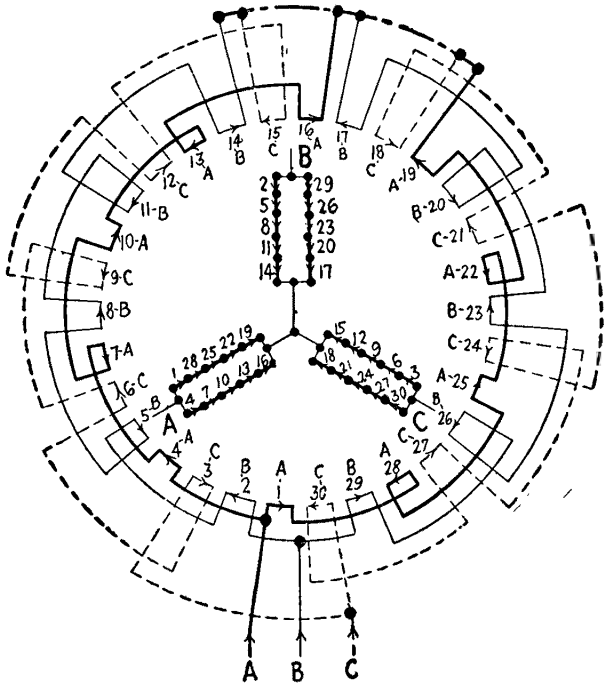


FIG. 371.—Ten-pole, three-phase, two-parallel star, top to top connection.

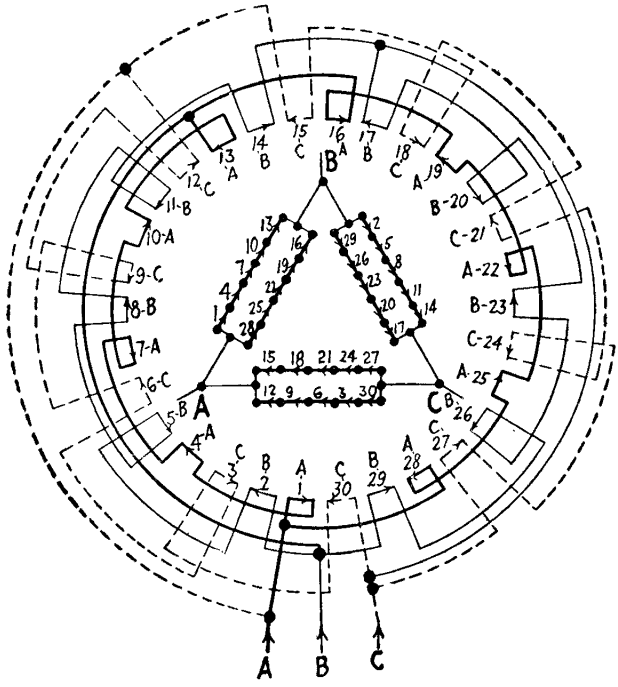


FIG. 372.—Ten-pole, three-phase, two-parallel delta, top to top connection.

TABLE XXV.—THREE-PHASE—12 POLES

Number of slots	Pole-phase groups																																Possible connections as grouped
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
42	2	1	1	1	1	1	1	2	1	1	1	1	1	1	2	1	1	1	2	1	1	1	1	1	1	2	1	1	1	1	1	2	1	1	1	Series and two parallel
54	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2	Series and two parallel
64	2	2	2	1	2	2	2	1	2	2	2	1	2	2	2	1	2	2X	2	1	2	2	2	1	2	2	2	1	2	2	2	1	2	2	2	1	Series
80	3	2	2X	2	2	2	2	3	2	2	2	2	2	2	3	2	2	2	3	2	2X	2	2	2	2	3	2	2	2	2	2	2	3	2	2	2	Series and two parallel
90	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	3	2	Series, two parallel, three parallel and six parallel
112	3X	3	3	3	3	3	3	3	3	3X	3	3	3	3	3	3	3	3	3X	3	3	3	3	3	3	3	3	3X	3	3	3	3	3	3	3	3	Two parallel, three parallel, four parallel, six parallel and twelve parallel
112	4	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3	3	3	3	3	3	4	3	3	3	3	3	3X	3	3	3	3	3	3	3	3	Series only
120	4	3	3	3	4	3	3	3	4	4	3	3	3	4	3	3	3	4	4	3	3	3	4	3	3	3	4	4	3	3	3	4	3	3	3	4	Series, two parallel; and four parallel
128	3X	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4X	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	3	4	Series, two parallel three parallel and six parallel
132	3	4	4	4	3	4	4	4	3	3	4	4	4	3	4	4	4	3	3	4	4	4	3	4	4	4	3	3	4	4	4	3	4	4	4	3	Series and two parallel
156	5	4	4	4	5	4	4	4	5	5	4	4	4	5	4	4	4	5	5	4	4	4	5	4	4	4	5	5	4	4	4	5	4	4	4	5	Series and two parallel
168	4	5	5	5	4	5	5	5	4	4	5	5	5	4	5	5	5	4	4	5	5	5	4	5	5	5	4	4	5	5	5	4	5	5	5	4	Series and two parallel
192	6	5	5	5	6	5	5	5	6	6	5	5	5	6	5	5	5	6	6	5	5	5	6	5	5	5	6	6	5	5	5	6	5	5	5	6	Series and two parallel
228	7	6	6	6	7	6	6	6	7	7	6	6	6	7	6	6	6	7	7	6	6	6	7	6	6	6	7	7	6	6	6	7	6	6	6	7	Series and two parallel
240	6	7	7	7	6	7	7	7	6	6	7	7	7	6	7	7	7	6	6	7	7	7	6	7	7	7	6	6	7	7	7	6	7	7	7	6	Series and two parallel
264	8	7	7	7	8	7	7	7	8	8	7	7	7	8	7	7	7	8	8	7	7	7	8	7	7	7	8	8	7	7	7	8	7	7	7	8	Series and two parallel

X indicates dead coil in location shown. Numbers indicate coils per group. Connections indicated are proper for either star or delta.

Diagrams for connecting:

Fig. 326 for series star.
Fig. 373 for two-parallel star.
Fig. 328 for three-parallel star.
Fig. 329 for four-parallel star.
Fig. 330 for six-parallel star.
Fig. 331 for twelve-parallel star.
Fig. 332 for series delta.
Fig. 374 for two-parallel delta.
Fig. 334 for three-parallel delta.
Fig. 335 for four-parallel delta.
Fig. 336 for six-parallel delta.
Fig. 337 for twelve-parallel delta.

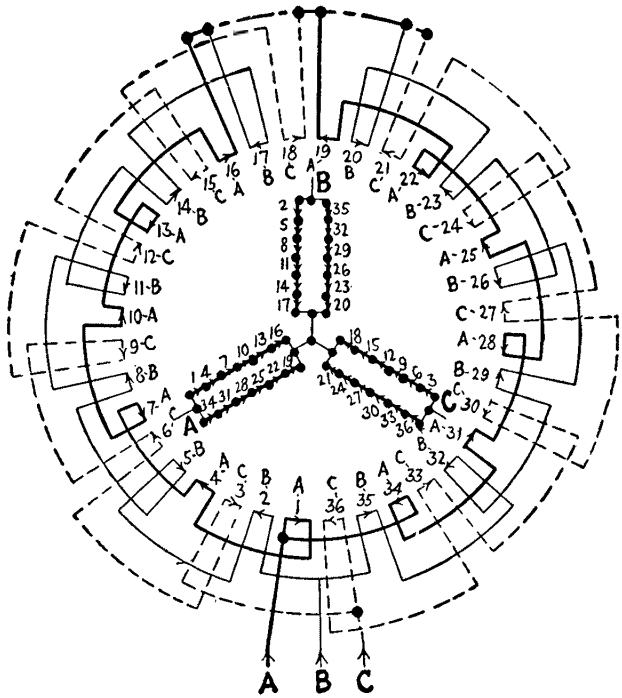


FIG. 373.—Twelve-pole, three-phase, two-parallel star, top to top connection.

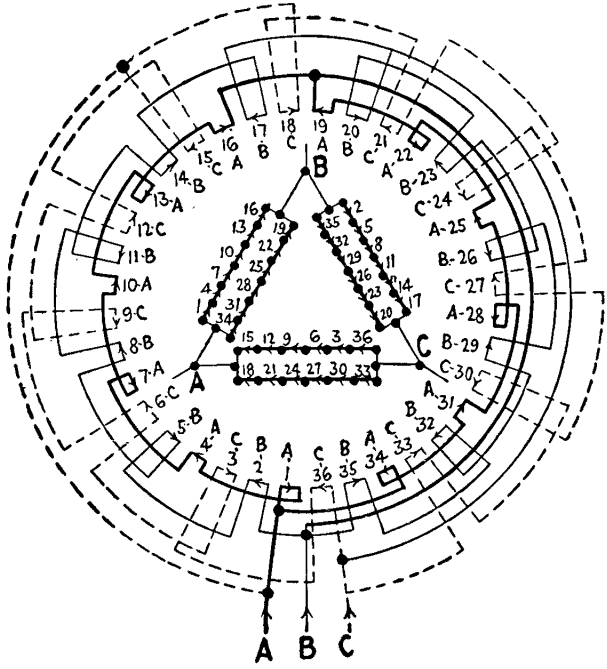


FIG. 374.—Twelve-pole. three-phase, two-parallel delta, top to top connection.

TABLE XXVI.—THREE PHASE—14 POLES

| Number of slots | Pole-phase groups | | |

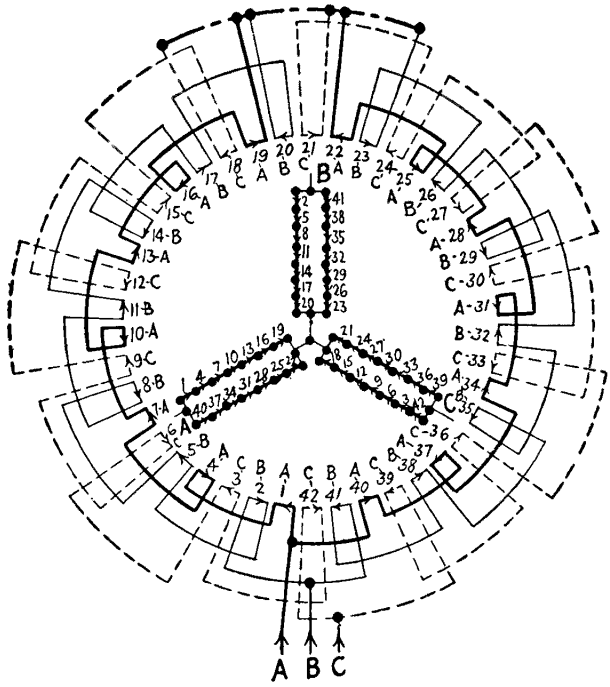


FIG. 375.—Fourteen-pole, three-phase, two-parallel star, top to top connection.

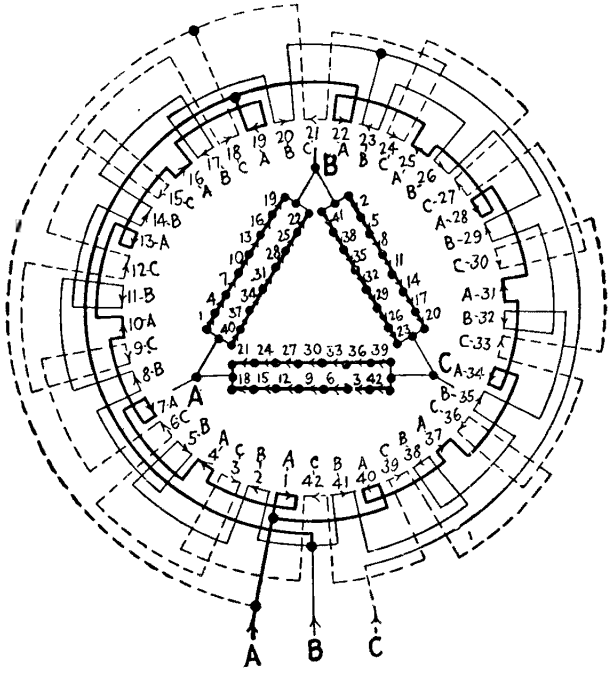


FIG. 376.—Fourteen-pole, three-phase, two-parallel delta, top to top connection.

CHAPTER XIX

WAVE DIAGRAMS

With the exception of one or two diagrams briefly mentioned in Chapter III practically all the diagrams discussed in the book and those shown in Chapter XVII are of the type usually employed for the stator winding. These could be used for the rotor also so far as any electrical considerations are concerned. It will be noticed, however, when the cross connections are considered that they are not arranged with mechanical symmetry around the machine and, hence, if a diagram of this type were used on the rotor there would be a tendency toward mechanical unbalance which would set up mechanical vibration when the rotor was running at full speed. In addition to this objection, cross connections of this type are difficult to arrange and secure in place on the rotor on account of their irregular shape and the considerable space which they occupy. For this reason, so-called "wave" diagrams, as shown in Figs. 377 to 387 inclusive, are ordinarily employed on the rotor. They are of the old, well known direct-current armature type sometimes called "progressive" or "retrogressive" windings. On examination they will be found to be very regular mechanically and distributed with practically perfect symmetry around the machine. They have also the advantage of requiring a minimum of cross connections—these being reduced to the three leads to the collector rings, one jumper joining the two halves of each phase winding and in case of a star connection the additional "star ring" with three taps, one to each phase.

The rotor winding is practically always three phase and may be connected either star or delta depending on the voltage which is desired between the collector rings. A star connection would give 1.73 times the voltage between rings that would exist with a delta connection. This would mean a smaller current with consequently smaller rings and brushes but would, in turn, require insulation for the higher voltage throughout the winding and between collector rings.

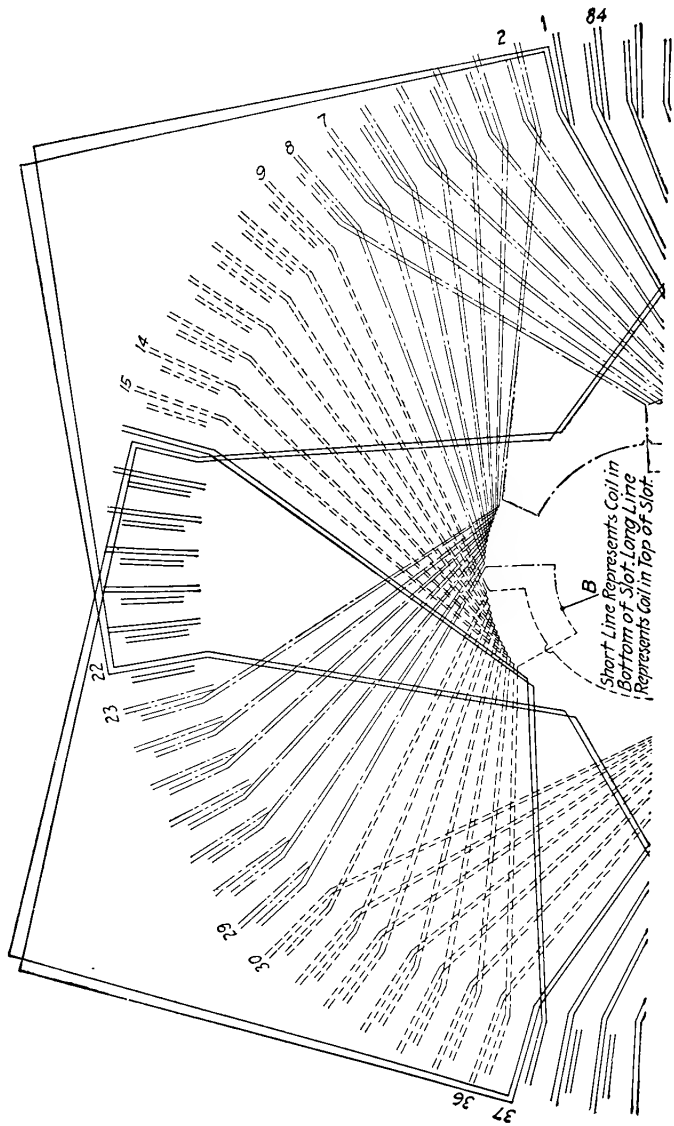
A two-phase winding is practically never used on the rotor as it would require four collector rings and an added set of brushes. When the rotating magnetic field is set up by the primary winding it is practically the same whether created by two-phase or three-phase current and is the same as if it were set up by direct current as described in Chapter II. Hence, when the field is set up it can act on a three-phase rotor as well as a two-phase rotor and advantage is taken of this fact to reduce the required number of collector rings and brush holders to a minimum.

In checking over these wave diagrams it will be noticed that the number of slots is always a multiple of the number of phases times the number of poles and hence is an even figure, whereas a true "progressive" or "retrogressive" winding as ordinarily used on direct current for a two coil per slot winding must satisfy the expression

$$\frac{\textit{Number of slots} \pm 1}{\textit{Pairs of poles}} = \textit{an integral number}$$

in order that the conductor after passing around the machine may fall into the slot adjacent to the one in which it started. In the diagrams, Figs. 377 to 387, this is avoided mechanically in the following way: Since the total number of slots is a multiple of the number of poles and since the throw of the coil on a rotor is exactly pitch the natural result would be that after once passing around the rotor the conductor would fall again into slot 1 in which it started. For example, assume a 72-slot rotor wound for eight poles. Starting in the bottom of slot 1 the conductor passes successively through the top of slot 10, bottom of 19, top of 28, bottom of 37, top of 46, bottom of 54, top of 63 and would again fall into the bottom of slot 1. However, the winder at this point bends the coil to one slot shorter throw and arbitrarily places it in the bottom of slot 72 and again around the rotor when he throws it in slot 71 and winds a third time around the rotor and stopping when he comes out of the top of slot 61. He then leaves the two ends of this section of winding, viz., bottom of slot 1 and top of slot 61. This completes one-sixth of the winding and he proceeds to complete the other five-sixths in the same manner. At the finish there are left six complete sections and 12 loose ends or leads. The winder then takes the lead from the top of slot 61, described above, and looks for the

(Text continued on page 398.)


84
1
2
7
8
9
14
15
22
23
29
30
36
37
B
Short Line Represents Coil in Bottom of Slot. Long Line Represents Coil in Top of Slot.

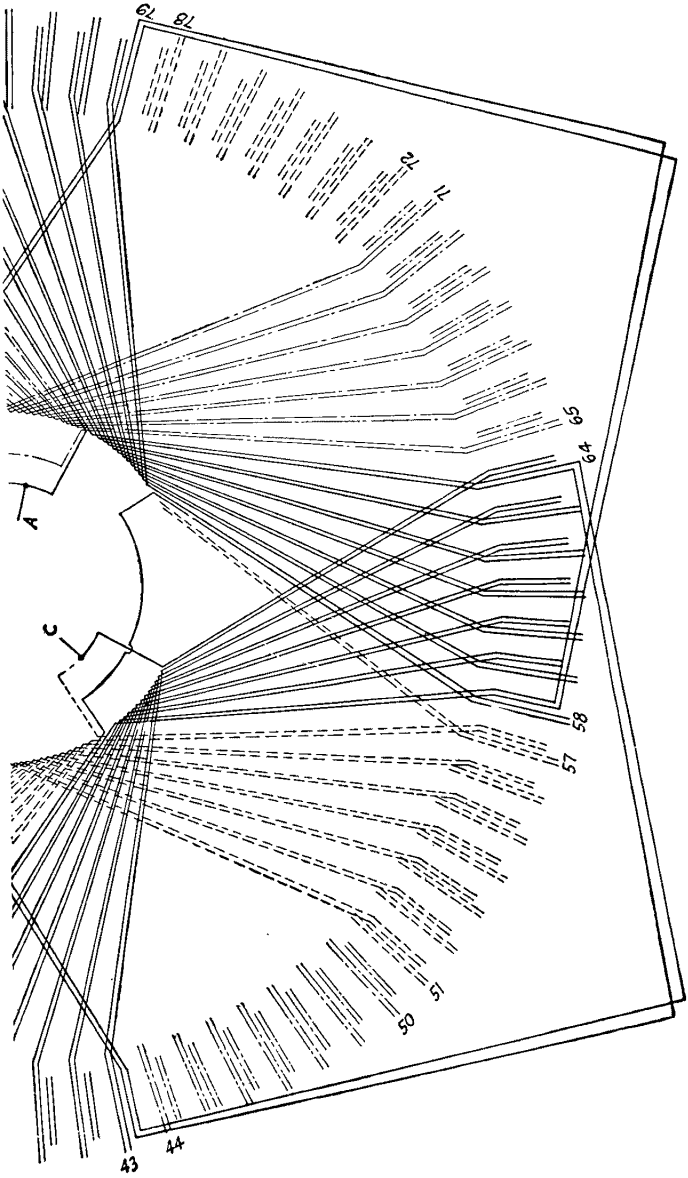
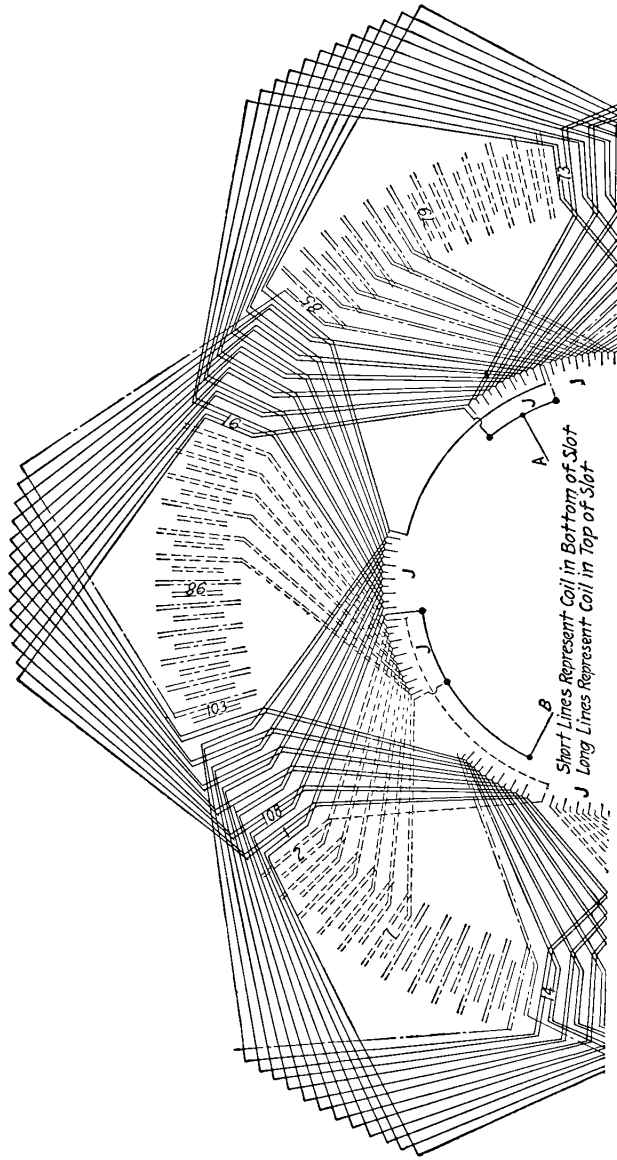


FIG. 377.—Three-phase, four-pole, series delta, wave diagram for 84 slots.


Short Lines Represent Coil in Bottom of Slot
Long Lines Represent Coil in Top of Slot
A
B
J
73
62
52
16
86
103
105
2
4

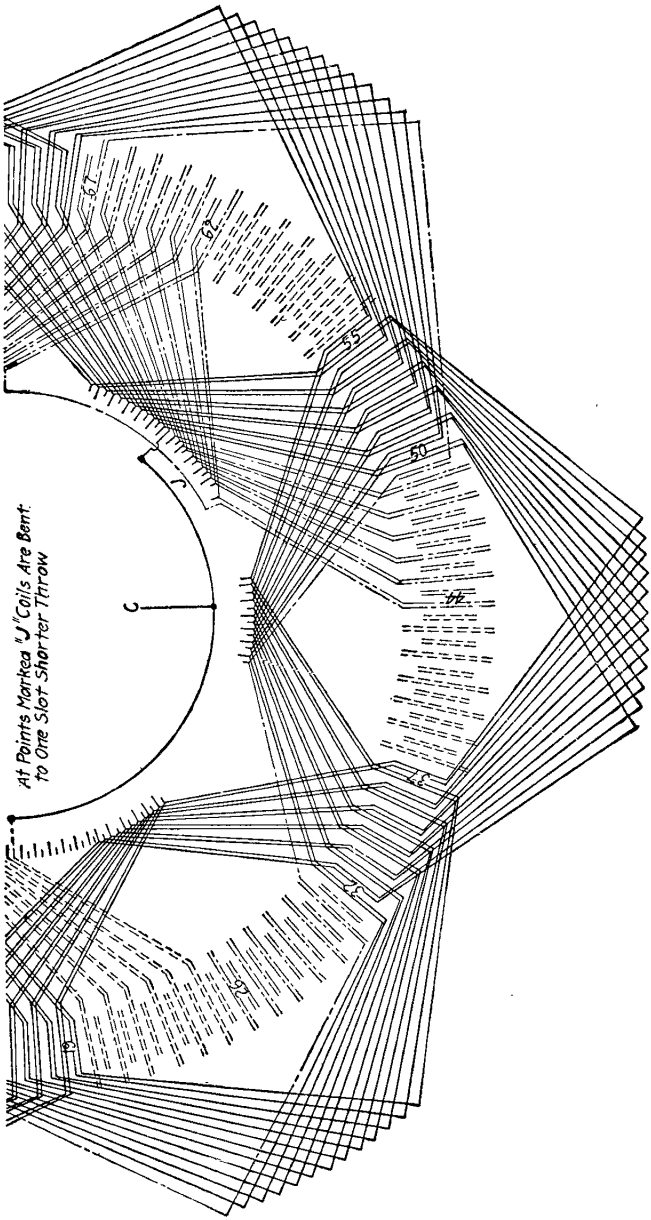
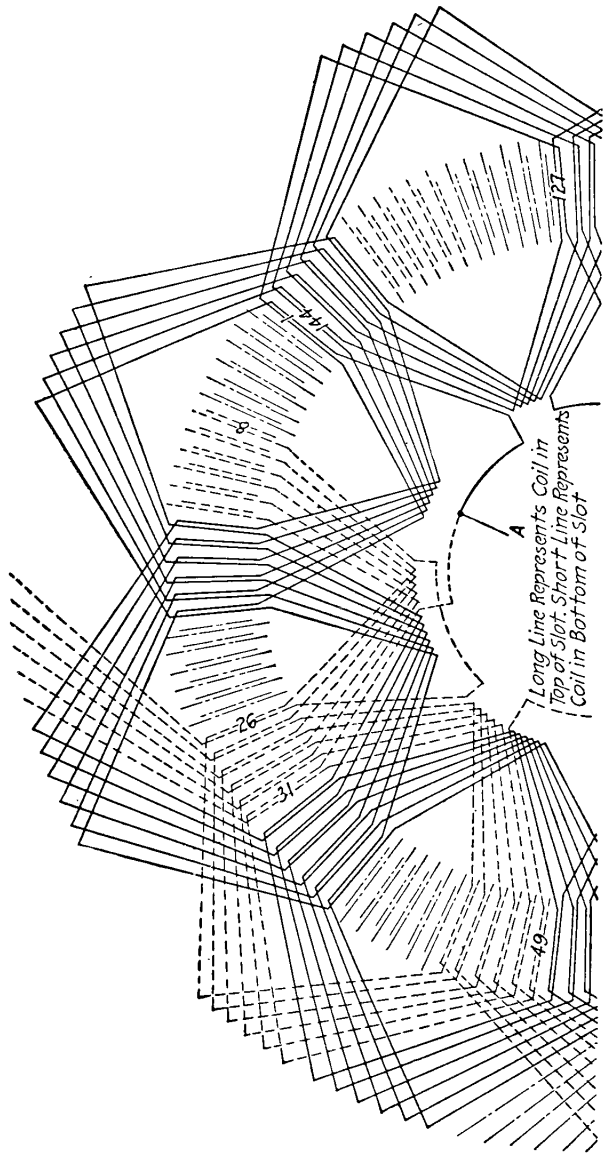


FIG. 378.—Three-phase, six-pole, series delta, wave diagram for 108 slots.


127
44
8
26
31
49
A
Long Line Represents Coil in Top of Slot. Short Line Represents Coil in Bottom of Slot

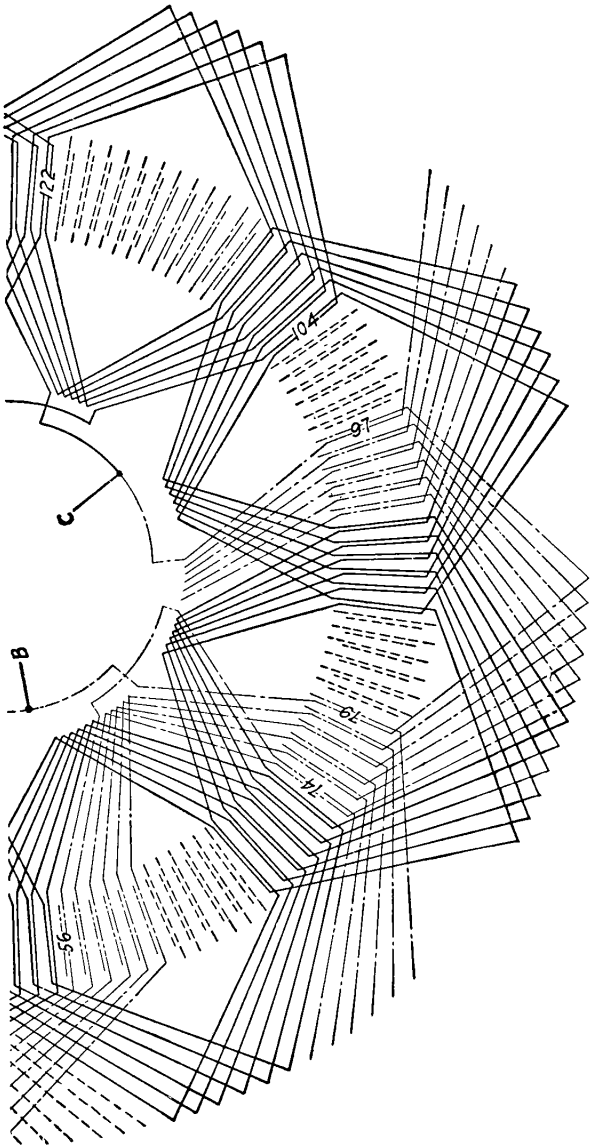
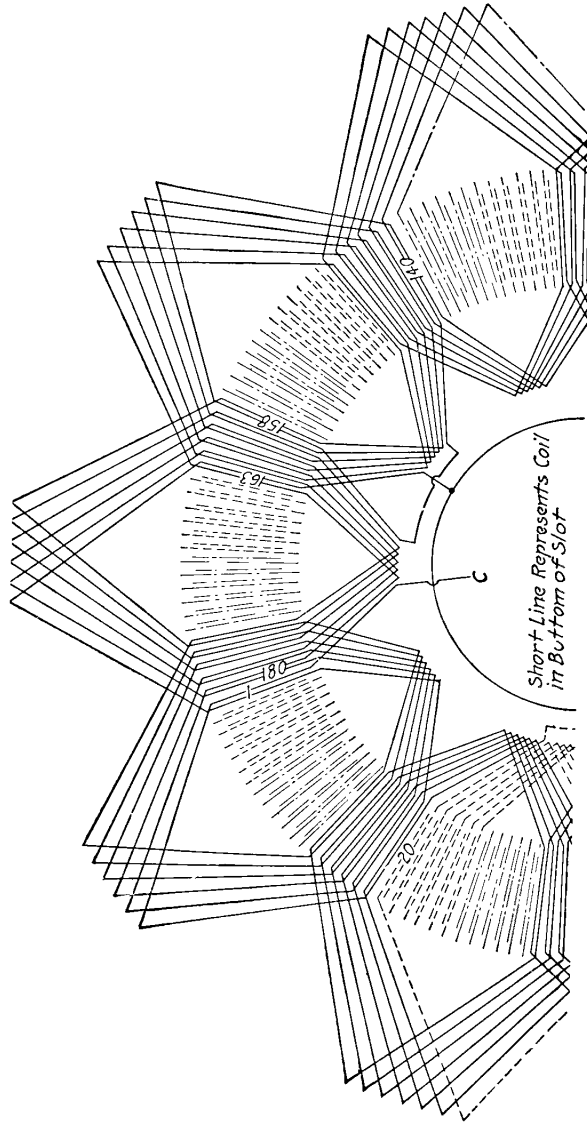


FIG. 379.—Three-phase, eight-pole, series delta, wave diagram for 144 slots.


140
158
163
180
20
C
Short Line Represents Coil
in Bottom of Slot

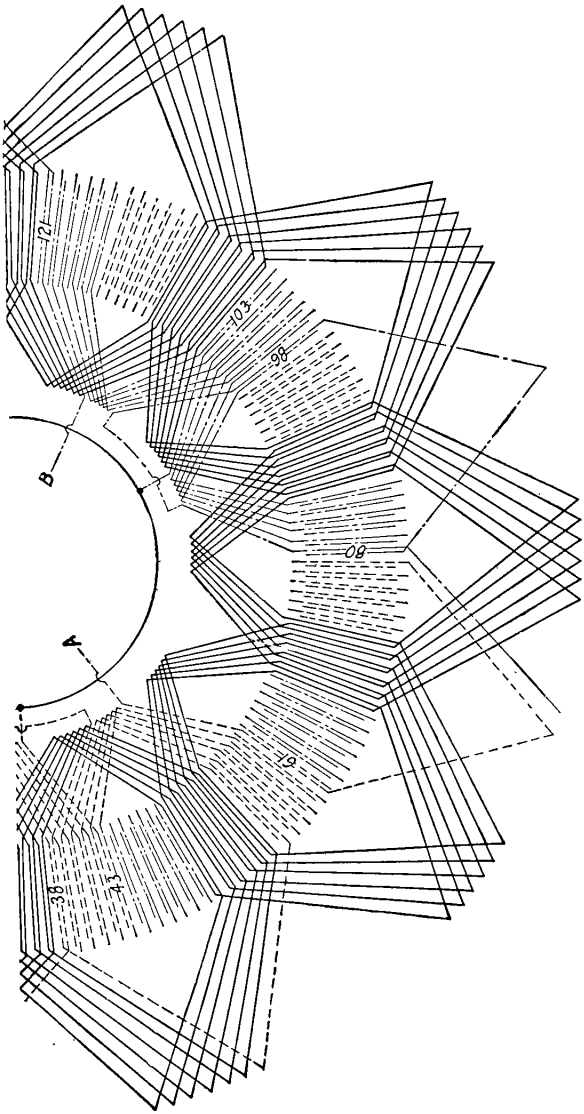
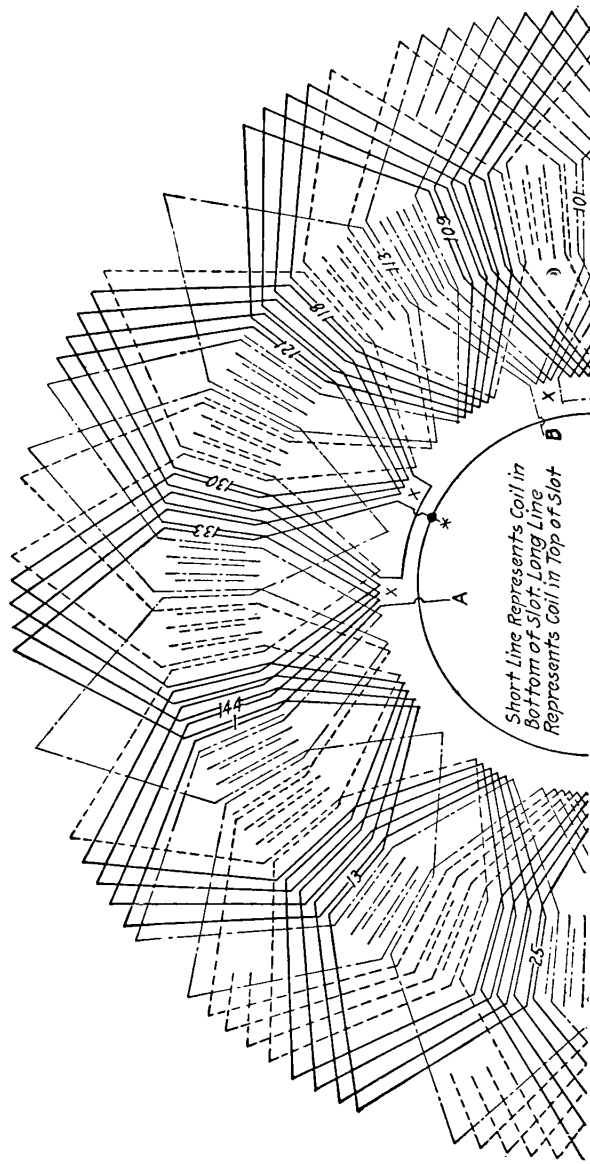


FIG. 380.—Three-phase, ten-pole, series star, wave diagram for 180 slots.


Short Line Represents Coil in Bottom of Slot. Long Line Represents Coil in Top of Slot
A
B
X
X
X
X
144
133
130
121
118
114
109
25

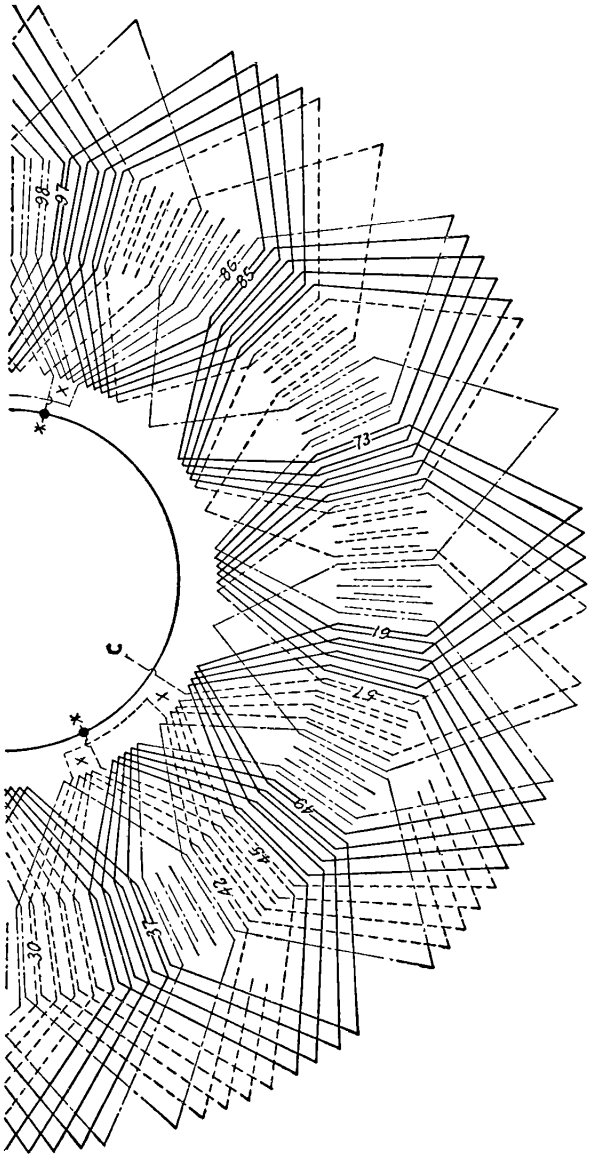
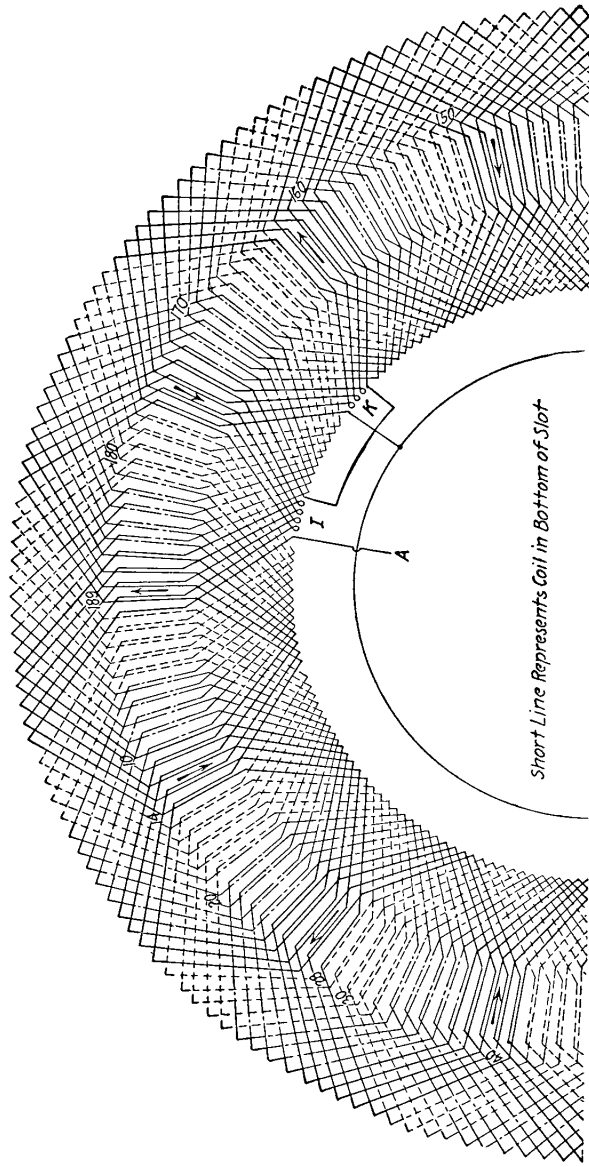


FIG. 381.—Three-phase, twelve-pole, series star, wave diagram for 144 slots.


Short Line Represents Coil in Bottom of Slot
A
I
K
1
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190

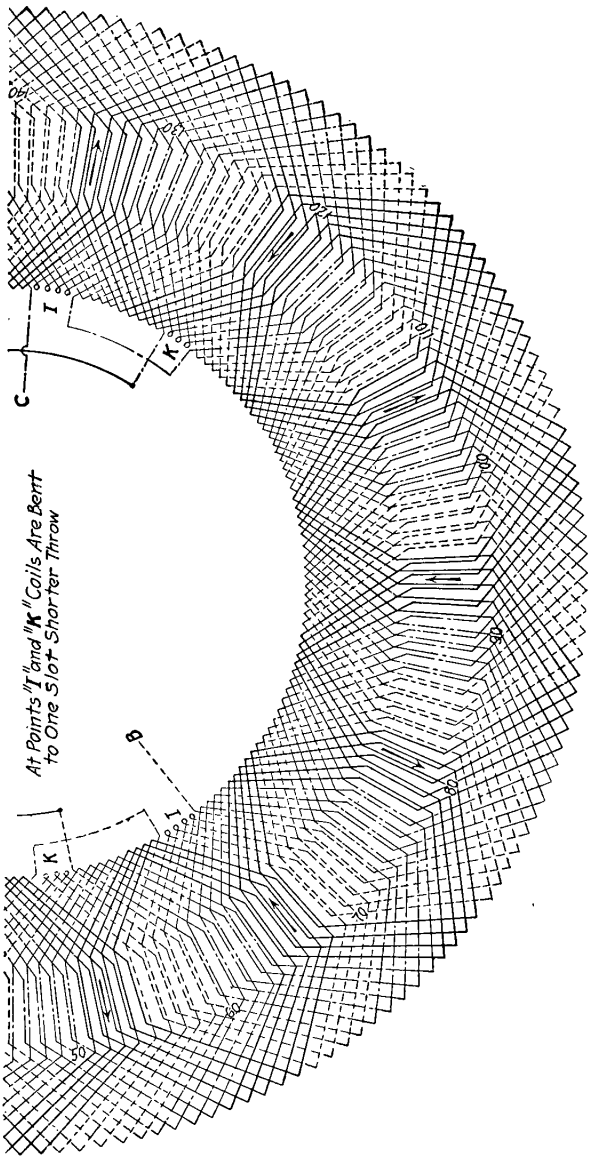
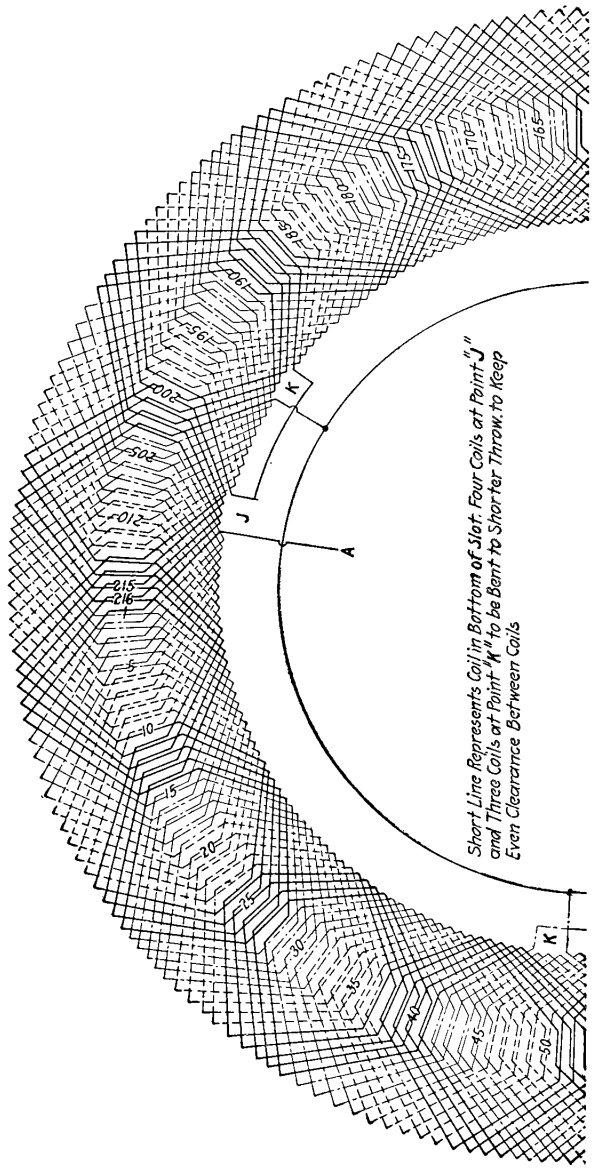


FIG. 382.—Three-phase, fourteen-pole, series star, wave diagram for 189 slots.


Short Line Represents Coil in Bottom of Slot. Four Coils at Point "J" and Three Coils at Point "K" to be Bent to Shorter Throw, to Keep Even Clearance Between Coils
J
K
A
K
165
170
175
180
185
190
195
200
205
210
215
216
5
10
15
20
25
30
35
40
45
50

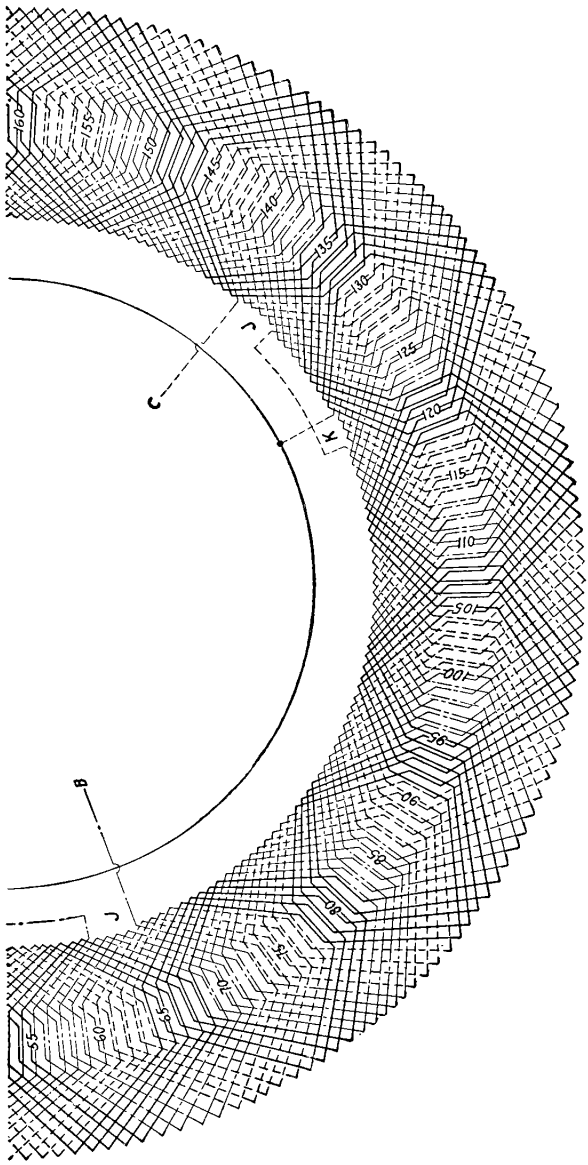
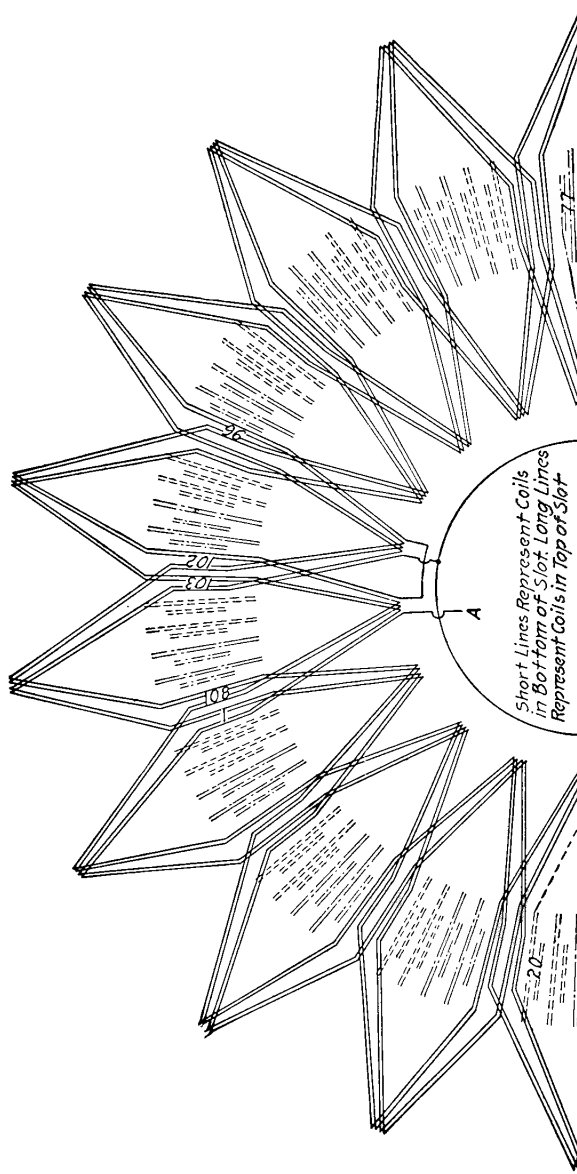


FIG. 383.—Three-phase, sixteen-pole, series star, wave diagram for 216 slots.


Short Lines Represent Coils in Bottom of Slot. Long Lines Represent Coils in Top of Slot
A
96
101
103
108
20
77

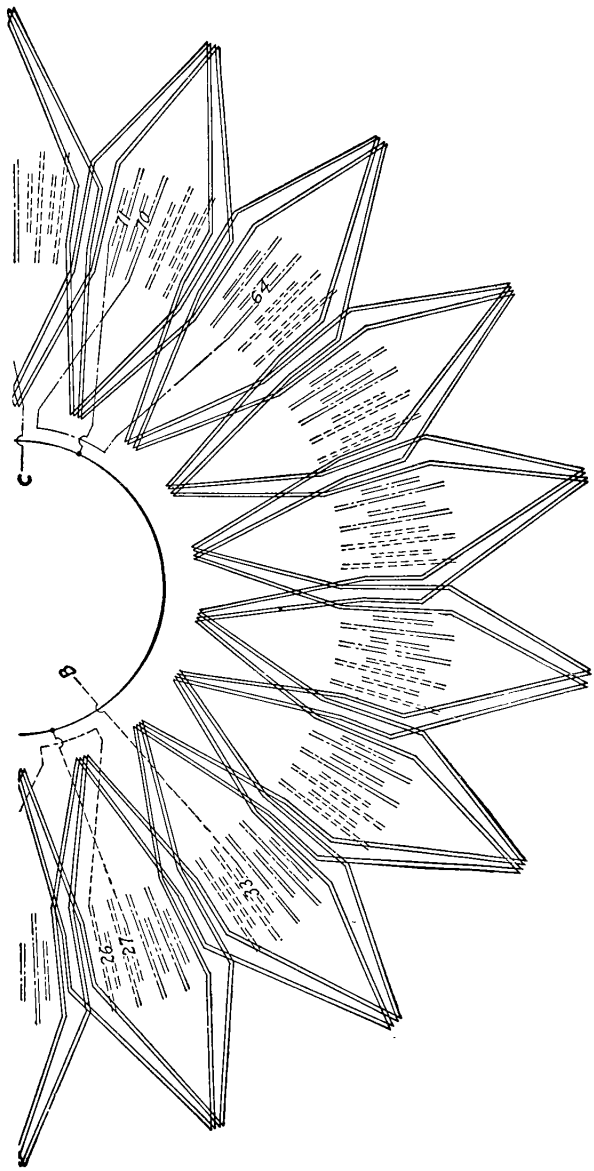
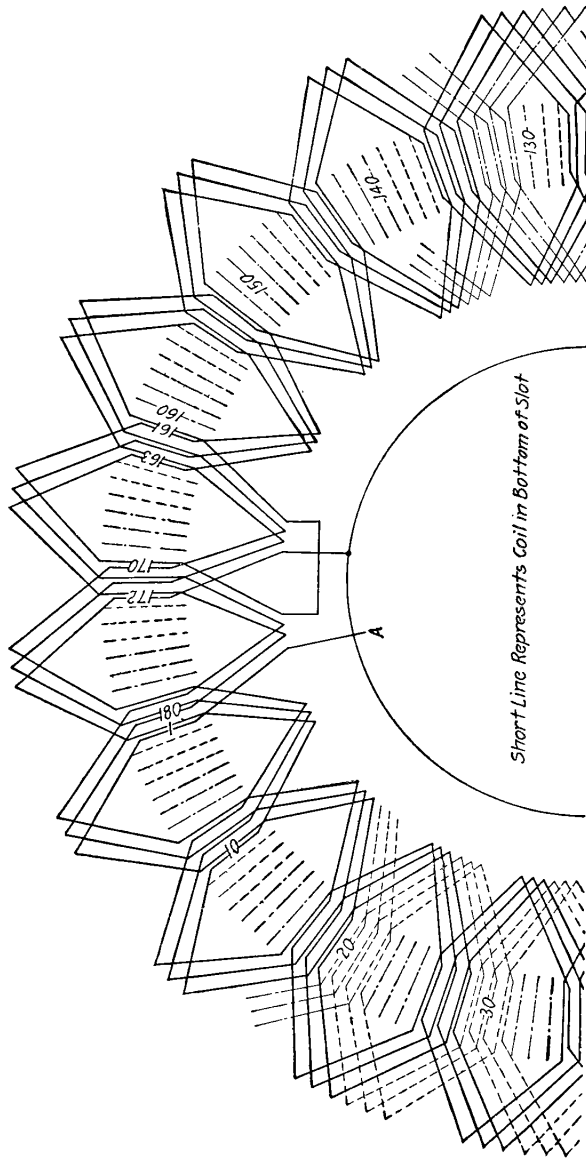


FIG. 384.—Three-phase, eighteen-pole, series star, wave diagram for 108 slots.


130
140
150
160
162
170
172
180
10
20
30
A
Short Line Represents Coil in Bottom of Slot

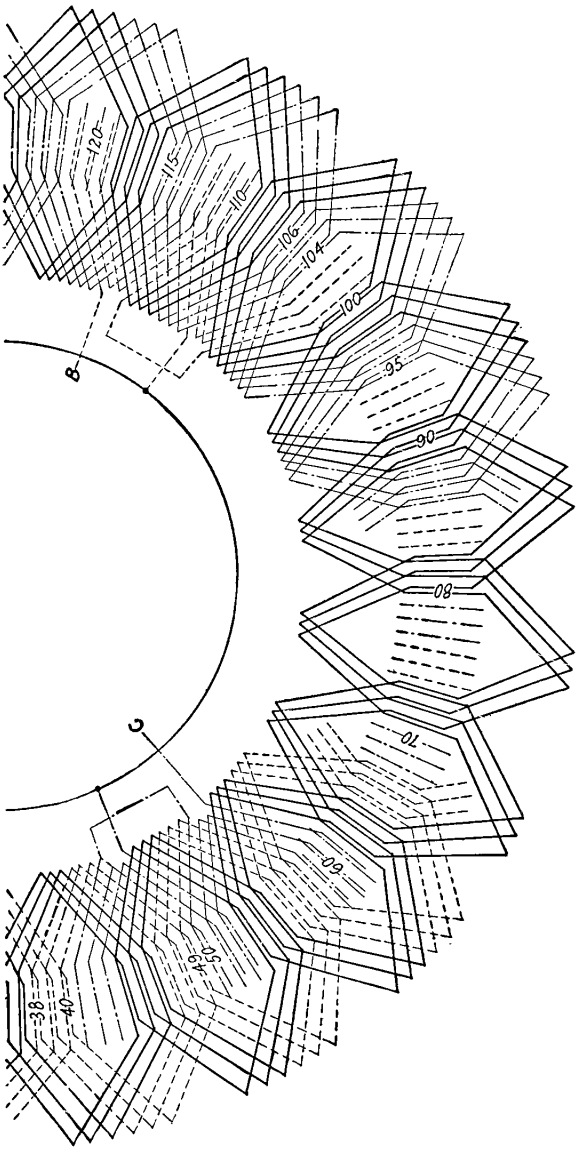
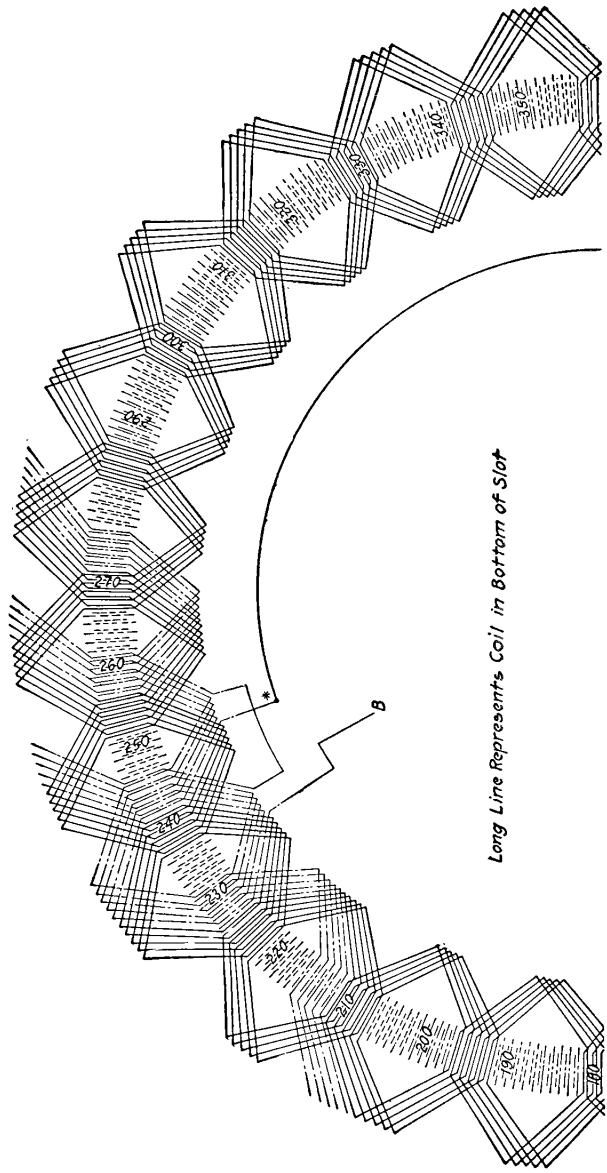


FIG. 385.—Three-phase, twenty-pole, series star, wave diagram for 180 slots.


Long Line Represents Coil in Bottom of Slot
B
180
190
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360

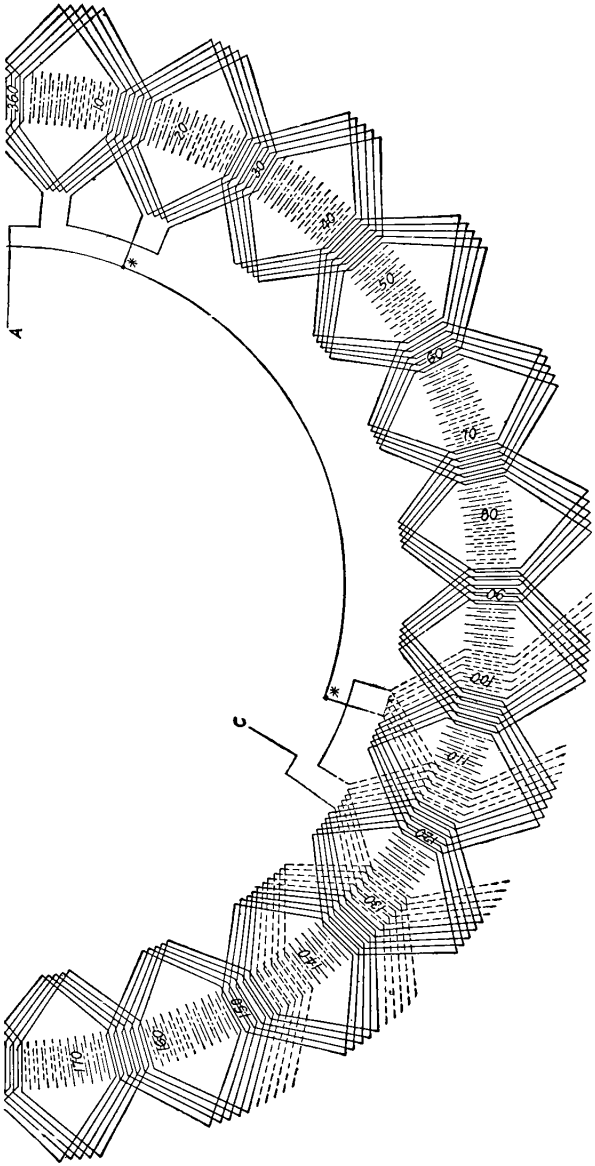
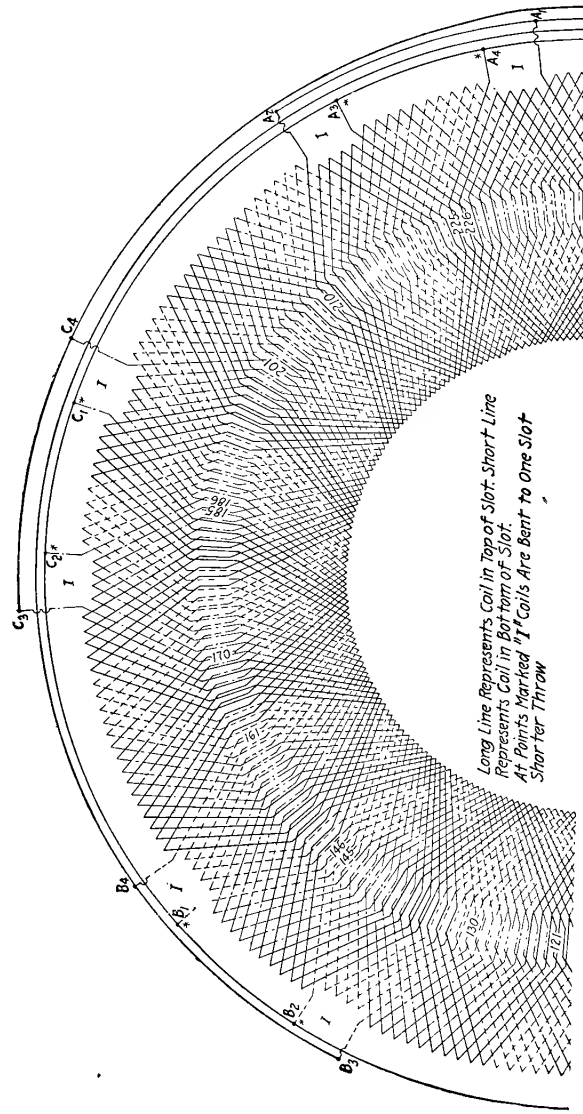


FIG. 386.—Three-phase, twenty-four-pole, series star, wave diagram for 360 slots.


Long Line Represents Coil in Top of Slot. Short Line Represents Coil in Bottom of Slot.
At Points Marked "1" Coils Are Bent to One Slot Shorter Throw
A1
A2
A3
A4
B1
B2
B3
B4
C1
C2
C3
C4
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36

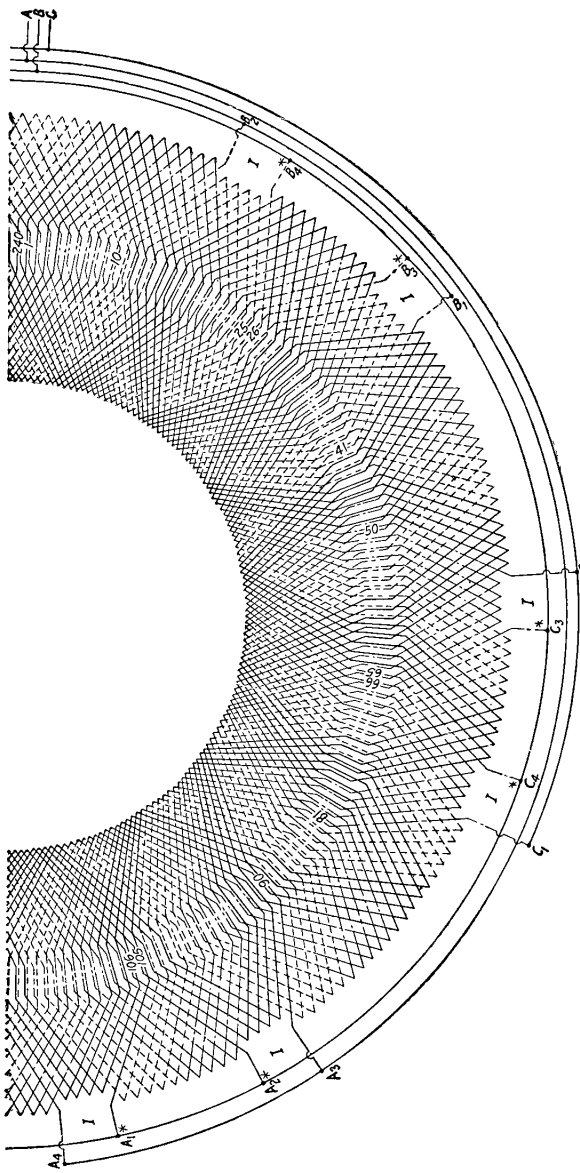


FIG. 387.—Special diagram illustrating how a wave winding is paralleled; three-phase, sixteen-pole, 240 slots connected four-parallel star.

section of the winding which lies in the tops of slots 1–72–71, etc. Having located this section, which may be called section 4, the end of section 1 is connected to the end of 4 so that the completed phase will have passed three times around the armature clockwise and three times counterclockwise. This is very similar to the case explained by Figs. 45, 46, and 47 in Chapter III. A little study of the diagrams, Figs. 377 to 387, will show how this is done. After the three separate phases are complete they are connected in star or delta and the leads brought out as shown in the diagrams.

CHAPTER XX

PERFORMANCE CALCULATION

Chapter XV of previous editions of this book contained some suggestions as to how to figure a winding for a given horsepower, voltage or speed when nothing was available except the bare core from which the windings had been stripped. The information in Chapter XV seems to have been widely used and for this reason is retained with practically no change in this edition. As a result of its use, there have been received by the author several requests for a method of making a complete calculation of the motor which could be made by anyone using arithmetic as a mathematical tool. This idea has been very attractive but has been attended with great difficulty. The author has experimented with a number of short cuts and empirical formulae alleged to give rough and ready results nearly enough correct for the needs of the average operator or service engineer. After much work, the conclusion was reluctantly reached that these simplified methods were not sufficiently accurate to be safe and that, on the other hand, any method good enough to publish as a general guide involved as much detailed calculation as is shown in this chapter. Finally the author decided to publish, with some abbreviations and approximations, a method that he used for a number of years in the calculation of commercial motors.

For this method the author is indebted to Rudolf E. Hellmund, with whom he has been associated for many years. It should be understood at this point that, while Mr. Hellmund is credited with this beautiful and simple method, he is not to be held responsible for such approximations as are here introduced in the attempt to make his method available to the man without formal technical training as well as to the college graduate. To students getting their engineering training in colleges, it is believed that this chapter will be suggestive and helpful in the mastering of the theoretical conditions that lie behind the comparatively simple arithmetical calculations involved.

At the start let it be said that this is a graphical method involving a circle diagram with three circles. To those who have tried to master the usual circle diagram and who have been troubled by the apparent maze of relations covered by one set of lines, it should be said that here is something different and much easier to keep clearly in mind. This is shown in the author's book, "Induction Motor Practice,"[1] with this very important difference, that in the latter case it was drawn from values actually obtained from tests after the motor was built, whereas in this chapter it is drawn from calculated values on paper before the winding exists anywhere except in the head of the man making the calculation.

Briefly, the current circle is located by knowing two points on its circumference and locating its center. From this circle the second or torque circle is drawn by calculating the stator copper loss at a load chosen at random. From the torque circle the third or output circle is drawn by calculating the rotor copper loss at the same load. It is further necessary to figure the no-load losses, which are principally iron loss, bearing friction and windage. When this no-load loss line is established, together with the three circles, there is available the complete story of what the motor will do in the way of efficiency, power factor and current at any load. In fact, it is the whole story of the motor's performance except heating. Nothing is said about noise, since that is not subject to exact calculation. It can be avoided only by knowing what combinations of numbers of stator and rotor slots are good or bad, or what magnetic densities combined with tooth shapes may give trouble. In other words, the noise problem is in the province of experience rather than in that of self-help in practice.

At this point the statement in Chapter XV should be repeated, that this material is not intended to produce polished designs to a finished nicety, but is intended to help the interested worker to get a clearer understanding of the tools with which he is working and the results of making changes in the stator and rotor windings of induction motors.

The method involves calculating a few very definite things. These are:

[1] Dudley, A. M., "Induction Motor Practice," Chap. X, McGraw-Hill Book Company, Inc., New York.

1. The no-load current.
2. The relation between the no-load current and the locked current which would exist if the motor had reactance only and no resistance.
3. The resistance of the stator and rotor windings.
4. The iron loss, bearing friction and windage.

Of these, let it be admitted at once, the second is the difficult and tricky one and the stumbling-block of many people who try to calculate induction motors. Even when one calculates carefully, as to arithmetic errors and method, it has a nasty habit of coming out somewhat differently when the motor is tested. There is a good reason for this, for one is working with

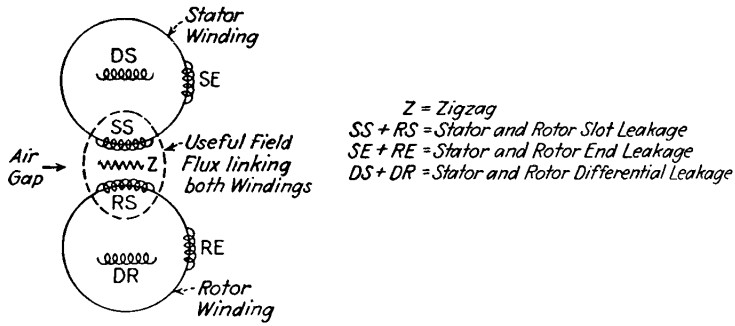


FIG. 388.—Schematic representation of the various ways the magnetic flux leaks off to cause reactance.

that part of the magnetic field which strays off through devious paths, completing its circuit without linking both windings, as does the useful or working field. Refer to Fig. 388, which shows in a fanciful way the stator winding and the rotor winding, indicating in how many ways the magnetic lines of force tend to stray off and sneak through without threading both these windings. The small curls *SS* and *RS* show some of the field leaking across the stator and rotor slots around the individual conductors; the line *Z* shows some of the field zigzagging along from the tops of stator teeth to the tops of the rotor teeth and back, continuing the action several times, until it finally sneaks through between the stator and rotor windings for a whole pole span; *RE* and *SE* show the small fields set up around the ends of the windings which are outside the iron core and hence not useful; finally, *DR* and *DS* show the so-called "belt," or differential

leakage, which is due to the fact that the rotor is slipping all the time with reference to the rotating field; hence the phase bands of current in the stator and rotor sometimes help and sometimes hinder one another. The useful field, as shown by the dotted line, links both circuits. A measure of these leakage fields could be obtained on any motor, on test, by applying to it just enough voltage to cause normal full-load running current to flow in the stator winding at standstill. Since most motors have a locked current at standstill and full voltage of six or eight times full-load current, it would take one-sixth, or one-eighth of normal voltage to cause full-load current to flow at standstill. Expressed in percentage, this would be $16\frac{2}{3}$ per cent, or $12\frac{1}{2}$ per cent of normal voltage. In passing, it might be mentioned that in the early days of induction motor design there was an empirical rule which said that 40/percentage of leakage = number of times full-load torque which the motor would develop running without pulling out. In other words, the motor just mentioned would have a maximum torque of $40/16\frac{2}{3} = 2.4$ times or $40/12\frac{1}{2} = 3.2$ times. This is not mentioned as a reliable guide, but it is a rough indication.

Some further comments might be made at this point as to what is involved in the calculation work of finding these four quantities:

1. The no-load current. This involves a number of contributing factors:

a. The increase in the magnetic air gap or clearance between rotor and stator iron, due to the openings in the slots. Referring to Fig. 389, it will be readily seen that the average length of the path of a magnetic line between the tops of the stator and rotor teeth is increased, owing to the iron which is cut away to make the slot openings, so that this average length is increased from the clearance *a* to something more like dimension *b*. It is necessary, then, to figure a factor for this increase for the stator slot and another for the rotor slot. The final effective magnetic gap is the mechanical clearance multiplied by the product of these two factors.

b. The magnetic flux per pole.

c. The magnetic density in the core back of the slots, in the teeth and in the air gap. This involves calculating the cross section of these parts of the magnetic path and dividing it into

the flux per pole, remembering that the teeth require an increase factor.

d. The length of the magnetic path in the stator and rotor cores back of the slots; in the stator and rotor teeth; and in the air gap or clearance between stator and rotor, as discussed under *a* above.

e. The ampere turns or driving force necessary to force the magnetic field around its circuit through cores, teeth and air gap.

f. The so-called "saturation factor," or the increase in the total magnetizing ampere turns over those required for the air gap alone.

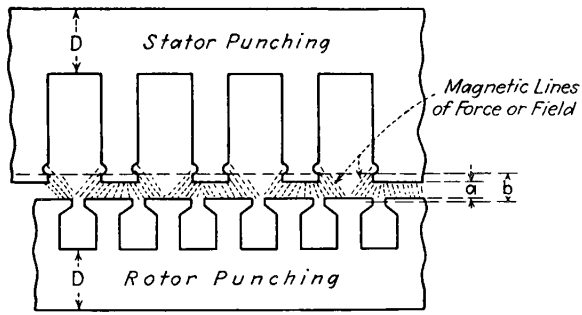


FIG. 389.—Effect of slot openings in increasing magnetic air gap.

2. The leakage coefficient or leakage reactance or, as, it is sometimes called, the "dispersion factor." This has already been discussed. This factor is called *sigma* and in this chapter is designated by the capital letter *S*. This requires more calculation than any other factor as it has seven parts, caused, respectively, by stator and rotor slot, coil-end and belt leakage and zigzag.

3. The resistance of the stator and rotor windings. This is a simple calculation involving the length and cross section of one conductor multiplied by the number of conductors per phase.

4. The iron loss, bearing friction and windage. The ordinary iron loss in the stator core and teeth is figured from the volume of iron in those parts and the density and frequency of the magnetic field. In the actual machine this would be increased to an amount frequently two or three times this value because the slot openings cause high-frequency pulsations in the teeth of the rotor. However, for the sake of simplicity this factor is approximated by multiplying the ordinary iron losses by the square of

the air-gap factor found under (1), *i.e.*, the product of the stator-gap factor and the rotor-gap factor. There is no iron loss in the rotor core or teeth, owing to the main magnetic field, because the frequency (cycles) in the rotor is only three or four per cent (per cent slip) of the stator frequency. This is because the rotor mechanically travels nearly as fast as the rotating magnetic field; hence there are comparatively few reversals of magnetism in the rotor iron as compared to the stator iron.

The author wishes to state definitely that he does not, in any sense, guarantee the results obtained by using this method, but to the sincere student, approaching the problem with the idea of increasing his understanding of what is involved, the method is offered in the hope that it may be helpful.

1. Magnetic Field and Magnetizing Current.—*a. Field per pole, i.e., per magnetic circuit.*

$$F = \frac{45{,}000{,}000 \times \textit{volts per phase}}{\textit{Cycles} \times \textit{conductors per phase} \times \textit{chord factor} \times \textit{distribution factor}}$$

See page 270, Chapter XV, for the complete explanation of this factor.

Caution: Look out for wires in parallel in the coil or for groups in parallel in the cross connection.

The number of conductors used should be those in "series" across the voltage, which is used in the numerator.

b. Sections of Magnetic Circuit.—Since the field calculated above is per pole, all sections are to be figured per pole.

Stator and Rotor Core Sections.—These cross sections of the magnetic path are found by multiplying twice the depth below slots *D* in Fig. 389 by the width of the core parallel to the shaft, by the fill factor. The fill factor takes care of the fact that there is core-plate varnish on the laminations and hence the core is not solid steel. In all cases use 0.93 for the fill factor.

Caution: Subtract openings due to air ducts. Referring to Figs. 390 and 391 the formulae become, respectively,

$$2 \times W \times D \times 0.93$$
$$2 \times (W_1 + W_2 + W_3) \times D \times 0.93$$

Twice the cross section is used as explained in Chapter XV because in coming from the rotor to the stator the field divides and goes both ways.

Figure the core cross section separately for stator and rotor.

Stator- and Rotor-tooth Cross Sections.—The section in the stator or in the rotor teeth is the width of the tooth, at one-half the tooth depth, times the core width times fill factor times number of teeth per pole. Referring to Fig. 390 this would be:

$$(P_2 - X) \times W \times 0.93 \times \frac{\textit{Total number of stator slots}}{\textit{Number of poles}}$$

P_2, or the pitch of the tooth at one-half its depth, is found by adding the slot depth L to the stator bore, multiplying by 3.1416 and dividing by the total number of stator slots. For example, suppose the bore was 16 inches, the core width 5 inches, the slot

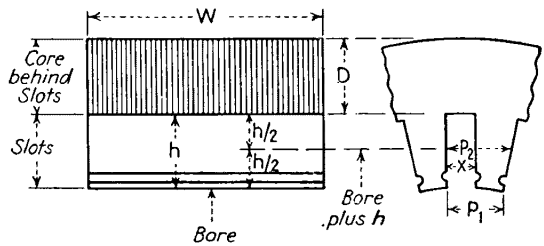


FIG. 390.—Area of stator core and teeth.

depth 1¼ inches and the slot width 3/10 inches; assume 72 slots and 6 poles; then the width of the stator tooth at one-half its depth would be

$$\frac{(16 + 1\tfrac{1}{4})3.1416}{72} - 0.3 = 0.45 \text{ inch}$$

and the cross section of the stator teeth would be

$$0.45 \times 5 \times 0.93 \times 72\tfrac{2}{6} = 24.6 \text{ square inches.}$$

Figure the cross section of the rotor teeth similarly, only remember in this case that, to get the tooth pitch at one-half the slot depth, the tooth height is subtracted from the rotor diameter instead of added as in the case of the stator.

Air-gap Section.—The cross section of the magnetic path in the air gap is the effective surface area covered by one pole. It is the product of the pole pitch and the effective width of the core

parallel to the shaft. The pole pitch =

$$\frac{\textit{Diameter of stator bore} \times 3.1416}{\textit{Number of poles}}.$$

The effective width of the core is the actual width of iron (do not reduce for fill factor) multiplied by 1.03, to take care of spreading

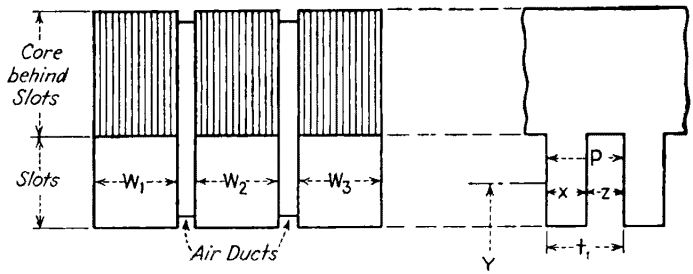


FIG. 391.—Stator core with air ducts.

of the field at the ends of the core and in the ducts. In the case of the machine in the preceding paragraph this would be

$$\left(\frac{16 \times 3.1416}{6}\right) \times 5 \times 1.03 = 43 \text{ square inches.}$$

c. Lengths of Magnetic Paths and Gap Factor.—All paths are to be figured per pole as indicated in Fig. 392. The length of paths in the primary and secondary cores is the average diameter of the core body times 3.1416 divided by twice the number of poles. Twice the number of poles is used because the field is coming in from the teeth, all the way from the center of the pole to the edge, so that the average length of path is only one-half the pole pitch in the core.

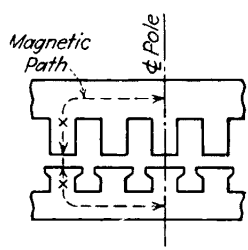


FIG. 392.—Magnetic path in air gap, teeth and cores.

The length of paths in the stator and rotor teeth is the depth of the tooth in each case.

The length of the air gap is the distance from the tops of the rotor teeth to the tops of the stator teeth multiplied by a factor taking account of the slot openings. It should be understood that the mechanical gap is the clearance between stator and rotor. It is the stator bore minus the rotor diameter, divided by two. The total air-gap factor is the product of two factors, one for the

stator and one for the rotor. Each of these factors is found from the following formula:

$$G_1 \text{ or } G_2 = \frac{1}{1 - \frac{Z_{r_1}}{t}}$$

This equation gives the value G_1 if r_1, t and Z are taken for the stator-slot opening, while G_2 is obtained by taking corresponding values for the rotor-slot opening. The values follow from the sketch and curve of Fig. 393.

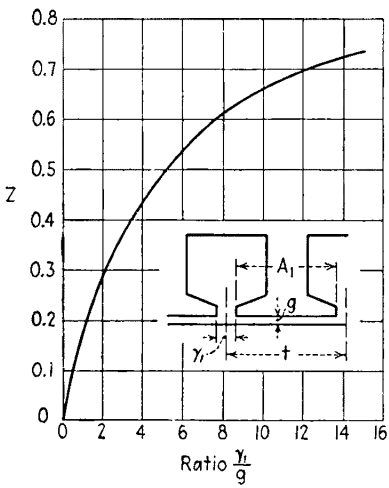


FIG. 393.—Carter's fringing curve. The equation of this curve is very nearly $Z = \frac{r_1}{g} \div \left(\frac{r_1}{g} + 5\right)$.

d. Densities in the Magnetic Circuit.—The average densities are found by dividing the field per pole by the cross section of the various parts of the path. In the stator- and rotor-core portions, the distribution of the magnetic field over the section is practically uniform, and hence the average density also represents the maximum density.

In the teeth and in the air gap, the space distribution of the field is according to the sine law. One must, therefore, distinguish between the average density and the maximum density. For this reason the density in the teeth must be divided by 0.636 (2/3.1416) before one figures the ampere turns required to force the magnetism through the teeth.

In connection with the calculation of the magnetizing current, it is immaterial whether average or maximum values of the field are used if the use is consistent. However, for the calculation of the iron losses in the teeth the maximum values of the magnetic density are more convenient. For this reason, it is proposed to use the maximum values for the teeth, and the curves given for the teeth are based on maximum densities. In the air gap the average density is used.

e. Ampere Turns.—The unit of magnetizing force is called the "ampere turn" and it represents the magnetizing ability pos-

sessed by one turn of wire through which is flowing 1 ampere. Hence, to arrive at the no-load or magnetizing current of a motor, a calculation is made of the ampere turns required to drive the magnetic field through the various parts of the circuit.

In figuring the ampere turns for the two cores, stator and rotor, it is not correct to use a normal magnetization curve for the grade of iron used because such a curve would give much too high values for the ampere turns. It should be kept in mind that the maximum density in the yoke or core exists only over a

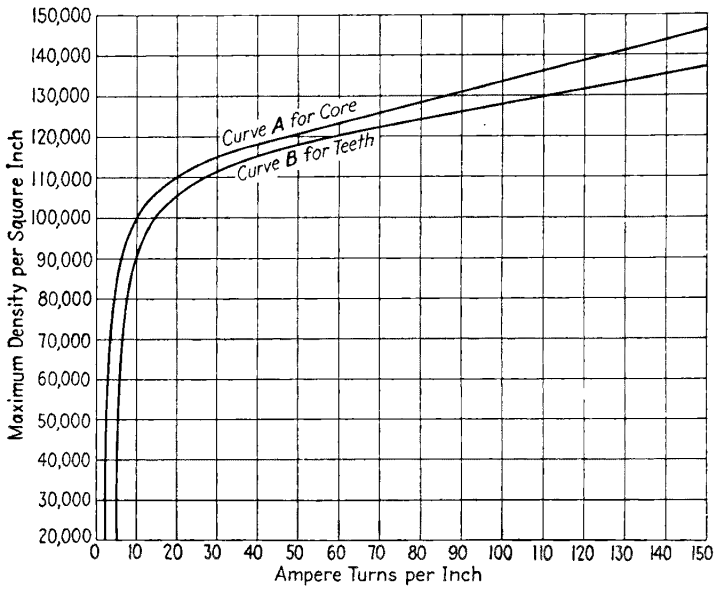


FIG. 394.—Magnetization curves for core and teeth.

short length of path. Curve *A* in Fig. 394 takes this fact into account, and gives the ampere turns in the cores for each inch of the length of path previously determined. Even this curve gives somewhat too high values for very high densities. This is because there is in every machine of this kind some flux going through the frame, the end plates and spider. While the densities in the core may be only slightly reduced thereby, it affects the ampere turns considerably, for with high densities a small change in the densities means a considerable change in the magnetizing ampere turns. For practical cases, the curve gives, therefore, very safe values.

Similar considerations apply to the teeth, for which curve *B* of Fig. 394 is used.

Ampere Turns in the Air Gap.—The greatest portion of the total required ampere turns are needed for the air gap, since the reluctance of air is many times that of iron. The formula for the ampere turns in the air gap is

$$AT_a = 0.8\frac{B_a}{6.45}2.54g \times G = 0.315B_a \times g \times G$$

where B_a = average density in the gap per square inch.
g = mechanical clearance between stator and rotor.
G = product of stator and rotor air-gap factors calculated above under *c*.

Ampere Turns for Cores and Teeth.—It is convenient, in calculating the total ampere turns, to make up a small table which

Part of Path	Area	Length of Part	Flux Density per Square Inch	Ampere Turns per Inch	Ampere Turns Total per Part
Stator Teeth	(2)	(3)	(4)	(5)	(3) x (5)
Stator Core					
Rotor Teeth					
Rotor Core					
Air Gap					
				Total	

FIG. 395.—Table for calculating magnetizing ampere turns.

shows the sections of the different parts of the path; the magnetic density in each section; the length of path; ampere turns per inch at the required density; and ampere turns total for the part. The summation of the ampere turns then gives the total required for the entire magnetic circuit. Such a table is illustrated as Fig. 395.

The ratio $\frac{\textit{Ampere turns total}}{\textit{Ampere turns for air gap only}}$ is called the "saturation factor" and is used later in connection with the leakage calculation.

f. Magnetizing Current.—The magnetizing current is found directly from the total ampere turns from the formulae:

$$i_m = 0.96\frac{\textit{Ampere turns total} \times \textit{number of poles}}{\textit{Turns per phase} \times \textit{chord factor}} \quad \text{(for two-phase)}$$

and

$$i_m = 0.605\frac{Ampere\ turns\ total \times number\ of\ poles}{Turns\ per\ phase \times chord\ factor} \quad \text{(for three-phase)}.$$

$$Turns\ per\ phase = conductors\ per\ phase \div 2$$

2. Leakage Coefficient.—The value of the magnetizing current, which has just been calculated, together with the leakage coefficient, fixes the diameter of the current circle and very largely determines the performance of the motor.

While the leakage calculation seems long to one unfamiliar with it, it is fairly simple and not difficult. Care is needed to

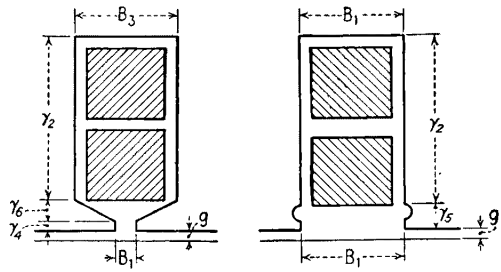


FIG. 396.—Stator- and rotor-slot leakage factor.

keep the arithmetic straight, in which case no trouble is experienced.

The leakage coefficient is made up of seven parts: *a.* Stator slot, *b.* rotor slot, *c.* zigzag, *d.* stator-coil end, *e.* rotor-coil end, *f.* stator belt, *g.* rotor belt. Belt is also referred to as differential.

These are developed as follows:

a. Primary slot constant (see Fig. 396).

$$L_1 = \frac{r_2}{3B_3} + \frac{2r_6}{B_1 + B_3} + \frac{r_4}{B_1} \quad (for\ closed\ slots)$$

$$= \frac{r_2}{3B_1} + \frac{r_5}{B_1} \quad (for\ open\ slots)$$

The rotor-slot constant L_2 is calculated, using the corresponding dimensions from the rotor slot. The dimensions used in all cases in this chapter are in inches or fractions thereof.

b. Zigzag Leakage.—This is the term which takes care of the leakage flux which crosses and recrosses the air gap from the tops of stator to the tops of rotor teeth and back, managing to

travel across the face of a complete magnetic pole without linking both stator and rotor windings. Its formula is:

$$S_z = \frac{K_0 K_z}{N_1 N_2}$$

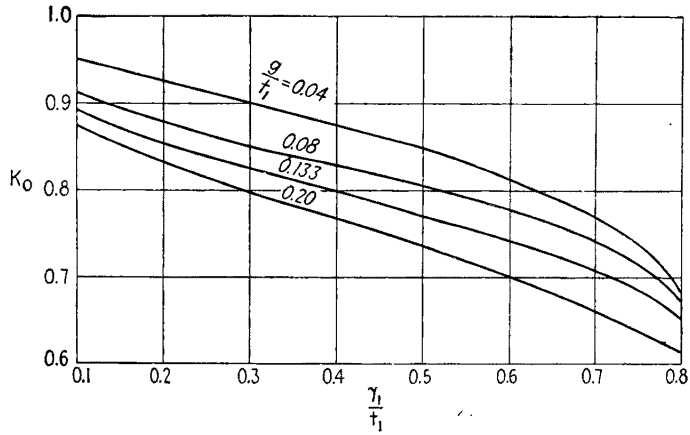


FIG. 397*a*.—Zigzag leakage curves.

where K_0 = 0.92 for partially closed slots in both stator and rotor, or 0.85 where the stator has open slots and the rotor partially closed slots.

N_1 = number of stator slots per pole.

N_2 = number of rotor slots per pole.

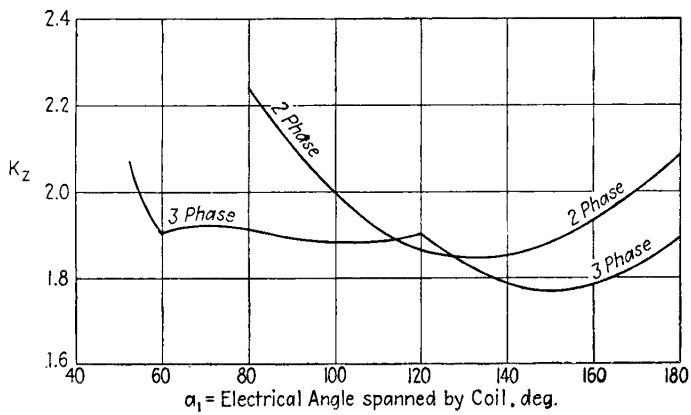


FIG. 397*b*.—Zigzag leakage curves.

These factors are calculated by dividing the total number of stator or rotor slots by the number of poles in the motor. K_z is

taken from Fig. 397*b* for the number of electrical degrees corresponding to the throw of the stator coil. This is explained in Chapter VI. One example would be a 72-slot stator wound for six poles with a coil throw of slots 1 to 11. In this case 180 electrical degrees corresponds to 72/6 = 12 slots; hence one slot is 15 deg. The coil spans 10 slots, which would be 10 × 15 = 150 deg., and the value of K_s from Fig. 397 would be 1.77 for a three-phase motor, since that is the value on the three-phase curve at 150 deg.

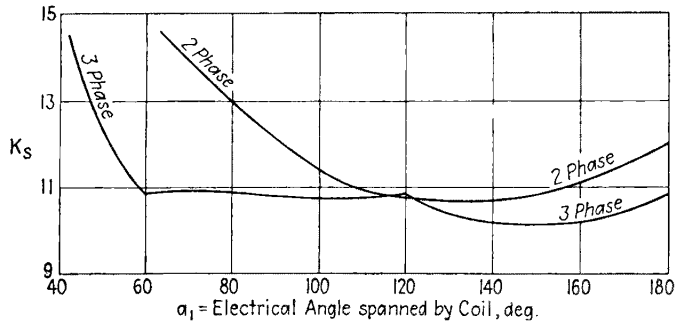


FIG. 398.—Slot leakage pitch constant.

c. Stator-slot Leakage.—Under (*a*) above was calculated the stator-slot constant, now combined with other factors to give the stator-slot leakage according to the formula:

$$S_{1s} = \frac{K_s g G L_1}{N_1 t}$$

where K_s is to be taken from Fig. 398 for the proper throw of the stator coil, in electrical degrees.

g = air gap in inches between stator and rotor.
G = total air-gap factor (stator factor × rotor factor) as calculated under (1*c*) above.
L_1 = stator-slot constant, (2*a*) above.
N_1 = number of stator slots per pole.
t = pole pitch = stator bore in inches × 3.1416 ÷ number of poles.

d. Rotor-slot Leakage.

$$S_{2s} = \frac{K_s g G L_2}{N_2 t}$$

where g, G and t are the same as for the stator, but where K_s, L_2 and N_2 are values taken from the rotor. K_s is to be taken from Fig. 398, using the stator-coil throw if the rotor is squirrel-cage

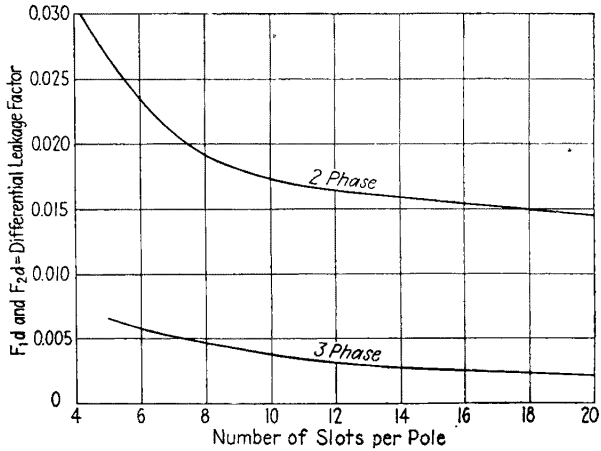


FIG. 399.—Differential leakage factor F.

type. K_s is 11 if the rotor has a three-phase-coil winding. This means that for a squirrel-cage rotor K_s is the same as for the stator and that for a wound-rotor motor K_s is 11.

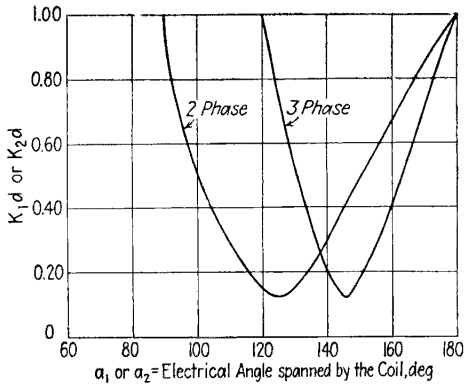


FIG. 400.—Differential leakage factor K.

e. Stator Differential or Belt Leakage.

$$S_{1d} = F_{1d} \times K_{1d}$$

where F_{1d} is taken from Fig. 399 for the correct number of stator slots per pole and K_{1d} from Fig. 400 for the proper throw of the stator coil.

f. Rotor Differential or Belt Leakage.—For a squirrel-cage rotor there is no rotor differential leakage, so that $S_{2d} = 0$ for squirrel-cage rotor; for a wound rotor, $S_{2d} = F_{2d}$, from Fig. 399 for the correct number of rotor slots per pole.

g. Stator-coil End Leakage.

$$S_{1e} = \frac{C_1 G g L_1}{lt(chord\ factor)^2}$$

where C_1 is from Fig. 401.

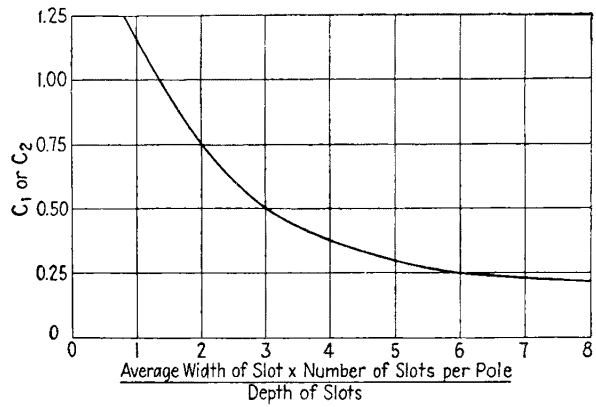


FIG. 401.—End-connection leakage factor.

g and G = the gap and gap factor, as explained above.
l = length of the core parallel to the shaft, minus the air ducts (if any) × 1.03 as calculated under (1*b*) above.
t = pole pitch, as under (2*c*).
L_1 = length of the coil end on one side of the core.

The *chord factor* is according to Chapter VI or Chapter XIX.

For diamond coils the following formula is close enough to use:

$$L_1 = 1.5 \times t \times \frac{N_t}{N_1} + 3.5 \text{ inches},$$

where t = pole pitch, as under (2*c*) above.
N_t = number of slots spanned by the coil; *e.g.*, if the coil throw is 1 and 8, then $N_t = 7$.
N_1 = total number of slots divided by number of poles.

The term "3.5" is common in all cases and is found to work out about right over a wide range of coils.

h. Rotor-coil End Leakage.—For a wound rotor,

$$S_{2e} = \frac{C_2 g G L_2}{lt}$$

where C_2 is from Fig. 401 and L_2 is found for the rotor coil as L_1 was found for the stator.

For squirrel-cage rotors,

$$L_2 = t + 2f$$

where t = pole pitch, as in (2*c*) above.

f = distance between the end of the core and the center of the short-circuiting ring at the ends of the rotor bars.

Also for squirrel-cage rotor the following equation is close enough to use:

$$C_2 = 0.45$$

i. Total Leakage Coefficient.—The complete leakage coefficient is the sum of all the factors calculated under (*b*), (*c*), (*d*), (*e*), (*f*), (*g*) and (*h*) multiplied by the saturation factor or *Ampere turns total/Ampere turns in the air gap*, as explained under magnetizing current (1*g*), above. The formula, then, would be

$$S = \frac{AT_t}{AT_g}(S_z + S_{1s} + S_{2s} + S_{1d} + S_{2d} + S_{1e} + S_{2e})$$

and its value will be somewhere in the range of 0.03 to 0.08 for motors from 1 hp. up. The larger figure goes with the smaller horsepower and vice versa. Later in the chapter are given sample calculations for moderate- and large-sized motors.

3. Iron Losses. *a. Losses in the Stator Core or Yoke.*—(1) Calculate the cubic inches in the stator core. Refer to Fig. 251. The area of the core is $C \times L$, where C = depth of iron back of the slots and L = width of core, minus air ducts, if any. There are none in Fig. 251. The volume of the core will then be $C \times L \times$ [*bore diameter* + (2 × *depth of stator teeth*) + *depth of iron back of slots*] × 3.1416. By adding once the depth of iron below slots, the diameter found is that at the middle of the core depth or at $C/2$.

(2) From the magnetic density in the core, found in calculating the ampere turns for magnetizing current, and from the curve of Fig. 402, find the loss per cubic inch at this density. Multiply this figure by the cubic inches in the core, just found, and the product is the iron loss in watts in the stator core.

b. Iron Loss in the Stator Teeth.—(3) Calculate the cubic inches of iron in the total number of stator teeth. The width of the tooth at half its depth was found in calculating magnetizing current. Multiply this by the core length (minus ducts, if any), by the height of the stator tooth and by the total number of

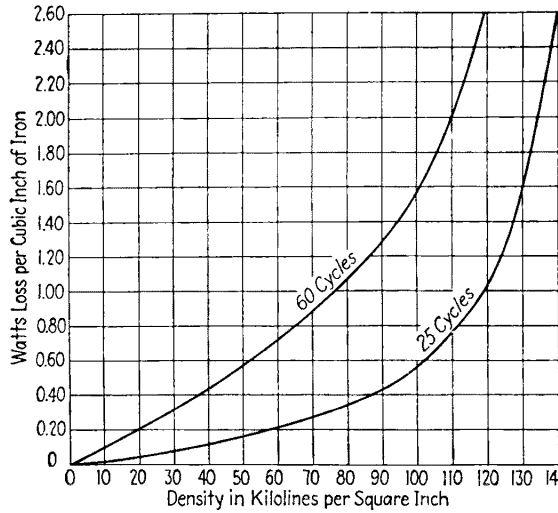


FIG. 402.—Iron-loss curve for teeth and core.

teeth. The product is the cubic inches of iron in the stator teeth.

(4) From the maximum density (*Average*/0.636) in the teeth as found under magnetizing current and from Fig. 402, find the watts loss per cubic inch in the stator teeth. Multiply this by the cubic inches in the teeth just found. The product is the watts loss in the stator teeth.

The losses found under (2) and (4) are the straight or legitimate iron losses in the machine, but they are greatly increased by high-frequency pulsations caused by the openings in the slots.

IRON LOSSES IN THE ROTOR SURFACE

In connection with the magnetizing current, it was shown how the maximum density in the air gap could be found. The value

found there, however, was under the assumption that there are no slot openings. (The influence of the latter was considered by increasing the gap length). For the present case, we are interested in the actual maximum magnetic density in the middle of the tooth, which for the instant is opposite the middle of the face of the magnetic pole. The latter is found as follows: Figure the surface of the primary tooth tops per pole:

$$F_1 = A_1 \times l \times N_1$$

where l = effective length of the core as previously defined, *i.e.*, actual length of iron (minus air ducts) $\times$ 1.03
For A_1, see Fig. 393. The actual maximum density in the gap is then

$$B_m = \frac{Flux\ per\ pole}{F_1 \times 0.636}$$

Now find the following value:

$$f = \frac{B_m \times B_1 \times N_1}{g \times 100{,}000}$$

where B_1 = stator-slot opening, from Fig. 396.
N_1 = number of stator slots per pole.
g = mechanical gap between stator and rotor
$= \frac{stator\ bore - rotor\ diameter}{2}$.

For this value f read from Fig. 403 the losses per square inch of the rotor surface at 25 cycles or 60 cycles as required by the problem in hand.

Find the total rotor surface in square inches, *i.e.*, the actual top surface of one rotor tooth times the total number of rotor teeth, and multiply this by the value found from Fig. 403.

c. Total Core Losses.—The total core losses are found by adding the above calculated three parts, *i.e.*, (*a*) stator core, (*b*) stator teeth and (*c*) rotor surface. There is practically no iron loss in the rotor core or teeth while the motor is running, because the frequency in the rotor is only the percentage slip times the line frequency and is only two or three cycles, which produces only a negligible iron loss. For hurry-up preliminary estimating, the author has often calculated the loss in the stator core and teeth

only and has multiplied it by the square of the air-gap factor as calculated under magnetizing current. This is often a close enough approximation to the additional losses on the surface of the rotor and elsewhere.

d. Constant Losses.—The so-called "constant losses" are the sum of the core losses and the bearing friction plus windage. The friction and windage losses for a given frame at a given speed are usually taken from tests on similar machines at the same speed. In order to make this chapter self-contained, the following data are given for estimating friction and windage.

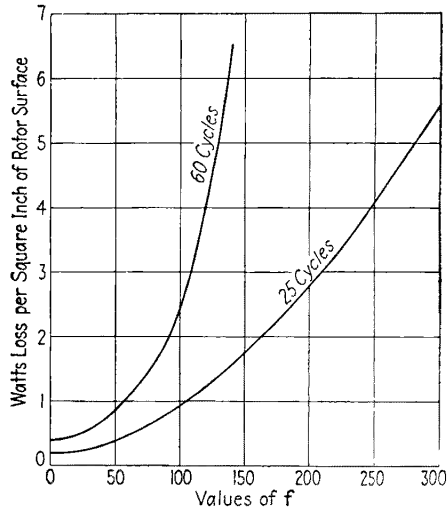


FIG. 403.—Iron loss on surface of rotor.

The best complete formula is one used in both Europe and America and for this reason it is given here as information, if the student can use it. The exponents $3\frac{1}{2}$ and $5\frac{1}{2}$ involve logarithms but can be done arithmetically, in the case of $5\frac{1}{2}$, by multiplying the number five times and then extracting the square root, or, in the case of $3\frac{1}{2}$, by multiplying it three times and extracting the square root. The formula is:

$$\textit{Watts friction and windage} = \frac{252 \times D^{5\frac{1}{2}} \times l \times (r.p.m.)^{3\frac{1}{2}}}{100{,}000{,}000}$$

where D = diameter of the rotor in inches.
l = length of rotor core, parallel to the shaft.
$r.p.m.$ = revolutions per minute.

If this formula is too unwieldy, Fig. 404 gives a curve plotted in percentage of the motor rating in kilowatts (hp. × 0.746) against peripheral speed in feet per minute, which is:

$$S = \frac{\textit{Rotor diameter in inches} \times 3.1416 \times \textit{r.p.m.}}{12}$$

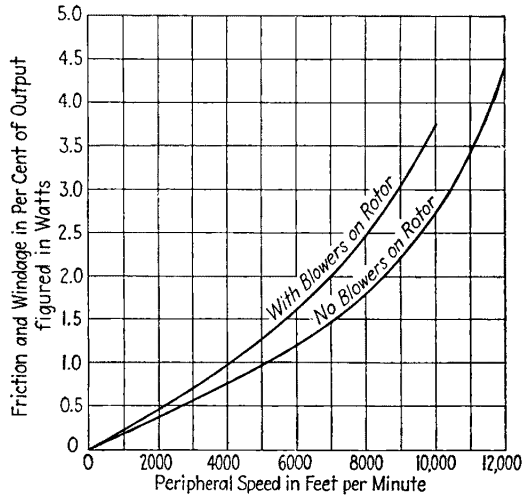


FIG. 404.—Hansen's curve for friction and windage on large machines.

This curve was primarily drawn for fairly large machines. For smaller machines the following table was drawn up from a series of test results.

Rotor Diameter, Inches	Friction and Windage, Watts
5 to 5½	$\frac{0.422 \times S^{3\frac{1}{2}}}{1000}$
6 to 7	$\frac{0.532 \times S^{3\frac{1}{2}}}{1000}$
8½ to 9½	$\frac{0.893 \times S^{3\frac{1}{2}}}{1000}$
11 to 12	$\frac{1.148 \times S^{2}}{100{,}000}$
13½ to 14½	$\frac{1.863 \times S^{2}}{100{,}000}$
16 to 18	$\frac{2.95 \times S^{2}}{100{,}000}$
20 to 22	$\frac{3.985 \times S^{2}}{100{,}000}$

where S = peripheral speed in feet per minute as calculated on page 419.

Where the exponent is $\frac{3}{2}$, one can dodge the use of logarithms by cubing the number (multiplying it by itself three times) and extracting the square root of the product.

As a rough check, if the full-load efficiency of the motor is to be 90 per cent, the iron losses plus friction and windage will be about 5 per cent of the motor rating in kilowatts and the friction and windage will be from 1 to 3 per cent, depending on the speed.

When the friction and windage loss is found or calculated, add it to the iron losses. The sum is the so-called "constant losses" and is used later in drawing the constant-loss line in the circle diagram.

4. Ohmic Resistances.—The only values remaining to be calculated, before drawing the circle diagram and estimating the complete motor performance, are the resistance of the stator and rotor winding. It is easy to see on the stator or a coil-wound rotor that this value is the resistance between leads or collector rings. On a squirrel-cage rotor this seems a little harder to handle but it is only the resistance of one rotor bar plus a piece of the short-circuiting ring at each end.

These resistances are calculated as follows:

a. Stator or Primary.—The resistance of the stator is to be figured per phase. That is, if it is a three-phase motor, the resistance per phase will be one-third the resistance of the entire winding. If it is a star connection, it will be one-half the resistance between any two leads. If it is a delta connection, it will be the resistance of one side of the delta. Great care must be used to take account of the wires in parallel in the coil to form the conductor and to take account of parallels in the group connections of the completed winding. The first step is to figure the length of one conductor including the coil ends. In the leakage calculation, the length of the coil-end connection at one end of the core was calculated as:

$$L_1 = 1.5 \times t\frac{N_t}{N_1} + 3.5$$

where t = pole pitch = stator bore $\times$ 3.1416 $\div$ number of poles.

N_1 = number of slots per pole.

N_t = number of teeth spanned by the coil.

If it is a 72-slot stator, four-pole, and the coils lie in slots 1 and 16, $N_1 = 72/4 = 18$ and $N_t = 15$. When the length of one coil end has been found, this is added to the width of the core to give the length of one conductor, which is one-half the length of a complete turn of the coil.

When the length of one conductor has been calculated and the number of conductors in series per phase is known, the resistance can be figured from the length and the cross section of the conductor. The latter factor will usually be found to be expressed in wire sizes as one No. 10 wire or three No. 12 wires, etc. A wire table will give the resistance per 1000 feet of that wire size and from the length, figured as length of one conductor in feet times number of conductors per phase, the cold resistance or resistance at 25°C., per phase, is found direct. The hot resistance, or resistance at 40°C. rise or 65°C. total, is about 114 per cent of the cold resistance. The hot value should be calculated and used in drawing the circle diagram as it will then represent the performance when running under load.

b. Rotor or Secondary.—If the rotor is phase wound, the resistance per phase is figured the same way as in the stator, by calculating the length of one conductor, including the slot and end portion and multiplying by the number of conductors per phase for the total length. The resistance per phase is then:

$$\textit{Resistance per phase} = \frac{\textit{Inches per conductor} \times \textit{number of conductors per phase} \times 6.9}{\textit{Cross-section area of conductor in square inches} \times 10{,}000{,}000}$$

This value will be the cold resistance per phase.

c. Squirrel-cage Rotor.—Chapter IV shows the method of figuring the resistance of the bar plus the ring plus the contact drop. In the present case, the value of the rotor resistance is not reduced to the stator and also with modern motors the bar is usually brazed or welded to the short-circuiting ring, so there is no contact drop as is the case with a bolted joint.

For the present purpose the cold resistance of the bar and ring is calculated as follows:

Take the length of the bar in inches between the centers of the two resistance rings. If it is copper, the resistance is:

$$\frac{\textit{Length in inches} \times 6.9}{\textit{Bar cross section in square inches} \times 10{,}000{,}000}$$

To this should be added the resistance of a piece of the ring between two adjacent bars. To find this resistance take the diameter of the ring at one-half its radial depth, multiply by 3.1416 and divide by the total number of rotor bars. This is the length of the ring used in the following formula:

$$\textit{Resistance of ring} = \frac{\textit{Length} \text{ (just found)} \times 6.9}{\textit{Cross section of the ring} \times \textit{Conductivity} \times 10{,}000{,}000}$$

All dimensions are given in inches.

The factor conductivity is expressed as a decimal. If the ring is rolled copper, the value is 1; if cast aluminum, about 0.5; if brass, with 80 per cent of the conductivity of copper, 0.8, etc., depending on the conductivity of the material used. The value of this ring resistance just found should be multiplied by a factor adding it to the bar resistance. This factor is:

$$\frac{2 \times (N_2^2 + 1)}{10}$$

where N_2 = number of rotor slots per pole. In a 10-pole machine with 171 rotor bars, N_2 would be 17.1. When the bar resistance is multiplied by this factor, it is added to the bar resistance and the total is multiplied by 1.14 to get the hot resistance value.

When the values have been calculated, as indicated above for (*a*) the no-load or magnetizing current, (*b*) the leakage coefficient, (*c*) the stator and rotor resistances, (*d*) the constant losses, *i.e.*, iron plus friction and windage, it is a simple matter to draw three circles which then give the complete story of currents, torques, power factor, efficiencies and r.p.m. at any and all loads. In the section immediately following, a general description is given of this construction. This is followed by the actual construction from the calculation of two motors which were built and tested. The comparison is shown between the calculated and tested values. From this it can be seen that the method is close enough to furnish interesting calculations if the reader is careful and patient and wishes to increase his familiarity with the induction motor. This motor is the most widely used of all motors and contains in its calculation practically all the phenomena of dynamo-electric design, except commutation. By

the time the reader has attempted and carried through, to some degree of satisfaction, a calculation of the performance of an

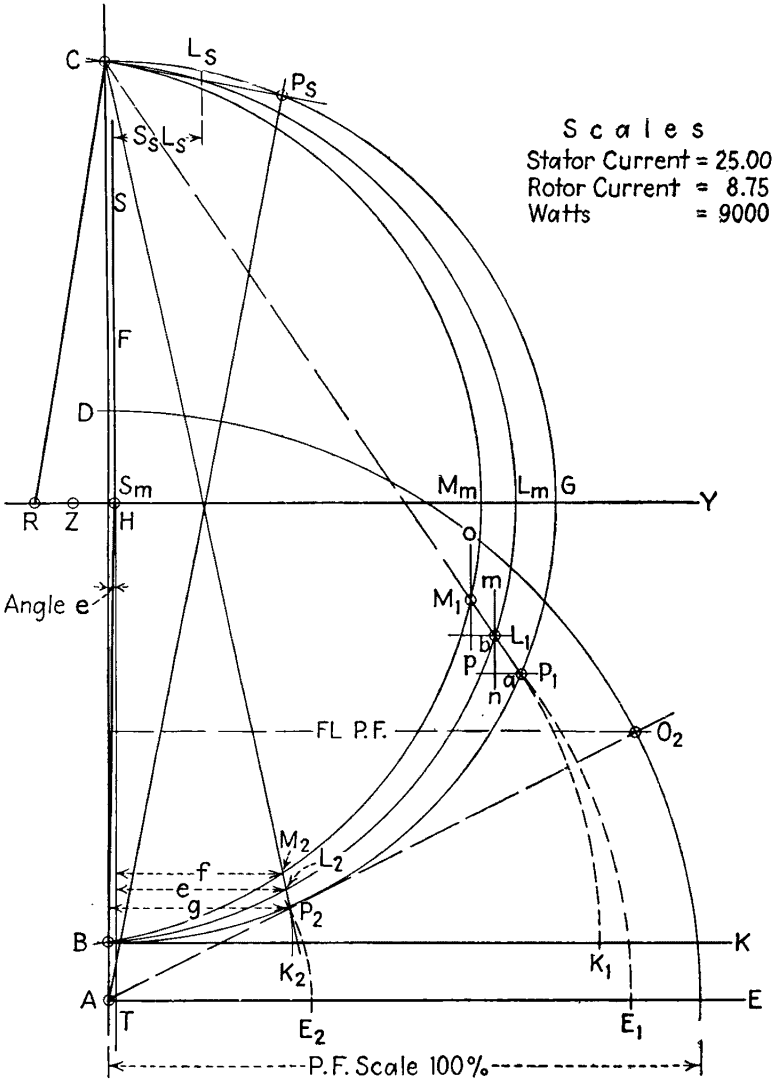


FIG. 405.—Circle diagram for 350-hp. wound rotor motor.

induction motor, he will be quite satisfied that it does not have a commutator.

5. Construction of the Circle Diagram. Select a length for the diameter of the circle to a convenient scale. For example, on a sheet of Cornell cross-section paper, assume that the diameter

will be 15 large squares or 150 small squares. This will be 7½ inches long and will be a fairly convenient scale. This is the line BC on Fig. 405. The distance AB is found from BC through the use of the leakage coefficient (calculated as S) in the formula:

$$AB = BC\frac{S}{1 - S}$$

For example, in the case worked out later, if $S = 0.06$, then $AB = BC\frac{0.06}{0.94} = BC \times 0.064$ and, since BC was 150 spaces, $AB = 150 \times 0.064 = 9.6$ spaces. Therefore lay out A the proper distance from B and, when the point A is fixed, the current for any load is determined by the distance from A to the load point on the current circle.

Draw a quarter circle around A, with the radius AD so chosen that it divides into 100 parts, since power factors for various loads are read from this scale. Now we know that the circle CP_SGB passes through C and B and we know that its center lies on the line PG, at right angles to ABC, but we do not know yet just where. The center of the circle is located by drawing the line AF at an angle e with AC. e is found from the expression

$$\sin\frac{e}{2} = \frac{i_m r_1}{E_p}$$

where i_m = the magnetizing current per phase calculated above.
r_1 = hot resistance of stator winding per phase.
E_p = voltage per phase.

It will be seen later how this is calculated. Since the angle e is very small, the sine can be placed equal to the arc and a point is located one or two spaces along the arc DKE, according to the product of $2 \times \frac{i_m r_1}{E_p}$. If it is 0.01, lay off DX one space; if 0.02, then $DX = 2$ spaces. Draw the line AX. Its intersection of RG at H is the center of the current circle.

Draw the circle CGB with its center at H. This is the current circle.

a. Scales.—The various current and power scales are determined as follows:

Primary- or Stator-current Scale (Stator Current Per Phase).—$C_1 = i_m/AB$, *i.e.*, the magnetizing current, as calculated, divided by the number of divisions in the line AB. If the magnetizing current was 24 amperes and $AB = 9.6$ spaces, then each space represents $24/9.6 = 2.5$ amperes $= C_1$.

Secondary- or Rotor-current Scale, for Phase-wound Rotor.

$$C_2 = C_1 \frac{T_{p1} \times a_1 \times K_{p1} \times C_1}{T_{p2} \times a_2 \times K_{p2} \times C_2}\left(1 + \frac{S}{2}\right)$$

where T_{p1} = number of turns per phase in the stator = conductors per phase $\div$ 2.

T_{p2} = number of turns per phase in the rotor = rotor conductors per phase $\div$ 2.

a_1 and a_2 = number of phases in stator and rotor, respectively.

K_{p1} and K_{p2} = distribution factors for stator and rotor, respectively. They can always be put equal to 0.905 for two-phase and 0.955 for three-phase.

C_1 and C_2 = stator- and rotor-chord factors, respectively, where C_2 is practically always equal to one, since the rotor winding is seldom if ever chorded.

S = leakage coefficient, as calculated (always of the order of 0.02 to 0.10 or in that range).

For Squirrel-cage Secondaries.

$$C_2 = C_1 \frac{2 \times T_{p1} \times a_1 \times K_{p1} \times C_1}{N_2 \times p}\left(1 + \frac{S}{2}\right)$$

where the quantities in the numerator are the same as the preceding paragraph and N_2 equals the number of rotor slots per pole and p equals the number of poles, so that $N_2 \times p$ = total number of rotor slots.

Watt Scale.

$$C_w = C_1 \times E_p \times A_1$$

where C_1 = stator-current scale

E_p = stator volts per phase.

A_1 = number stator phases.

Power-factor Scale.

$$C_p = \frac{100 \text{ per cent}}{AD}$$

The meaning of these various scales is that one small division on the cross-section paper is equal to C_1, C_2, or C_w stator amperes, rotor amperes or watts, respectively.

b. The Torque and Output Circles.—The one circle which has been drawn represents the stator and rotor currents and could be made to represent torques, watts and output also, but it introduces so many lines that they are confusing. It is simpler and easier to draw two additional circles CL_mB and CM_mB, which represent these quantities very neatly and clearly.

Assume at random a point P_1, somewhere below the center line RY of the circle CGB, and draw the line CP_1; also draw the lines BK and AE. Draw a circle around A, from P_1 to E_1, and a second circle around B, from P_1 to K_1.

The stator current for the point P_1 is equal to $AE_1 \times C_1$, and the stator copper loss for this load is represented by

$$(AE_1 \times C_1)^2 \times r_1 \times a_1$$

where r_1 = hot stator resistance per phase.
a_1 = number of stator phases.

Divide this value by the watt scale C_w and find the horizontal distance a of a vertical line mn from P_1, as shown in Fig. 405. The vertical line mn intersects CP_1 in L_1. Draw a circle through BL_1C with its center on HY. Locate the center in the usual way, by drawing a straight line from C to L_1 and erecting a perpendicular at its center which intersects the line HY at Z, the center from which the circle BL_1C is struck. This is the torque circle, showing starting torque, pull-out torque and torques for all loads.

c. The Output Circle.—The rotor current for the load point P_1 is expressed by $BK_1 \times C_2$ and the secondary copper losses by

$$(BK_1 \times C_2)^2 \times r_2 \times a_2$$

where r_2 = the hot resistance of the rotor per phase or per bar as it is phase-wound or squirrel-cage.
a_2 = number of rotor phases (nearly always three if phase-wound).
a_2 = total number of rotor bars if squirrel-cage.

Divide this value by the watt scale C_w and find the horizontal distance b of a vertical line op from L_1, as shown in Fig. 405. The vertical line op intersects CP_1 in M_1. Draw a circle through BM_1C with its center on HY. The center of this circle is R. Draw the line RC and the line CP_s at right angles thereto. Draw a line ST parallel to AC such that the distance AT between ST and AC equals *Constant losses*$/C_w$. This completes the construction of the diagram.

6. Use of Circle Diagram. *a. Output or Load.*—If the performance for any required load of M hp. is to be determined, we find the output in watts to be $M \times 746$, which corresponds to $\frac{M \times 746}{C_w}$ divisions on the diagram. Draw the line f horizontally between the line ST and the circle BM_1C (output circle) so that it is equal to $\frac{M \times 746}{C_w}$, then f represents the output expressed in watts.

b. Torque.—Draw the line CM_2P_2. The horizontal distance e, between the line ST and the circle CL_2B, represents the torque. The torque is:

$$T_2 = eC_w \text{ synchronous watts} = \frac{eC_w}{746} \text{ synchronous hp.}$$

The expressions "synchronous watts" and "synchronous hp." mean the output the motor would develop if it ran at synchronous speed $\left(\frac{cycles \times 120}{poles}\right)$ instead of running three or four per cent less owing to the "slip."

$$\frac{eC_w \times 5250}{746 \times n} = \text{pounds torque at 1 foot radius}$$

where n = the synchronous speed $\left(\frac{cycles \times 120}{poles}\right)$.

c. Input.—The horizontal distance g, between the line BC and the circle CGB, represents the input. The input is $g \times C_w$ expressed in watts.

d. Stator Current.—Draw a circle around A, through P_2 to E_2. The distance $AP_2 = AE_2$ represents the stator current. The latter is therefore $AE_2 \times C_1$.

e. Rotor Current.—Draw a circle around B through P_2 to K_2. The distance $BP_2 = BK_2$ represents the rotor current. This is therefore $BK_2 \times C_2$.

f. Power Factor.—Draw a line through A and P_2 until it intersects the power factor circle DO_2E at O_2. The line PF represents the power factor. It equals $PF \times C_p$. If the line $AD = AE$ was chosen as suggested, 100 divisions long, the point O_2 can be projected vertically downward on AE and the power factor read direct. For example, on the diagram as shown, $PF = 89$ per cent, since $AE = 100$ divisions and $O_2 = 89$ divisions horizontally from ABC. Therefore $89 \div 100 = 89$ per cent.

g. Stator Copper Losses.—The stator copper losses are represented by the horizontal distance between P_2 and L_2. It is preferable, however, to calculate the losses from the square of the stator current and the resistance of all phases of the winding, as explained for the point P_1 when constructing the diagram. For the point P_2 this equals $(AE_2 \times C_1)^2 \times r_1 \times a_1$, as previously explained.

h. Rotor Copper Losses.—The rotor copper losses are represented by the horizontal distance between L_2 and M_2, but it is better to figure them as just explained under stator copper losses for the point P_1.

i. Efficiency.—The efficiency may be found from the ratio of output divided by input or *Line f/Line g* from the diagram. It is better, however, to figure the input as the sum of the output plus stator copper loss plus rotor copper loss plus fixed losses and to divide the output by this sum.

j. Maximum Output (in Horsepower).—The maximum output is represented by the distance S_mM_m. If f is the rated output, we find the maximum output in terms of the rated output from S_mM_m/f. S_m is the intersection of lines ST and RY.

k. Maximum Torque or Pull-out.—The pull-out torque is represented by the distance S_mL_m. If e is the rated torque, we find the pull-out torque in terms of the rated torque as S_mL_m/e.

l. Starting Torque.—The starting torque at full voltage is represented by the distance S_sL_s, where the straight line CP_s is drawn at right angles to the radius CR of the output circle. If e is the rated full-load torque, the starting torque in terms of full-load torque is S_sL_s/e.

m. Starting Current.—The starting current or locked current at full voltage is $AP_s \times C_1$.

n. Slip.—It is possible to draw a line in the diagram from which the slip may be read graphically, but it is preferable to calculate it from the following equation:

$$Slip = \frac{Rotor\ copper\ losses}{Output + rotor\ copper\ losses}$$

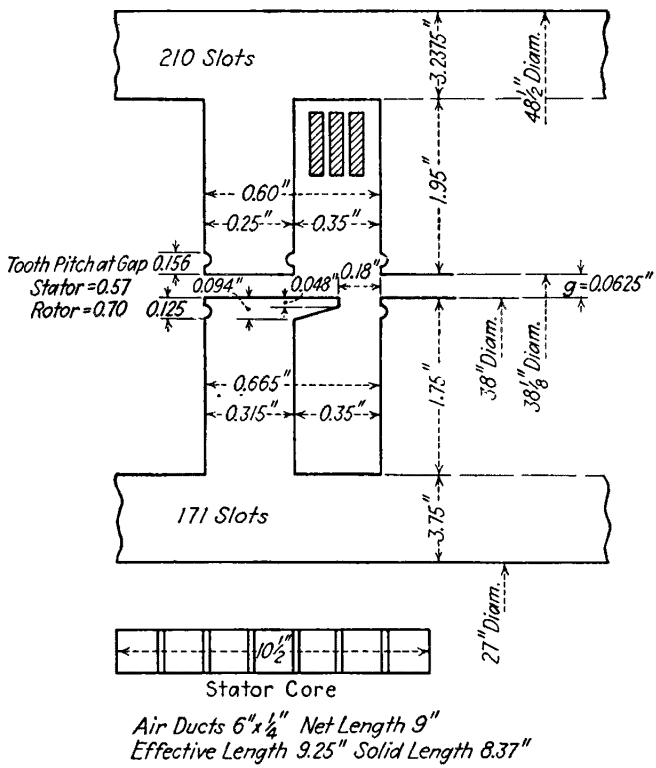


FIG. 406.—Core and slot dimensions of 350-hp. wound rotor motor.

7. Practical Examples of Calculation.—In order to show how the method of calculation explained in this chapter is applied, the rest of the chapter takes up the calculation of a large wound rotor motor and a smaller squirrel-cage motor. These motors

were actually built and tested and the calculated performance can be checked against the tested results.

SAMPLE CALCULATION

Rating: 350-hp. three-phase, 60-cycle, 10-pole, 2080-volt, 705-r.p.m. wound-rotor induction motor.

The dimensions of the core and punchings, together with the number of stator and rotor slots, are shown in Fig. 406. Since there are six $\frac{1}{4}$-inch air ducts, the net length of the core is $10\frac{1}{2} - (6 \times \frac{1}{4}) = 9$ inches; multiplying by 0.93, on account of the varnish on the laminations, leaves $9 \times 0.93 = 8.37$ inches of solid iron. Adding about three per cent for flare gives $9 \times 1.03 = 9.27$ inches as the effective length for the leakage calculation.

Make a table like this:

Slot insulation		Windings	Stator	Rotor
Width of stator slot, in.		Volts per phase	1200	341
Copper	0.165	Connections	Series star	Series star
Insulation	0.185	Series conductors per phase	420	114
		Conductors per coil	1×3	1×2 in parallel
Total	0.350	Coils per slot	2	2
		Copper	0.055×0.750	$\frac{1}{16} \times \frac{5}{8}$
Depth of stator slot, in.		Area of conductor	0.0406 sq. in.	
Copper $2 \times 0.75 =$	1.500	Throw of coil	1 to 18	1 to 18
Insulation	0.294	Mean length of conductor	29 in.	31 in.
Wedge	0.156	Length of one end connection	$3\frac{1}{2} + 15 = 18.5$ in.	$3\frac{1}{2} + 17 = 20.5$ in.
Total	1.950	Hot resistance 65°C. per phase	0.267 ohm	0.0225 ohm
Width of rotor slot, in.				
Copper	0.200			
Insulation	0.150			
Total	0.350			
Depth of rotor slot, in.				
Copper	1.250			
Insulation	0.260			
Wedge	0.140			
Total	1.650			

Distribution Factors, Chord Factors, Air-gap Factor, Slot-leakage Factors.—Make a small table like this:

	Distribution	Chord	Slot leakage	Air gap
Stator	0.955	0.96	2.155	1.48
Rotor	0.955	1.00	2.22	1 10
Slot pitch (stator)	At ½ slot depth, 0.60; at air gap, 0.57			
Thickness of tooth at ½ depth	0.60 − 0.35 = 0.25 inch			
Total air-gap factor	1.48 × 1.10 = 1.628 inches			
Pole pitch at air gap	12 in.			

This is the calculation for the figures in this table. Since both stator and rotor are three-phase, the distribution factor is 0.955 in both cases. The chord factor for the stator is calculated as explained in Chapter XV.

$$\sqrt{\frac{21^2 - 2(4)^2}{21^2}} = 0.96$$

The chord factor for the rotor is 1, because it is wound full pitch.

Stator Slot-leakage Factor.—Slot depth below wedge 1.950 − 0.156 = 1.794 slot width = 0.35.

$$\frac{1.794}{3 \times 0.35} = 1.71$$

$$\frac{0.156}{0.350} = 0.445$$

$$\text{Total slot factor} = 2.155$$

Rotor Slot-leakage Factor.—Slot depth below wedge 1.75 − 0.14 = 1.61. Slot width = 0.35. The slanting portion under the overhang is 0.046 deep and is divided by the average width between 0.35 and 0.18. The total factor is figured thus:

$$\frac{1.61}{3 \times .35} = 1.53$$

$$\frac{0.046 \times 2}{0.35 + .18} = 0.17$$

$$\frac{0.094}{0.18} = 0.52$$

$$\text{Total slot factor} = 2.22$$

Air-gap-factor Stator.—Single air gap or clearance between stator and rotor = 0.0625. Slot opening = 0.35 and 0.35/0.0625 = 5.6. From Carter's curve in Fig. 393 for 5.6 the value z = 0.53.

$$\frac{1}{1 - \frac{0.53 \times 0.35}{0.57}} = 1.48$$

where 0.57 is the stator-slot pitch at the air-gap surface.

Rotor Air-gap Factor.—Slot opening = 0.18 and 0.18/0.0625 = 2.9. From curve, Fig. 393, for 2.9, z = 0.37

$$\frac{1}{1 - \frac{0.37 \times 0.18}{0.70}} = 1.1$$

Total air-gap factor is $1.48 \times 1.1 = 1.628$. This means that the actual length of the magnetic path across the air gap is not the physical length of 0.0625 inch, but is a magnetic length of $1.628 \times 0.0625 = 0.102$ inch, called the *effective gap* or the greater distance, which the magnetism has to travel in crossing the gap, since the slot openings are cut away. The stator-slot pitch is $\frac{38.125 \times 3.14}{210} = 0.57$ at the air gap or $\frac{(38.125 + 1.95) \times 3.14}{210}$ = 0.60 halfway down the tooth. The pole pitch at the air gap = $\frac{38.125 \times 3.14}{10} = 12$ inches.

Magnetic-field Flux Per Pole and the Magnetizing Current.—The magnetic-field flux per pole is:

$$\frac{45{,}000{,}000 \times 1200}{60 \times 420 \times 0.96 \times 0.955} = 2{,}340{,}000 \text{ lines}$$

where 45×10^6 = a constant.
1200 = volts per phase (2080 ÷ 1.73).
60 = cycles.
420 = stator conductors per phase.
0.96 = chord factor.
0.955 = the distribution factor.

To figure the magnetizing current, which is for all practical purposes the no-load current, make a table like this:

Part of path	Cross section of path, sq. in.	Magnetic density in part	Length of path, in.	Ampere turns required per inch from curve	Ampere turns for the part
Stator teeth.....	44	83,500	1.95	9	18
Stator core.....	54.25	43,000	7.1	2.5	18
Rotor teeth.....	45	81,750	1.75	8	14
Rotor core.....	62.5	37,500	4.82	2	10
Air gap.....	108	22,500	0.102	...	730
				Total....	790
Saturation factor....	$\frac{790}{730} = 1.08$				
Magnetizing amperes	$\frac{0.605 \times 790 \times 10}{210 \times 0.96} = 23.9$ amperes				

Cross Section of Path.—*a.* The *cross section of the path in the stator teeth* is the width of one tooth at half its height (0.60 − 0.35 = 0.25) times the solid length of the core times number of slots per pole:

$$0.25 \times 8.37 \times 21 = 44 \text{ square inches}$$

b. Stator-core Area.—Depth below slots times solid length of core times 2, because the flux divides and goes both ways when it gets back to the core:

$$3.24 \times 8.37 \times 2 = 54.25$$

c. Rotor-teeth Area:

$$(0.665 - 0.35 = 0.315) \times 17.1 \times 8.37 = 45$$

d. Rotor-core Area:

$$3.75 \times 8.37 \times 2 = 62.5$$

e. Air-gap Area per Pole.—Pole pitch times effective core length = $12 \times 9 = 108$ square inches.

Magnetic Densities in the Parts.—In the cores and the air gap the field flux per pole, which is 2,340,000 lines, is divided by the respective cross sections for the cores and the air gap.

However, for the teeth, the maximum density is used instead of the average; this gives

$$\textit{Density in stator teeth} = \frac{2{,}340{,}000}{44 \times 0.636} = 83{,}500$$

$$\textit{Density in rotor teeth} = \frac{2{,}340{,}000}{45 \times .636} = 81{,}750$$

The factor 0.636 is 2/3.1416.

Length of Path.—The length of path in the stator and rotor teeth is the depth of the slot, or rather height of the tooth. In the air gap it is the effective gap or mechanical clearance multiplied by the total air-gap factor, or 0.102, as explained above. In the stator and rotor cores the diameter is taken at the radial center of the core times 3.1416 divided by 2 times number of poles. For the stator this is

$$\frac{(48.5 - 3.2375) \times 3.1416}{2 \times 10} = 7.1$$

and for the rotor

$$\frac{(27 + 3.75) \times 3.1416}{2 \times 10} = 4.82$$

The accuracies of the arithmetic calculations are those of the ordinary 10-inch slide rule.

Ampere Turns Per Inch.—Curve *A* for the core and curve *B* for the teeth, from Fig. 394, give the figures for this column when checked at the density for the part. The last column shows the product of columns 4 and 5, *i.e.*, turns per inch times number of inches for the first four items. For the effective air gap, the ampere turns are figured from the formula:

$$0.315 \times \textit{density} \times \textit{effective gap}$$

For this motor the formula gives $0.315 \times 22{,}500 \times .102 = 730$. The total ampere turns required are the sum of those for all the parts, or 790, as shown. Immediately the observation is made that the air gap is responsible for over 90 per cent of the total. This impresses the fact that it is necessary to figure the air-gap factor very carefully, as errors in the gap calculation are much more serious than in the iron portion of the magnetic circuit.

The ratio of total ampere turns divided by ampere turns in the gap, or 790/730 = 1.08 in the present case, is called the *saturation factor*. This is the factor by which the leakage coefficient S is multiplied after it is calculated.

Calculation of the Leakage Coefficient S.—For the leakage calculation make a table like this:

Zigzag	$\frac{0.75 \times 1.78}{21 \times 17.1}$	= 0.0037
Stator slot	$\frac{10.2 \times 0.102 \times 2.155}{21 \times 12}$	= 0.0089
Rotor slot	$\frac{11 \times 0.102 \times 2.22}{17.1 \times 12}$	= 0.0122
Stator differential	$0.005 \times .16$	= 0.0008
Rotor differential	0.006×1.00	= 0.0060
Stator end connection	$\frac{0.40 \times 0.102 \times 18.5}{9.25 \times 12 \times .91}$	= 0.0075
Rotor end connection	$\frac{0.85 \times 0.102 \times 20.5}{9.25 \times 12 \times 1}$	= 0.0160
Total		= 0.0551

$$0.0551 \times 1.08 = 0.0595 - \textit{use } 0.06 = \textit{leakage coefficient } S$$

a. Zigzag.—To get the factors in the numerator for zigzag, first divide the stator-slot opening and the air gap (*i.e.*, the mechanical clearance) by the stator-slot pitch, thus: 0.35/0.57 = 0.615 and 0.0625/0.57 = 0.11. Now look at Fig. 397 and run along horizontally to 0.61. Since there is no curve marked 0.11, run up vertically, halfway between curve 0.133 and curve 0.08 and read the value on the right-hand margin. This is 0.75. For the second factor 1.78, look at Fig. 397 on the three-phase curve above 146°, because this is the electrical angle representing the stator coil throw.

$$180^\circ \frac{\textit{Slots throw}}{\textit{Slots pitch}} = 180^\circ \frac{17}{21} = 146^\circ$$

The value above 146° on the three-phase curve is 1.76. The two factors in the denominator of the zigzag are the number of stator and rotor slots per pole, 210 and 171, divided by 10.

b. Stator Slot.—10.2 is from Fig. 398 for 146°. 0.102 is the effective air gap as calculated from the air-gap factor. 2.155 is the stator-slot-leakage factor as calculated. 21 is stator slot per pole and 12 is the pole pitch as calculated.

c. Rotor Slot.—11 is from Fig. 398 for 180°, since the rotor coil is full pitch. 0.102 is again the effective air gap and 2.22 is the rotor-slot-leakage factor as calculated. 17.1 is the number of rotor slots per pole and 12 is the pole pitch in inches.

d. Stator Differential.—The factor 0.005 is from Fig. 399, for seven slots per pole per phase $\frac{210}{10 \times 3} = 7$. 0.16 is from Fig. 400 for three-phase and 146°.

e. Rotor Differential.—0.006 is from Fig. 399, for $\frac{171}{10 \times 3} =$ 5.7 slots per pole per phase. The factor 1.00 is from Fig. 400 for 180°. Rotor coil is full pitch.

f. Stator End Connection.—For the factor 0.40 see Fig. 401 and calculate the horizontal value $\frac{0.35 \times 21}{1.95} = 3.75$. From Fig. 401 the value on the curve vertically above 3.75 = 0.40. 0.102 is the effective air gap. 18.5 is the length in inches of one coil end connection calculated at 1½ times the throw of the coil plus 3½ inches. 9.25 is the effective length of the core plus 3 per cent for fanning out of the laminations. 12 is the pole pitch in inches and 0.91 is the square of the chord factor.

g. Rotor End Connection.—0.85 is from Fig. 401 where $\frac{0.18 \times 17.1}{1.75} = 1.7$ and the curve value above this is 0.85. 0.102 is the effective air gap. 20.5 is the length of one rotor-coil end connection, figured as in the stator except that, since the rotor coil is full pitch, the end connection is longer. In the denominator the last factor is 1, for the coil is full pitch.

The total of all the parts gives 0.0551 for the leakage factor. This, multiplied by the saturation factor, gives 0.0595. The value 0.06 is used as being more convenient and allowing a further small factor of safety in the right direction.

The Constant Losses.

a. Iron Loss.—For the iron loss make a table like this:

		Watts per cubic inch	Watts loss
Stator teeth	900 cu. in.	1.16	1044
Stator core	3850 cu. in.	0.48	1850
Rotor surface	820 sq. in.	(2.50 per sq. in.)	2050
Total	.....	.....	4944

The volume in cubic inches of the stator teeth is the cubic inches for one tooth times the total number of teeth. Thus: $0.25 \times 8.37 \times 1.95 \times 210 = 900$, *i.e.*, the width of tooth times the length of the tooth (width of core solid) times the height of the tooth times 210 teeth. For the core, take the radial depth of iron back of the stator slots times the solid core length times the diameter at the center of the core times 3.1416; or $3.2375 \times 8.37 \times (48\frac{1}{2} - 3.2375) \times 3.1416 = 3850$. The surface of the tops of all the rotor teeth equals the width of one tooth top (0.70 − 0.18) (tooth pitch minus slot opening) times 9.25 (effective core width) times 171 teeth = 820 square inches. The watts loss per cubic inch for teeth and core are from Fig. 402 and are 1.16 and 0.48, respectively, as shown in the third column. To find the watts loss per square inch in the rotor surface, figure first the surface of the stator tooth tops per pole. This is:

$$0.22 \times 9.25 \times 21 = 42.8 \text{ square inches.}$$

The maximum magnetic density at the stator tooth top is then:

$$\frac{2{,}340{,}000}{42.8 \times .636} = 86{,}000 \text{ lines per square inch}$$

Now find the following value

$$f = \frac{86{,}000 \times 0.35 \times 21}{0.0625 \times 100{,}000} = 100$$

where 0.35 is the stator-slot opening.

21 = the number of stator slots per pole.

0.0625 = the mechanical clearance between stator and rotor.

From Fig. 403, for 60 cycles find the value 2.5 directly above $f = 100$. This is the figure given in the third column. The fourth column or watts loss is the product of the factors in columns 2 and 3. The total is 4944. This is low because a wide-open slot stator creates pulsations in the magnetic wave which cause extra losses in the teeth of both stator and rotor. These losses are quite involved and difficult to calculate. Reference was made above to a short cut for getting the total iron loss by taking the tooth and core losses and multiplying them by the square of the air-gap factor. In the present case, this would be $(1044 + 1850) \times (1.63)^2 = 7700$ watts. This is usually high, so we may

be arbitrary and take the average of 4944 and 7700, which is 6322. Actually the iron loss on this motor tested 6200 watts. On a partly closed slot stator and rotor the calculation, as made above, will be nearly correct, but with an open slot stator an increase of 15 to 20 per cent will be more nearly correct.

b. Friction and Windage.—Since this would be classified as a "large motor," use Fig. 404 and the curve "No Blowers." The peripheral speed of this motor is

$$\frac{38 \times 3.1416 \times 720}{12} = 7200 \text{ feet per minute}$$

where 38/12 = rotor diameter in feet.
720 = the synchronous speed in r.p.m.

From Fig. 403, for 7200 feet peripheral speed the friction and windage are 1½ per cent of the output in watts (*i.e.*, 0.015) × 350 × 746 = 3917 watts. Actually the friction and windage on this motor tested 4000 watts.

Drawing the Diagram.—Turn again to Fig. 405. While this was used above as a general illustration it was constructed from the values actually calculated for the 350-hp. motor upon which we are now working. Starting with a blank sheet of Cornell cross-section paper, choose the line *BC*, equal to 150 small squares in length, as a convenient diameter for the circle. The point *B*, the point from which rotor currents are measured, and the point *A*, from which stator currents are measured, are now laid out. This is done by means of the leakage coefficient *S*, which was calculated and found to be 0.06. The equation is:

$$AB = \frac{BC \times 0.06}{1 - 0.06} = BC\frac{0.06}{0.94}$$

Since $BC = 150$ small squares, $AB = \frac{150 \times 0.06}{0.94} = 9.6$, so the line *AB* is made equal to 9.6 small squares and *A* is located. Now we know that the current circle passes through *B* and *C* but its center is not on *BC*. It is, however, on a line *RHGY* drawn perpendicular to *BC* at its midpoint. The center *H* is located by drawing another line *AF* upon which the center also lies.

This line is drawn at an angle with BC which is found by taking twice the magnetizing current times the stator resistance per phase and dividing by the voltage per phase; or $2 \times 23.9 \times 0.267 \div 1200 = 0.01$. This expression is really in trigonometry the sine of the angle which is sought but it can be handled by going 100 spaces from B toward C to D and there stepping up 1 space and drawing the line AF through the point so found. This line crosses RY at H and locates the center of the current circle. With H as a center, draw the half circle CP_sGP_1B. Upon this circle will lie the currents for all possible loads of the motor. With a center at A and a radius of 100 spaces, draw the quarter circle DO_2E which will determine the power factor for any load. For example, if the point P_2 is being considered, a line AP_2 is drawn and prolonged until it cuts the power-factor circle at O_2. Then the horizontal number of spaces from the line ABC to O_2 is the power factor. For O_2 this is close to 89, so the power factor for the load P_2 is 89 per cent.

The next step is to locate the torque circle CL_ML_2B. Anywhere at random below the line RY select some point P_1 as a trial construction point. With A as a center and a radius AP_1, strike an arc cutting the line AE at E_1; with B as a center and a radius BP_1, strike an arc cutting the line BK at K_1. Then the length AE_1 represents the stator current for the point P_1 and BK_1 represents the rotor current. To convert these into amperes we need the scales for the stator and rotor currents and for the watts. The stator-current scale comes immediately from the fact that we know AB equals 9.6 spaces and also equals 23.9 amperes, so the stator-current scale $C_1 = 23.9/9.6 = 2.5$ amperes per space. The rotor-current scale follows from the stator, thus:

$$C_2 = C_1 \frac{T_{p1} \times A_1 \times K_{p1} \times C_1}{T_{p2} \times A_2 \times K_{p2} \times C_2}\left(1 + \frac{S}{2}\right)$$

where T_{p1} and T_{p2} = turns per phase $\left(\frac{conductors}{2}\right)$.

A_1 and A_2 = number of stator and rotor phases.

K_{p1} and K_{p2} = distribution factors.

C_1 and C_2 = chord factors for stator and rotor, respectively.

Thus

$$2.5\frac{420 \times 2 \times 3 \times 0.955 \times 0.96}{2 \times 114 \times 3 \times 0.955 \times 1}(1.03) = 9.1 \text{ amperes per space.}$$

The watt scale C_w equals C_1 times volts per phase times number of phases $= 2.5 \times 1200 \times 3 = 9000$ watts per space.

Then, for the point P_1, the stator current AE_1, being 89.5 spaces is $2.5 \times 89.5 = 224$ amperes and the rotor current BK_1, being 83.5 spaces, is $9.1 \times 83.5 = 760$ amperes. From the point C draw a line through P_1 upon which will be located two points L_1 and M_1, through which the torque and output circles pass. Find the stator-copper loss for the point P_1 by squaring the current and multiplying it by the hot resistance per phase and the number of phases:

$$(224)^2 \times 0.267 \times 3 = 40{,}000 \text{ watts}$$

Lay off to the left horizontally from P_1 a distance a of 4.45 spaces. One space equals 9000 watts from the watt scale C_w, so 40,000 watts $= 40{,}000/9{,}000 = 4.45$ spaces. At 4.45 spaces to the left of P_1, draw a vertical line cutting CP_1 at the point L_1. This is a point on the torque circle. Join B and P_1 and erect a perpendicular at the center of the line BP_1. By geometry this intersects the line RY at Z, which becomes the center of the torque circle. With a center at Z and a radius ZC, draw a half circle through C, P_1 and B. This is the torque circle. To the left of the point L_1, lay off a horizontal distance equal to the rotor copper loss. This is the rotor current squared times the rotor resistance per phase times the three phases; or $(760)^2 \times 0.0225 \times 3 = 39{,}000$ watts. The horizontal distance b to the left of L_1 is $39{,}000/9{,}000 = 4.35$ spaces. At the left-hand end of b erect a vertical line cutting the line CP_1 at the point M_1, which then becomes a point on the output circle. Since the circle passes through C, M_1 and B, find the center as in the case of the torque circle. It is found to be the point Z. With this center and a radius CZ draw the circle CM_1B. This is the output circle.

The Constant-loss Line.—Draw the line ST parallel to ABC and distant by a space equal to the amount of iron loss plus friction and windage, divided by the watt scale; or $10{,}200/9{,}000 = 1.1$ spaces.

Using the Diagram.—All the lines are now drawn to show the complete performance for any current or load.

Full-load Point.—First locate the full-load point by finding the line or distance f between the vertical line ST and the output circle CL_MB, which is equal to the full-load watts output of the motor. Since the motor is rated 350 hp., the watts output is $350 \times 746 = 261{,}100$ watts. Dividing this by C_W (*i.e.*, 9,000 on the watt scale) gives $261{,}100/9{,}000 = 29$ spaces. Setting a divider at 29 spaces, the point M_2 is found to be the full-load point because it is on the output circle and is 29 spaces away from the line ST. From C draw the line CP_2 through the full-load point M_2, intersecting the torque circle at L_2 and the current circle at P_2. Then the full-load torque is the line e between the line ST and L_2. Then, since the full-load torque is 30 spaces, it can be used to measure the maximum or pull-out torque and the starting torque in terms of the full-load torque.

Maximum Torque.—The maximum torque is the greatest distance between the line ST and the torque circle CL_MB. This is obviously on the diameter and is the distance from the line ST to the point L_M. This is 60 spaces so the pull-out torque is $60/30 = 2$ times full-load torque.

Starting Torque.—The starting torque is measured at the standstill or locked point P_s. A line from C to P_s cuts the torque circle at the point L_s, 14 spaces horizontally from the line ST. So the starting torque is 14 spaces, or $^{14}\!/_{30} = 46$ per cent of full-load torque. This is what would be expected of a wound rotor motor if started with the collector rings on the rotor short-circuited.

Full-load Current, Power Factor and Efficiency.—The full-load current is the distance $AP_2 = AE_2 = 35\frac{1}{2}$ spaces. Multiplying 35.5 by 2.5 (the stator-current scale) gives 88.75 amperes, which is the full-load stator current. The rotor current at full load is the distance $BP_2 = BK_2 = 32$ spaces. Multiplying by 8.75 (the rotor-current scale) gives $32 \times 8.75 = 280$ amperes, which is the rotor current per collector ring at full load. The full-load power factor is found by drawing a line from A through P_2, extending it until it cuts the power-factor circle DKE at the point O_2. When AE has been drawn equal to 100 spaces, the point O_2 is 89 spaces from the line ABC, so the full-load power factor is 89 per cent.

The full-load efficiency could be measured by dividing the length of the line f by the length of the line g. f is the distance from the line ST to the point M_2 and g is the distance from the line ABC to the point P_2. As nearly as can be easily measured, f equals 29 spaces and g equals 30.6 spaces, so the full-load efficiency would be $29/30.6 = 94.5$ per cent. This looks too high, owing to difficulties in constructing and measuring exactly, so it is better to figure the losses. These are the iron loss, the friction and windage, the stator copper loss and the rotor copper loss. The iron loss as figured was 6000 watts; the friction and windage, 4000 watts; the stator full-load current, as above equals 88.75 amperes, so the stator copper losses equal

$$(88.75)^2 \times 0.267 \times 3 = 6100 \text{ watts}$$

where 0.267 = the stator resistance per phase hot.
3 = three phases.

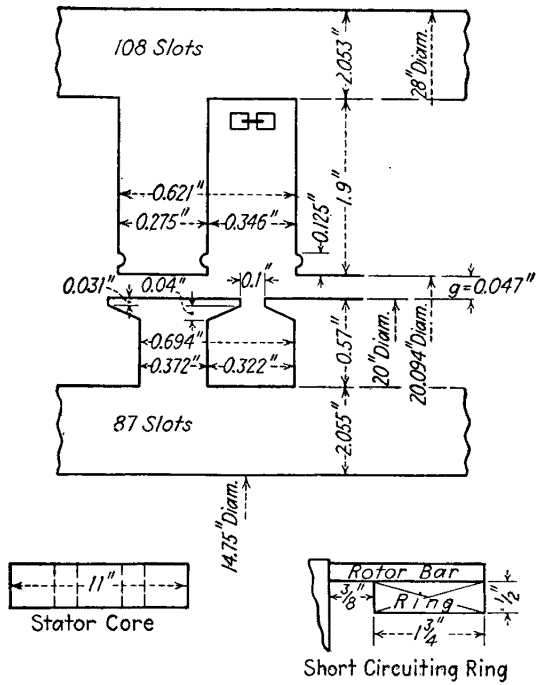
The rotor copper losses $= (280)^2 \times 0.0225 \times 3 = 5300$ watts. The total full-load losses are $6000 + 4000 + 6100 + 5300 = 21{,}400$ watts. The output in watts = 261,100. The input is $261{,}100 + 21{,}400 = 282{,}500$ watts and the full-load efficiency is $261{,}100/282{,}500 = 92.5$ per cent, which is a shade higher than the motor actually tested but near enough for all practical purposes.

Similarly other load points might be chosen at ½, ¾ or 1¼ load and the power factor and efficiency found. From a series of such points, the conventional complete curves at all loads of power factor and efficiency could be drawn.

Slip or Full-load R.P.M.—It is possible to draw a line in the diagram from which speeds can be read but this method is not so accurate as using the rotor copper loss. The rotor copper loss is a direct measure of the slip or drop from synchronous speed and the r.p.m. can be found by multiplying the synchronous speed $\left(\frac{cycles \times 120}{poles}\right)$ by a factor $\left(\frac{output\ in\ watts}{output + rotor\ copper\ loss}\right)$. In the case just checked, this equals $\frac{261{,}100}{261{,}100 + 5300} = 0.98$. The synchronous speed would be $\frac{120 \times 60}{10} = 720$ r.p.m., so the full-load speed is $720 \times 0.98 = 706$ r.p.m.

CALCULATION OF A SQUIRREL-CAGE MOTOR

The only thing not covered by the calculation of the wound rotor motor is the method of figuring the rotor resistance in the case of a squirrel-cage motor. For this reason a calculation will be given for a smaller squirrel-cage motor to illustrate the procedure in such a case.



Air Ducts - 2" x 3/8" Net Length 10.25"
Effective Length 10.5" Solid Length 9.55"

FIG. 407.—Core and slot dimensions of 200-hp. squirrel-cage motor.

It is assumed that by this time the reader is familiar with the details of how the formulae are built up, so that the arithmetic will be indicated and the results shown without explaining the procedure aside from the rotor resistance. Also the diagram will be shown but not explained in detail.

The motor is rated 200 hp., three-phase, 60 cycles, 2200 volt, 6 poles, 1160-r.p.m. The stator and rotor punchings and the core dimensions are shown in Fig. 407. The table for the slot insulation and windings is as follows:

Slot insulation		Windings	Stator	Rotor
Width of stator slot		Volts per leg	1270	
Copper	0.188	Connections	Series *Y*	
Insulation	0.157	Series conductors per leg	432	
Total	0.345	Conductors per coil	2 × 6 − 2 in parallel	
Depth of stator slot		Coils per slot	2	
Copper	1.512	Copper bare	0.081 × 0.114	5/16 × 1/2 bar
Insulation	0.257	Copper insulated	0.094 × 0.126	
Wedge	0.114	Area of conductor	11,100 C.M. − 0.0087 sq. in.	Short-circuiting ring made from rolled copper. 100 per cent conductivity. 1/2 × 1 3/4 in. Mean diameter of ring = 17.25 in. Mean length of rotor bar = 13.5 in.
Total	1.883	Resistance per 1000 feet	1.14	
		Weight per 1000 ft	34.6	
		Throw	1 to 14	
		Mean length of conductor	27.1	
		Feet total	5850	
		Weight total	203	
		Hot resistance per leg	0.56	
		Tested	0.55	

The calculations are as follows for the slot and gap factors, the field flux per pole and the magnetizing current.

	Distribution	Chord	Slot-leakage factor	Air-gap factor
Stator	0.955	0.906	2.07	1.55
Rotor	1.00	1.00	1.03	1.05

Total air-gap factor (1.55 × 1.05) equals 1.63.

	At air gap, in.	At center-stator core, in.	At center-rotor core
Pole pitch	10.5	13.6	8.75 in.
Flux per pole	$\frac{45{,}000{,}000 \times 1270}{60 \times 432 \times 0.955 \times 0.906}$ = 2,540,000 lines		

	Sq. in.	Density per sq. in.	Length of path	Ampere turns per inch	Column 3 × column 4; ampere turns for part
Stator teeth	47.2	84,500	1.9	9	18
Stator core	39.2	65,000	6.8	3.5	24
Rotor teeth	54	74,000	0.57	7	4
Rotor core	39.3	65,000	4.37	3.5	15
Air gap	110	36,800	0.0766	...	558
				Ampere turns total	= 619

Saturation factor equals $^{619}/_{558} = 1.11$. Ampere turns total equals 619. Magnetizing amperes per phase is $\frac{0.605 \times 619 \times 6}{216 \times 0.906}$ = 11.4 amperes. (The motor actually tested 11.3 magnetizing or no-load amperes.)

Leakage Calculation:

Stator slot	$\frac{10.8 \times 0.0766 \times 2.07}{18 \times 10.5}$	= 0.0091
Rotor slot	$\frac{10.8 \times 0.0766 \times 1.03}{14.5 \times 10.5}$	= 0.0057
Stator end	$\frac{0.46 \times 0.0766 \times 15}{10.5 \times 10.5 \times 0.83}$	= 0.0059
Rotor end	$\frac{0.5 \times 0.0766 \times 13.5}{10.5 \times 10.5 \times 1}$	= 0.0047
Zigzag	$\frac{0.74 \times 1.8}{18 \times 14.5}$	= 0.0051
Stator differential	0.006×0.24	= 0.0014
Rotor differential (zero for all squirrel-cage motors)		= 0.0000
Total		0.0319

Multiply by saturation factor: $1.11 \times .0319 = 0.0355 = S$ = leakage coefficient.

Iron-loss Calculation:

	Cu. in.	Density	Watts loss per cubic inch	Loss
Stator teeth	540	84,500	1.17	630
Stator core	1610	65,000	0.81	1300
Rotor surface	....	88,000	....	2160
Total	....	.....	....	4090

The rotor-surface loss calculation is as follows:

Total surface stator tooth tops per pole: $0.239 \times 10.5 \times 18 = 45.5$ square inches.

Maximum density in gap: $\frac{2{,}540{,}000}{45.5 \times 0.636} = 88{,}000$ lines per square inch.

$$f = \frac{88{,}000 \times 0.346 \times 18}{0.047 \times 100{,}000} = 116$$

From Fig. 403 the loss per square inch of rotor surface (tooth tops) = 3.8. Total surface rotor tooth tops is

$$0.622 \times 10.5 \times 87 = 570 \text{ square inches.}$$

Loss in rotor surface is $3.8 \times 570 = 2160$ watts.

Calculation of Friction and Windage.—Since the rotor diameter is 20, use the formula from the table.

$$F \text{ and } W = 3.985 \times S^2 \times 10^{-5}$$

where S = peripheral speed in feet per second due to rotation = $^{20}/_{12} \times 3.1416 \times 1160 = 6100$.
Hence

$$F \text{ and } W = \frac{3.985 \times (6100)^2}{100{,}000} = 1500 \text{ watts}$$

The fixed losses, *i.e.*, iron plus friction and windage, equal $4090 + 1500 = 5590$ watts. This illustrates the difficulty of calculating the fixed losses exactly, since from actual test the iron was 3500 and the friction and windage 1680 and their sum = 5193 watts. From this it will be seen that the iron in this case calculates too high and the friction and windage too low, but the sum is nearly correct and is on the safe side, a favorable comment on the method used.

The Stator and Rotor Resistance Calculation.—The stator resistance calculation is straightforward, consisting of multiplying the length of one conductor (27.1 inches) by the number of series conductors per phase (432) and multiplying this by the resistance of the conductor from a wire table. However, the rotor resistance is different. The resistance of one bar is figured. To it is added the resistance of a piece of the end ring of a length equal to the distance from center to center of adjacent bars. This value is multiplied by a factor, to reduce it to the same terms as the bar resistance, to which it is then added. The rotor is then treated as if it had the same number of phases as there are rotor bars. The copper loss is found by squaring the rotor current and multiplying it by the resistance per bar, as calculated, times the total number of bars. The calculation follows:

$$\textit{Bar resistance} = \frac{13.5 \times .00000069}{0.156} = 0.00006 \textit{ ohm}$$

where 13.5 = the bar length in inches from the center of one ring to the center of the other.

0.156 = the square inches of cross section of a bar ½ by 5/16 inch.

0.00000069 = the specific resistance of a cubic inch of copper at 25°C.

The outside diameter of the resistance (short-circuiting) rings is $20 - 2(0.071 + 0.50) = 18.86$ inches, where 0.71 is the tooth tip plus the slant height of the tip, and 0.50 inch is the height of the bar. The pitch from center to center of adjacent bars is $18.86 \times 3.1416 \div 87 = 0.680$ inch.

$$\textit{Ring resistance} = \frac{0.680 \times 0.00000069}{0.875} = 0.00000055$$

To reduce to the same terms as the bar, this must be multiplied by $0.2(14.5^2 + 1) = 42.2$, where 14.5 is the number of rotor slots per pole $= 87/6 = 14.5$.

Multiplying $0.00000055 \times 42.2 = 0.000023$.

This added to the bar is: $0.00006 + 0.000023 = 0.000083$ ohm at 25°C. This must be multiplied by 1.14 to give the value hot at 65°C. $0.000083 \times 1.14 = 0.000095$ ohm per bar hot; this is the value of rotor resistance that is used in drawing the circle diagram or in figuring the slip or full-load speed or the rotor copper losses.

This completes the data and the diagram for the motor is drawn in Fig. 408 similarly to that drawn for the wound rotor motor in Fig. 405.

Referring to Fig. 408, BC is again selected as 150 small squares of a sheet of Cornell paper. Since $S = 0.0355$, the number of squares in $AB = 150\frac{0.0355}{1 - 0.0355} = 5.5$ spaces. Stator current scale is 11.4 (magnetizing amperes)/5.5 = 2.07 amperes per space. Watt scale C_w is $2.07 \times 1270 \times 3 = 7887$ watts per space. Rotor current scale is

$$(2.07)\frac{432 \times 3 \times 0.955 \times 0.906}{14.5 \times 6}\left(1 + \frac{0.0355}{2}\right) =$$

25 amperes per space.

The small angle e, for the line AF on which the center of the current circle is located, is found by

$$\frac{2 \times 11.4 \times 0.56}{1270} = 0.01.$$

This means going out 100 spaces from A along the line ABC to D and then moving out 1 space to X. The line AF is then

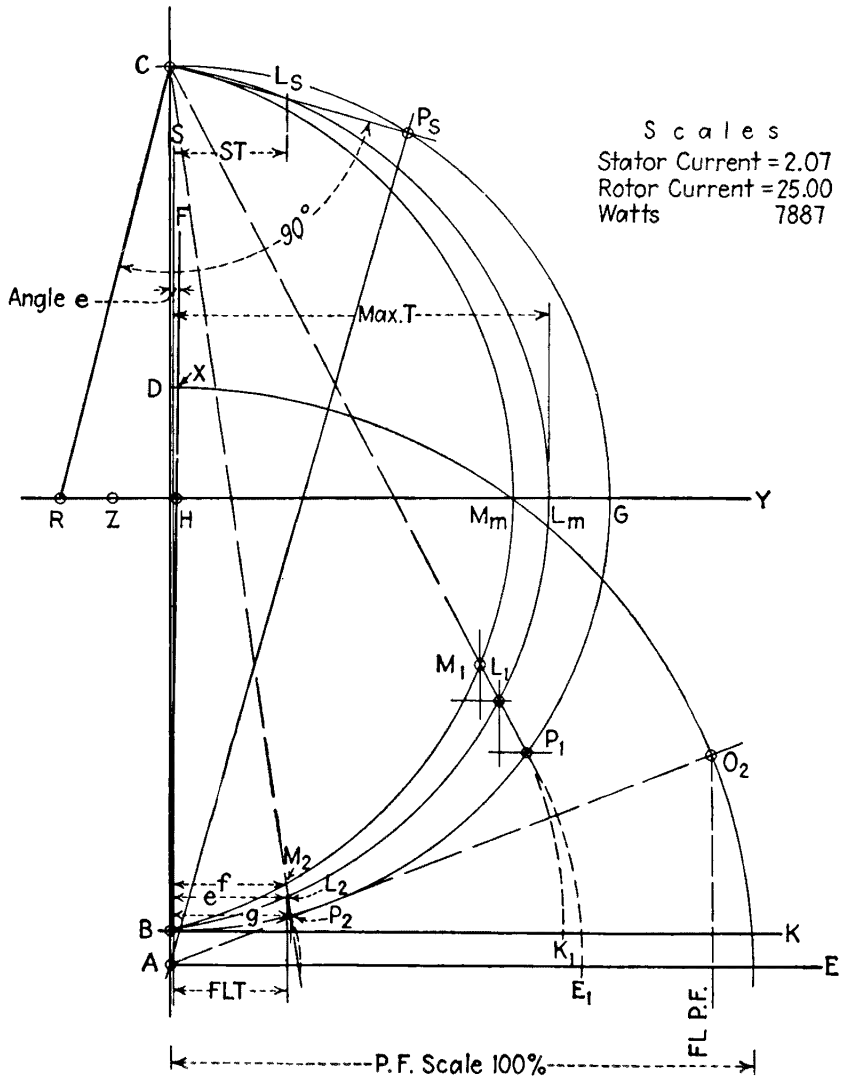


FIG. 408.—Circle diagram for 200-hp. squirrel-cage motor.

drawn through X and crosses the line RZY at H, which becomes the center of the current circle. When the current circle CP_sGB has been drawn, the line CP_1 is drawn at random and the points L_1 and M_1 on the torque and output circles are located by calculating the stator and rotor copper losses for the point P_1 as was done in the other case.

The locked or standstill point P_s is located by drawing at C a perpendicular to the radius CR of the output circle. This cuts the current circle at P_s and the torque circle at L_s, which gives the value of the locked current as AP_s and the value of the starting torque as the distance of L_s from the fixed loss line ST. This brings out the principal chance for error in the diagram performance as the starting torque L_s is only about full-load torque, whereas the motor actually tested 1.4 times full-load torque. The reason for this is that at standstill, when full frequency exists in the rotor, the deep rotor bars have eddy currents set up in them; this has the momentary effect of increasing the resistance 40 per cent and hence increasing the starting torque. However, after the motor is up to full speed and the rotor frequency is only perhaps 2 per cent of line frequency, the calculated value of the rotor resistance is correct and proper as shown in the diagram. There are ways of taking this into account but, since the error is on the safe side in actually testing more starting torque than is figured, it is close enough for all cases for which the method as illustrated is given.

Comparison of Calculated and Tested Values.

The normal full-load point is the point P_2 and the stator current is AP_2 times the scale C_1. As a matter of interest the actual tested values for the motor from brake test or the equivalent are compared with the diagram values.

	Calculated	Tested
No-load current	11.4	11.3
Locked current	310	322
Full-load current	45	45.3
Iron loss	4090	3500
Friction and windage	1500	1680
Full-load efficiency	93.2	93.1
Full-load power factor	93	93.1
Starting torque	1 × F.L.T.	1.4 × F.L.T.
Pull-out torque	3 33	3.53
Slip	1.54%	1.66%
Full-load r.p.m.	1182	1180

In closing this chapter, the author wishes to repeat the statement at its beginning that this chapter is intended solely to be helpful to those practical men and students who have become interested in the performance of induction motors and who wish to try their hand at making a complete calculation. It is a very fascinating problem and, while there seems to be a confusing amount of small arithmetical calculations, they are not difficult and the satisfaction following a successful venture along this line is great. The method is placed in the hands of men who have solved many difficult problems and they have the author's best wishes in their attempts to apply it.

It may help to consult the author's book on "Induction Motor Practice," Chapter X, which gives a detailed explanation of the construction of the circle diagram from test data as contrasted with the problem of constructing it from calculations.

GENERAL INDEX

A

Adams, C. A., 104
Air gap, effect on performance, 263, 264
  as increased by slot openings, 403
  factor, 406
Alternating currents, instantaneous values through cycle, 117
  representing single and polyphase, 10
Ampere turns, 407, 409
Annett, F. A., vii
Arrow check, 53, 54

B

Bache-Wiig, Jens, 104
Balance test, 250
Bar and end connector winding, 27, 71
Bearing friction and windage, 401, 403
  curve and formulae for, 419

C

Cabot, W. K., 104
Calculation, practical examples of, 429, 443
  350-hp. wound-rotor motor, 430–442
  200-hp. squirrel-cage motor, 443–450
Capacitor motors, 83
Changes in supply circuit all handled as voltage changes, 221
Changing volts, phase, poles, cycles, r.p.m., horsepower, 213
  examples of, 225, 228, 229, 231
Chord factor, 108, 137, 196, 198
  figured from coil throw, 131
  simple approximate formula for, 271
  table of values, 131
Chorded windings, 102
  advantages of, 104
  changing poles, 107
  effect of, 111, 200
    shown graphically, 115
    (*See also* Windings)

Circle diagram, 400
construction of, 423
current and power scales, 424
for 350-hp. wound-rotor motor, 423
torque and output circles, 426
for 200-hp. squirrel-cage motor, 448
use of, 427
Coils, concentric-coil winding, 41, 42
dead, 167
effect of throw, 102, 220
(*See also* Chorded Windings)
reversed, 242, 253
typical rotor, 70
voltage relation of, 40
volts per turn, 152
(*See also* Windings)
Compass test, 248
Conductors per phase, cross section of, 217, 276
formula for, 273
how calculated, 137
Connecting to correct side pull, 283
Connections, special, 201
Consequent-pole windings, 43, 48, 203
for two speeds, 203
Constant losses, 418
Core iron, cross section required, 217, 268
Core losses, 417
Counter e.m.f., 22, 196
Current, close-working approximation for, 277
full-load, formula for, 277
locked, 401
magnetizing, 404, 409
no-load, 401, 402

D

Defects in windings, order of locating, 260
ten most common, 238
(*See also* Faults)
Design, points considered in making, 265, 266
Diagrams (*see* special index, 461)
check for star point, 48
delta, 54, 346
how to lay out for any case, 50, 344, 345
master, for polar grouping, 33
open-slot, 45, 46
schematic, 48
series and parallel, 49

Diagrams, showing current bands in winding, 134
showing effect of chording graphically, 123
single-phase, 92 *ff.*
standard group, 301
three-phase star check, 53
top to bottom, 173
wave or progressive, 35, 36, 37, 374
formula for, 37, 375
how drawn, 375
modified, 78
Diamond-coil winding, 33
Distribution, single-phase winding, 88
Distribution factor, 132, 138
arithmetic formula for, 139
consequent-pole, 140
fully distributed single-phase winding, 142
geometric method of finding, 127
graphic solution for, 138
practical application of, 147
three-phase, 141, 208
two-phase, 140, 208
when less important, 111

E

Efficiency, 263, 264
Equalizer connections, 295
on direct-current armature, 298

F

Faults, connected for wrong voltage, 244, 256
grounds, 245
locating, 233, 259
open circuits, 245, 255, 258
reversed-coil, group, phase, 242, 253
short circuits, 241, 247
ten most common, 238
wrong grouping, 242, 253
wrong number of poles, 245, 255
(*See also* Defects)
Ferraris, Galileo, 6
Field (*see* Rotating magnetic field)
Figuring the complete performance, 399–450
Figuring a new winding, 262
general considerations, 265, 266
Flux per pole, 270, 402

Frequency of an alternating current, 8
  changes in, 228
  check speed on higher frequency, 183
  how it affects r.p.m., 183
  how it affects windings, 183
  relation to voltage, 185

G

Grounds in winding, 240, 245
Group reversed, 253, 254

H

Hand-wound coils, 30
Hellmund, R. E., 187, 399
Horsepower, change in, 231
  formula for, 214
  relation to torque and r.p.m., 185, 214

I

Induction motor, how it drives its load, 5
"Induction Motor Practice," 400
Insulation, checking for voltage, 148
  resistance, 151
  space required in slot, 278
  standard tests, 149
    (*See also* Phase Insulation)
Iron loss, 401, 403, 415
  curve for stator teeth and core, 416
  in rotor surface, 416
Irving, G. A., 104

K

Kincaid, C. W., v

L

Lamme, B. G., xv
Lap windings, 44
  with unequal coil groupings, 347
Leakage coefficient, 403, 410
  dispersion factor, 403
  sigma, 403
Leakage, differential or belt, 413
  end, 414
  kinds of, 401

Leakage, magnetic field, 401
slot, 410, 412
zigzag, 410, 411
Least-common-multiple grouping scheme, 347
Locating faults, 233, 259

M

Magnetic actions, representing alternating currents, 8
Magnetic equilibrium, 261, 284, 285
Magnetic field (*see* Rotating magnetic field)
Magnetic flux, formula for, per pole, 270
Magnetic noise, how distinguished from windage, 234
Magnetic side pull on rotor, 283
factors normally corrective, 300
helped by parallel connection, 289
not corrected by series connection, 288
not helped by "top-to-bottom" parallel connection, 294
Magnetization curves, 408
Maximum torque, 263, 264
Mechanical troubles caused by windings, 234–236
Middle leg or B phase, why reversed, 12
Miller, G. E., 106
Motors, single-phase, 80–101
capacitor, 83
general classification, 80
repulsion start, induction run, 81
shaded-pole, 83
split-phase, 80
universal, 84
Motors, matching rotation, 182

N

N.E.M.A. terminal markings, 82, 83, 85
Noise and vibration, 233, 234

O

Open circuits in windings, 245
Output coefficient, 267

P

Performance, calculation of, 399–450
how affected by unbalanced voltages, 157
how affected by winding, 263, 264
Peripheral speed of rotor, 198

Peripheral speed of rotor, safe value for, 184
Phase, changes in, 165, 225
and voltage, 176
how it affects windings, 165
reversed, 253, 256
rotation, 182
and voltage, master table, 177
Phase insulation, 45, 113, 114, 200
when important, 112
Pole pitch, 406
Poles, changing number, affects chord factor, 107
effect of, 197, 229
with same throw, 107
check points, 198
connected for wrong number of, 245, 255
how number affects cross section of iron below slots, 217
number of, and r.p.m., 191–200
Power factor, 263, 264
"Pull out" (*see* Maximum torque)
Pushed-through windings, 26, 28

R

Reconnecting an old winding, 213
limiting conditions, 215, 216
Repulsion motor, single-phase winding for, 91
Repulsion-start motor, 81
Reversal of part of winding, 242, 253, 254
Reversing rotation by reversing leads, 19, 21
Revolutions per minute, relation to cycles, 15
relation to horsepower and torque, 185, 214
relation to poles, 184, 191
Rotating magnetic field, building by stair-step pictures, 20, 22, 102-131
effect of chording on, 102
getting to, from current and winding, 119
graphical pictures of, 17, 115
(*See also* Rotating magnetic field, instantaneous picture of)
how it rotates, 18
how reversed, 19, 21, 23
how set up, by alternating current, 7
by three-phase currents, 16
by two-phase currents, 14
how single phase works, 24, 25
instantaneous picture of, 18, 19, 118, 121–123, 126, 128, 130
sine-wave shape, 11
Rotor windings, 55–79
connections, 72
why three phase, 23, 374
(*See also* Windings)

S

Schematic equivalent diagram, 48
Scott connection of winding, 168
Secondary voltage, 181
  how to figure, 278
Shaded-pole motors, 83
Short circuits in windings, 241, 247, 248
Single-phase motors (*see* Motors, single-phase)
Skein winding, for single-phase motors, 88
Slip, definition of, 191
Slots, common forms of, 22, 68
  effect of form of, 26
  not multiple of phase times poles, 201
Speed, changes in, 229
  frequency, necessity for checking, 183
"Split" groups, 208, 209
Split-phase motors, 80
Squirrel-cage rotor winding, calculating resistance of, 59
  current paths in, 58
  effect of changing number of poles, 59
  mechanical construction of, 61
  resistance in terms of stator, 60
Star connection, changing to delta, 346
Starting from generator at rest, 190
Starting squirrel-cage motor, 190
Starting current and starting torque, 263, 264
Stubbing and connecting, 343

T

"Tee" connection, 169, 170, 173
Tesla, Nikola, 6
Tests, balance and compass, 248–250, 252
  volts and watts, 236, 237
Tingley, E. M., 202, 347
Torque, how produced, 5
  relation to horsepower and r.p.m., 185, 214
Two-speed windings, 203
  three-phase diagram, 204, 205
  two-phase diagram, 207

U

Unbalanced voltages, effect on performance, 157
Unequal coil groupings, tables for, 354
Universal motor, 84
Unsymmetrical connections, 201

V

Vibration, mechanical, 235
Voltage, all kinds of changes considered as voltage changes, 221
between collector rings, how to figure, 279
equivalent between star and delta, 155
general tables comparing, 157
master table of voltage and phase changes with reconnection, 177
positive and negative sequence, 160
relation to frequency, 185
resolving unbalanced system into two balanced systems, 160
symmetrical components, 160
zero sequence, 164

W

Wasmund, James A., 163
Wave diagrams, 374
Weber, C. A. M., v
Werner, G. B., 187
Windage noise, 233
Windings, bar and end connector, 27, 71
best parallel arrangement, 295
changing throw of coils, 220
chorded, 102
concentrated, 132
concentric-coil, 41, 42
rearrangement of, 43
connected for different phase, 133
connecting for more than one voltage, 148
consequent-pole, 43, 48, 203
"diamond" coil, open-slot, 33, 34, 44
distributed, 132
effect on performance, 263
effect of voltage on, 148
equalizer connections, 295
fed in or dropped in, 27, 29
figuring new, 262, 265, 266
function of, on direct-current motor, 1, 2
on induction motor, 1, 3, 4
on synchronous motor, 3
generator, action of, 219
(*See also* Counter e.m.f.)
grounded, 240
hand-wound, 27, 29, 30
how affected, by frequency, 183
by number of phases, 165
lap, 44

Windings, lap, with unequal coil groupings, 347–373
least common scheme of, 347, 348
two- and three-phase check, 351, 352
will it parallel? 350
open-slot, 33
partly closed slot, 26
pushed-through, 26, 28
reconnecting for new conditions, 213
reversal of part, 242, 253
rotor, 32, 55, 67, 55–79
squirrel-cage, 57
Scott connection of, 169
short-circuited, 241
single-phase, 80–101
connections of main and starting, 92
distinction between chord and distribution factor, 146
distribution of, 88
forms of, 85
mould-winding method, 93
skein, 88
starting, 94
why concentric wound, 144
tables for unequal coil groupings, 354
"tee" connected, 168, 169
types of, 26
unequal coil groupings, 347
volts per turn in, 152
wave windings, 37, 44, 72
formula for, 37, 375
modified wave, 77
true wave, 74
how to lay out quickly, 74

INDEX OF TABLES

PAGE

Table I.—Chord Factors for Various Electrical Angles 125
Table II.—Table of Chord Factors from 45 to 90 Deg. 131
Table III.—How the Chord Factor Is Figured from the Throw of the Coil 131
Table IV.—Comparison of Motor Voltages with Various Three-phase Connections 156
Table V.—Comparison of Motor Voltages with Various Two-phase Connections 156
Table VI.—Comparison of a Two-phase Motor Connected "T" with a Normal Three-phase Winding 171
Table VII.—Comparison of Motor Voltages with Various Connections and Phases 177
Table VIII.—Characteristics of a Three-phase Motor. 195
Table IX.—Effects of Changing the Number of Poles in an Induction Motor Winding 197
Table X.—Factors, Due to Change in Number of Poles, Modifying Induction Motor Voltage 197
Table XI.—Comparison of a Two-phase Motor Connected "T" with a Normal Three-phase Winding 211
Table XII.—Output Coëfficient Values 267
Table XIII.—Unequal Coil Grouping: Two-phase—2 Poles 354
Table XIV.—Unequal Coil Grouping: Two-phase—4 Poles 355
Table XV.—Unequal Coil Grouping: Two-phase—6 Poles 356
Table XVI.—Unequal Coil Grouping: Two-phase—8 Poles 357
Table XVII.—Unequal Coil Grouping: Two-phase—10 Poles 359
Table XVIII.—Unequal Coil Grouping: Two-phase—12 Poles 360
Table XIX.—Unequal Coil Grouping. Two-phase—14 Poles. 362
Table XX.—Unequal Coil Grouping: Three-phase—2 Poles 363
Table XXI.—Unequal Coil Grouping: Three-phase—4 Poles. 363
Table XXII.—Unequal Coil Grouping: Three-phase—6 Poles 364
Table XXIII.—Unequal Coil Grouping: Three-phase—8 Poles. . . . 366
Table XXIV.—Unequal Coil Grouping: Three-phase—10 Poles . . . 368
Table XXV.—Unequal Coil Grouping: Three-phase—12 Poles 370
Table XXVI.—Unequal Coil Grouping: Three-phase—14 Poles . . . 372

INDEX OF DIAGRAMS

Single Phase Diagrams

Induction motor, running winding only, four pole, series and parallel, Fig. 101, p. 91.
Induction motor, main and starting winding, four pole, series, Fig. 103, p. 92.
Induction motor, main and starting winding, four pole, two parallel, Fig. 104, p. 93.
Induction motor, main and starting winding, two pole, series, Fig. 109, p. 96.
Induction motor, main and starting winding, six pole, series, Fig. 112, p. 98.
Induction motor, main and starting winding, six pole, two parallel, Fig. 113, p. 99.
Repulsion motor, stator winding, two pole, series or parallel, Fig. 114, p. 100.
Repulsion motor, stator winding, six pole, series or parallel, Fig. 115, p. 101.

Standard Polyphase Group Diagrams

Two-pole, two-phase, series, Fig. 269, p. 302.
Two-pole, two-phase, parallel, Fig. 270, p. 302.
Two-pole, three-phase, series star, Fig. 271, p. 303.
Two-pole, three-phase, parallel star, Fig. 272, p. 303.
Two-pole, three-phase, series delta, Fig. 273, p. 304.
Two-pole, three-phase, parallel delta, Fig. 274, p. 304.
Four-pole, two-phase, series, Fig. 275, p. 305.
Four-pole, two-phase, two parallel, Fig. 276, p. 305.
Four-pole, two-phase, four parallel, Fig. 277, p. 306.
Four-pole, three-phase, series star, Fig. 278, p. 307.
Four-pole, three-phase, two parallel star, Fig. 279, p. 307.
Four-pole, three-phase, four parallel star, Fig. 280, p. 308.
Four-pole, three-phase, series delta, Fig. 281, p. 308.
Four-pole, three-phase, two parallel delta, Fig. 282, p. 309.
Four-pole, three-phase, four parallel delta, Fig. 283, p. 309.
Six-pole, two-phase, series, Fig. 284, p. 310.
Six-pole, two-phase, two parallel, Fig. 285, p. 310, top to bottom.
Six-pole, two-phase, two parallel, Fig. 362, p. 354, top to top.
Six-pole, two-phase, three parallel, Fig. 286, p. 311.
Six-pole, two-phase, six parallel, Fig. 287, p. 311.
Six-pole, three-phase, series star, Fig. 288, p. 312.
Six-pole, three-phase, two parallel star, Fig. 289, p. 312, top to bottom.
Six-pole, three-phase, two parallel star, Fig. 367, p. 365, top to top.
Six-pole, three-phase, three parallel star, Fig. 290, p. 313.
Six-pole, three-phase, six parallel star, Fig. 291, p. 313.
Six-pole, three-phase, series delta, Fig. 292, p. 314.

Six-pole, three-phase, two parallel delta, Fig. 293, p. 314, top to bottom.
Six-pole, three-phase, two parallel delta, Fig. 368, p. 365, top to top.
Six-pole, three-phase, three parallel delta, Fig. 294, p. 315.
Six-pole, three-phase, six parallel delta, Fig. 295, p. 315.
Eight-pole, two-phase, series, Fig. 296, p. 316.
Eight-pole, two-phase, two parallel, Fig. 297, p. 316, top to bottom.
Eight-pole, two-phase, two parallel, Fig. 363, p. 358, top to top.
Eight-pole, two-phase, four parallel, Fig. 298, p. 317.
Eight-pole, two-phase, eight parallel, Fig. 299, p. 317.
Eight-pole, three-phase, series star, Fig. 300, p. 318.
Eight-pole, three-phase, two parallel star, Fig. 301, p. 318, top to bottom.
Eight-pole, three-phase, two parallel star, Fig. 369, p. 367, top to top.
Eight-pole, three-phase, four parallel star, Fig. 302, p. 319.
Eight-pole, three-phase, eight parallel star, Fig. 303, p. 319.
Eight-pole, three-phase, series delta, Fig. 304, p. 320.
Eight-pole, three-phase, two parallel delta, Fig. 305, p. 320, top to bottom.
Eight-pole, three-phase, two parallel delta, Fig. 370, p. 367, top to top.
Eight-pole, three-phase, four parallel delta, Fig. 306, p. 321.
Eight-pole, three-phase, eight parallel delta, Fig. 307, p. 321.
Ten-pole, two-phase, series, Fig. 308, p. 322.
Ten-pole, two-phase, two parallel, Fig. 309, p. 322, top to bottom.
Ten-pole, two-phase, two parallel, Fig. 364, p. 358, top to top.
Ten-pole, two-phase, five parallel, Fig. 310, p. 323.
Ten-pole, two-phase, ten parallel, Fig. 311, p. 323.
Ten-pole, three-phase, series star, Fig. 312, p. 324.
Ten-pole, three-phase, two parallel star, Fig. 313, p. 324, top to bottom.
Ten-pole, three-phase, two parallel star, Fig. 371, p. 369, top to top.
Ten-pole, three-phase, five parallel star, Fig. 314, p. 325.
Ten-pole, three-phase, ten parallel star, Fig. 315, p. 325.
Ten-pole, three-phase, series delta, Fig. 316, p. 326.
Ten-pole, three-phase, two parallel delta, Fig. 317, p. 326, top to bottom.
Ten-pole, three-phase, two parallel delta, Fig. 372, p. 369, top to top.
Ten-pole, three-phase, five parallel delta, Fig. 318, p. 327.
Ten-pole, three-phase, ten parallel delta, Fig. 319, p. 327.
Twelve-pole, two-phase, series, Fig. 320, p. 328.
Twelve-pole, two-phase, two parallel, Fig. 321, p. 328, top to bottom.
Twelve-pole, two-phase, two parallel, Fig. 365, p. 361, top to top.
Twelve-pole, two-phase, three parallel, Fig. 322, p. 329.
Twelve-pole, two-phase, four parallel, Fig. 323, p. 329.
Twelve-pole, two-phase, six parallel, Fig. 324, p. 330.
Twelve-pole, two-phase, twelve parallel, Fig. 325, p. 330.
Twelve-pole, three-phase, series star, Fig. 326, p. 331.
Twelve-pole, three-phase, two parallel star, Fig. 327, p. 331, top to bottom.
Twelve-pole, three-phase, two parallel star, Fig. 373, p. 371, top to top.
Twelve-pole, three-phase, three parallel star, Fig. 328, p. 332.
Twelve-pole, three-phase, four parallel star, Fig. 329, p. 332.
Twelve-pole, three-phase, six parallel star, Fig. 330, p 333.
Twelve-pole, three-phase, twelve parallel star, Fig. 331, p. 333.

Twelve-pole, three-phase, series delta, Fig. 332, p. 334.
Twelve-pole, three-phase, two parallel delta, Fig. 333, p. 334, top to bottom.
Twelve-pole, three-phase, two parallel delta, Fig. 374, p. 371, top to top.
Twelve-pole, three-phase, three parallel delta, Fig. 334, p. 335.
Twelve-pole, three-phase, four parallel delta, Fig. 335, p. 335.
Twelve-pole, three-phase, six parallel delta, Fig. 336, p. 336.
Twelve-pole, three-phase, twelve parallel delta, Fig. 337, p. 336.
Fourteen-pole, two-phase, series, Fig. 338, p. 337.
Fourteen-pole, two-phase, two parallel, Fig. 339, p. 337, top to bottom.
Fourteen-pole, two-phase, two parallel, Fig. 366, p. 361, top to top.
Fourteen-pole, two-phase, seven parallel, Fig. 340, p. 338.
Fourteen-pole, two-phase, fourteen parallel, Fig. 341, p. 338.
Fourteen-pole, three-phase, series star, Fig. 342, p. 339.
Fourteen-pole, three-phase, two parallel star, Fig. 343, p. 339, top to bottom.
Fourteen-pole, three-phase, two parallel star, Fig. 375, p. 373, top to top.
Fourteen-pole, three-phase, seven parallel star, Fig. 344, p. 340.
Fourteen-pole, three-phase, fourteen parallel star, Fig. 345, p. 340.
Fourteen-pole, three-phase, series delta, Fig. 346, p. 341.
Fourteen-pole, three-phase, two parallel delta, Fig. 347, p. 341, top to bottom.
Fourteen-pole, three-phase, two parallel delta, Fig. 376, p. 373, top to top.
Fourteen-pole, three-phase, seven parallel delta, Fig. 348, p. 342.
Fourteen-pole, three-phase, fourteen parallel delta, Fig. 349, p. 342.

Typical Wave Diagrams

Four-pole, three-phase, series delta, 84 slots, Fig. 377, pp. 376, 377.
Six-pole, three-phase, series delta, 108 slots, Fig. 378, pp. 378, 379.
Eight-pole, three-phase, series delta, 144 slots, Fig. 379, pp. 380, 381.
Ten-pole, three-phase, series star, 180 slots, Fig. 380, pp. 382, 383.
Twelve-pole, three-phase, series star, 144 slots, Fig. 381, pp. 384, 385.
Fourteen-pole, three-phase, series star, 189 slots, Fig. 382, pp. 386, 387.
Sixteen-pole, three-phase, series star, 216 slots, Fig. 383, pp. 388, 389.
Eighteen-pole, three-phase, series star, 108 slots, Fig. 384, pp. 390, 391.
Twenty-pole, three-phase, series star, 180 slots, Fig. 385, pp. 392, 393.
Twenty-four-pole, three-phase, series star, 360 slots, Fig. 386, pp. 394, 395.
Sixteen-pole, three-phase, special four parallel star connection for 240 slots, Fig. 387, pp. 396, 397.

Miscellaneous Wave Diagrams

Four-pole, three-phase, series delta, 84 slots, Fig. 39, p. 36.
Eight-pole, three-phase, series star, 96 slots, Fig. 40, p. 36.
Six-pole, three-phase, series star, 62 slots, Fig. 41, p. 37.
Eight-pole, three-phase, series delta, 144 slots, Fig. 89, p. 78.

Miscellaneous Group Diagrams

Two-pole, two-phase, series, Fig. 63, p. 50
Two-pole, two-phase, two parallel, Fig. 64, p. 50.
Two-pole, three-phase, series star, Fig. 65, p. 51.

Two-pole, three-phase, two parallel star, Fig. 66, p. 51.
Two-pole, three-phase, series delta, Fig. 67, p. 51.
Two-pole, three-phase, two parallel delta, Fig. 68, p. 51.
Four-pole, three-phase, series star, Figs. 54, 55, 59, pp. 41, 44, and Figs. 54a, 54b, and 58, pp. 45, 48.
Four pole, three-phase, two parallel star, Figs. 59, 244, pp. 49, 257.
Four-pole, three-phase, four parallel star, Fig. 60, p. 49.
Six-pole, two-phase, series, Figs. 174, 177, pp. 167, 171.
Six-pole, three-phase, three parallel star, Fig. 61, p. 50.
Six-pole, three-phase, six parallel star, Fig. 62, p. 50.
Eight-pole, three-phase, two parallel star, Fig. 196, p. 202.
Eight-pole, three-phase, four parallel star, Fig. 217, p. 229.
Ten-pole, three-phase, two parallel delta, Fig. 218, p. 230.

Developed Winding Diagrams—Showing All Coils

Four-pole, two-phase, series, Figs. 116, 117, 125, pp. 103, 105, 113.
Four-pole, two-phase, series, Fig. 159, p. 153.
Four-pole, two-phase, two parallel, Figs. 160, 182, 211, pp. 153, 179, 222.
Four-pole, two-phase, two parallel, Fig. 216, p. 227.
Four-pole, two-phase, four parallel, Fig. 161, p. 153.
Four-pole, three-phase, series star, Figs. 57, 124, 183, 190, 212, pp. 46, 113, 179, 192, 222.
Four-pole, three-phase, two parallel star, Fig. 244, p. 257.
Four-pole, three-phase, series delta, Fig. 215, p. 227.
Six-pole, three-phase, series star, Figs. 137, 138, 162, 189, 213, pp. 124, 154, 192, 226.
Six-pole, three-phase, two parallel star, Figs. 187, 194, 214, pp. 188, 199, 226.
Six-pole, three-phase, series delta, Fig. 188, p. 188.
Six-pole, three-phase, three parallel delta, Fig. 163, p. 154.
Eight-pole, two-phase, series, Fig. 126, p. 114.
Eight-pole, two-phase, four parallel, Fig. 185, p. 181.
Eight-pole, three-phase, series star, Figs. 127, 184, 191, pp. 114, 181, 193.
Ten-pole, three-phase, series star, Fig. 192, p. 193.
Twelve-pole, three-phase, series star, Fig. 193, p. 194.

Special Diagrams

Two-speed, three-phase, four and eight poles, parallel and series star, for constant torque, Fig. 197, p. 204.
Two-speed, three-phase, four and eight poles, parallel star and series delta, for constant horsepower, Fig. 198, p. 205.
Two-speed, two-phase, four and eight poles, parallel and series, same distribution factor on both speeds, Fig. 200, p. 207.
"Split group," three-phase, six poles, in four parallels, Fig. 201, p. 208.
"Consequent pole," three-phase, twelve-pole, series star, Fig. 195, p. 199.
"Tee," three-phase, six poles, "top to bottom," Fig. 178, p. 172.
"Tee," three-phase, six poles, "top to top," Fig. 181, p. 174.
"Dead coil," three-phase, six pole, series, Fig. 175, p. 167.